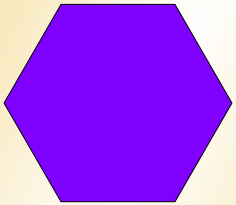
Dokazi bez riječi

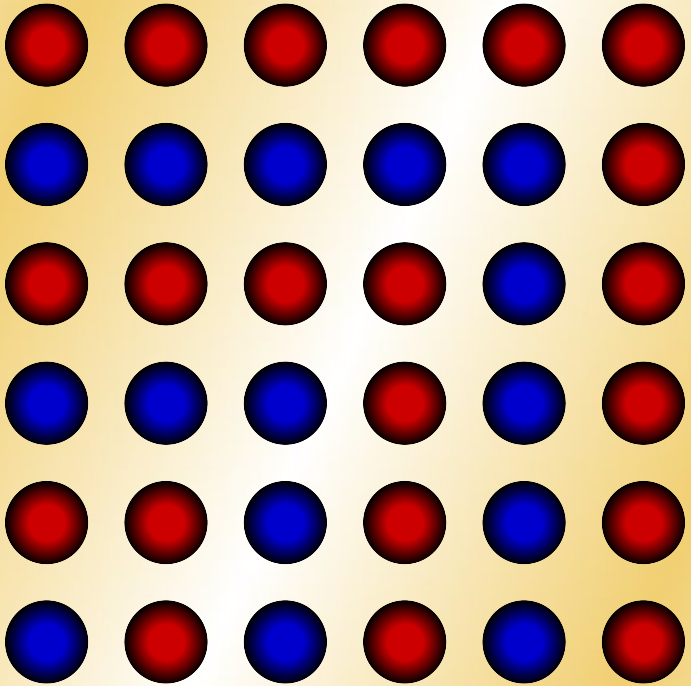


doc.dr.sc. Julije Jakšetić

julije@math.hr

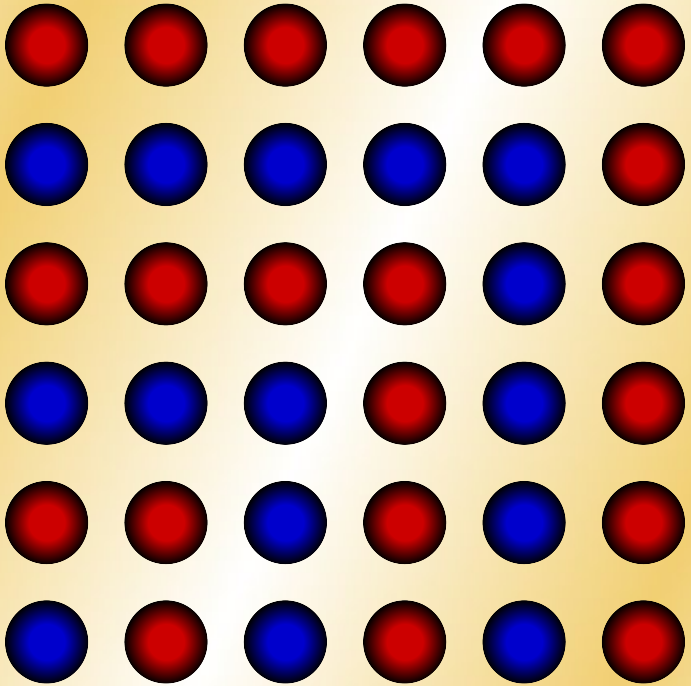
Sume

Suma prvih n neparnih brojeva



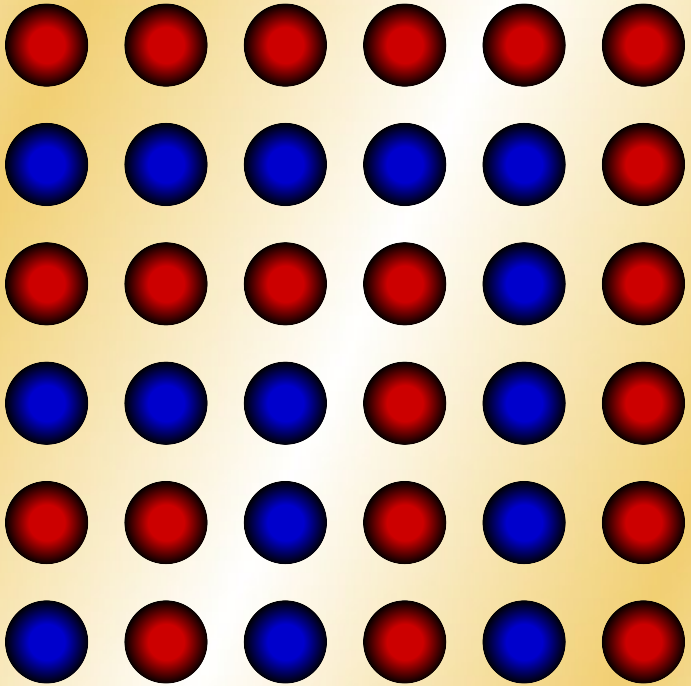
$$1 + 3 + \cdots + (2n - 1) = n^2$$

Suma prvih n neparnih brojeva



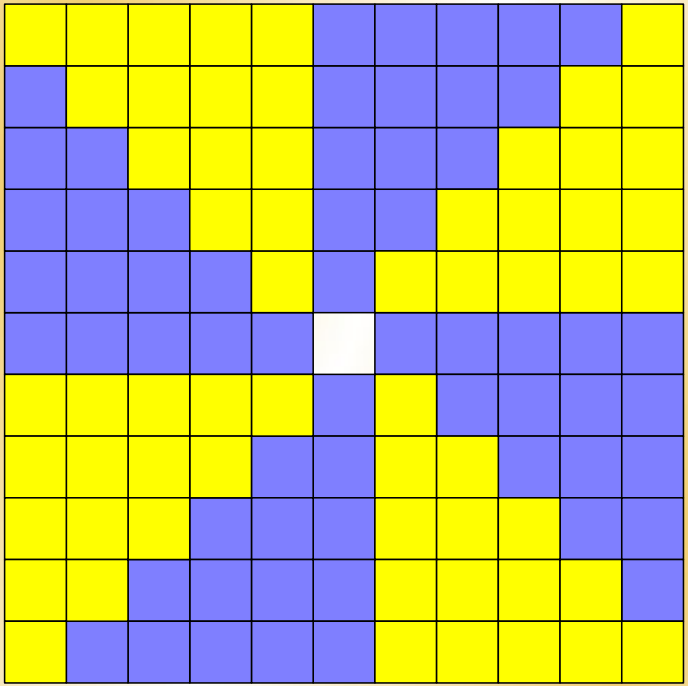
$$1 + 3 = 2^2$$

Suma prvih n neparnih brojeva



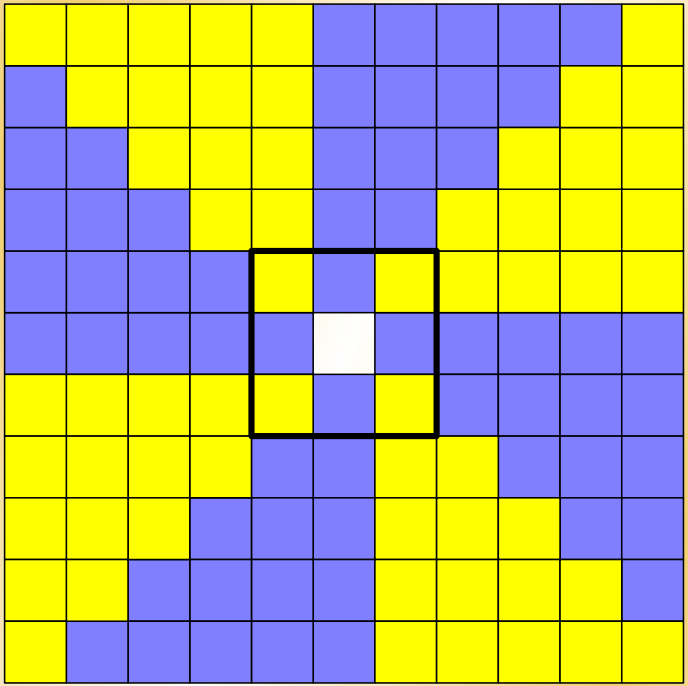
$$1 + 3 + 5 = 3^2$$

Identitet za sumu prvih n brojeva



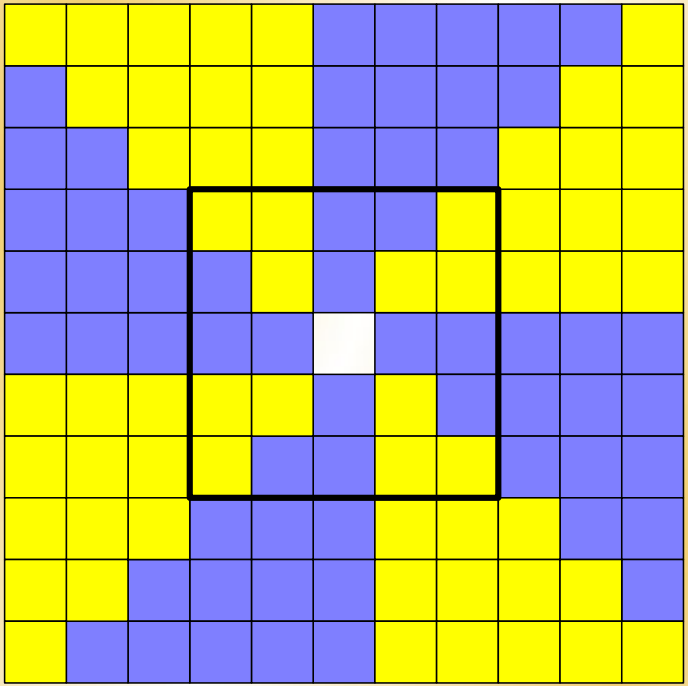
$$S_n = 1 + 2 + \cdots + n$$
$$8S_n + 1 = (2n + 1)^2$$

Identitet za sumu prvih n brojeva



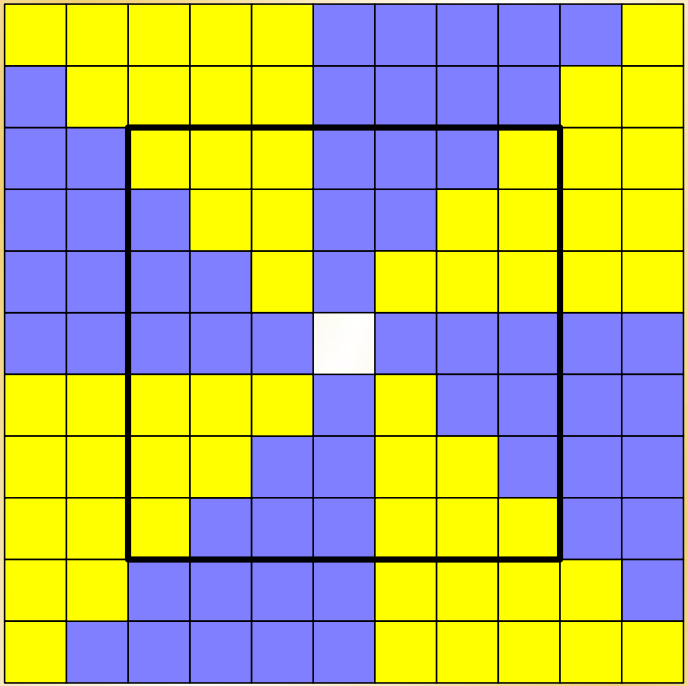
$$8 \cdot 1 + 1 = 3^2$$

Identitet za sumu prvih n brojeva



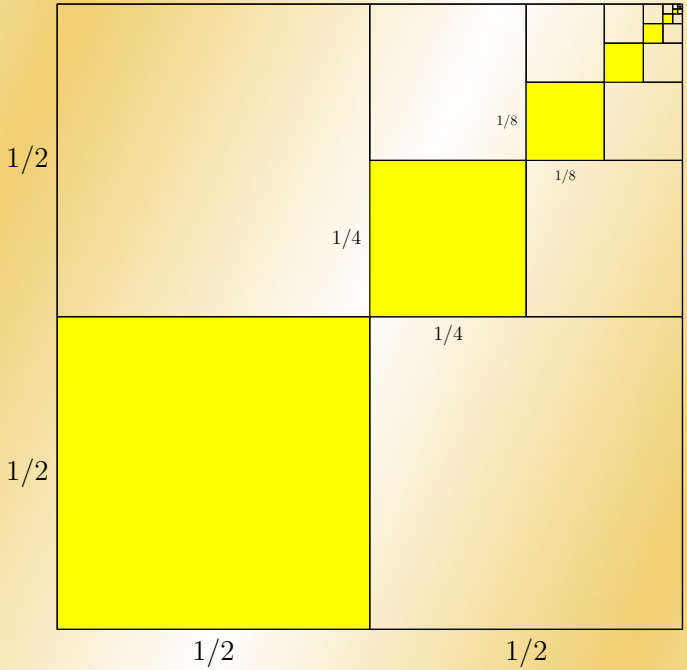
$$8 \cdot (1 + 2) + 1 = 5^2$$

Identitet za sumu prvih n brojeva



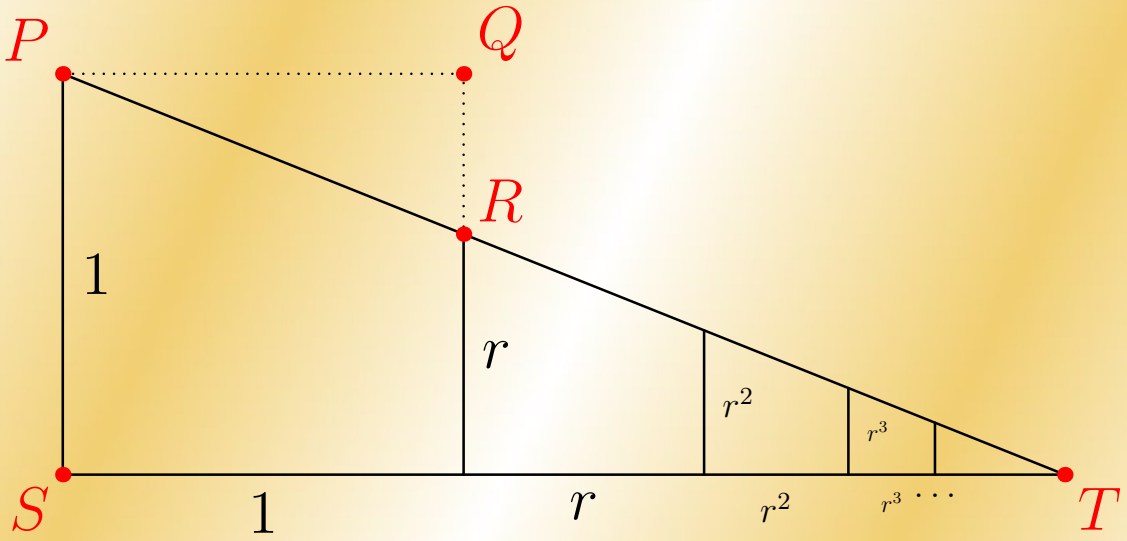
$$8 \cdot (1 + 2 + 3) + 1 = 7^2$$

Suma geometrijskog reda I



$$\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+\cdots=\frac{1}{3}$$

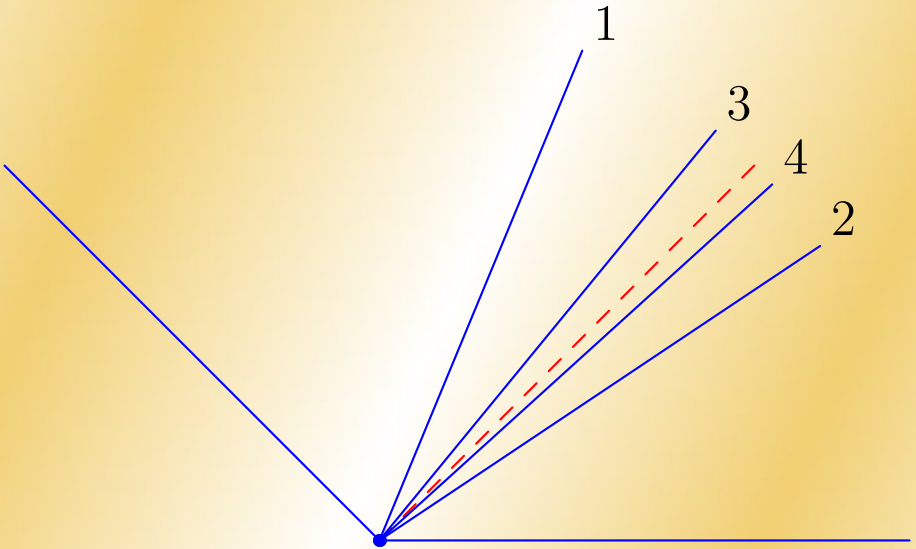
Suma geometrijskog reda II



$$\triangle PRQ \sim \triangle STP$$

$$1 + r + r^2 + \cdots = \frac{1}{1 - r}$$

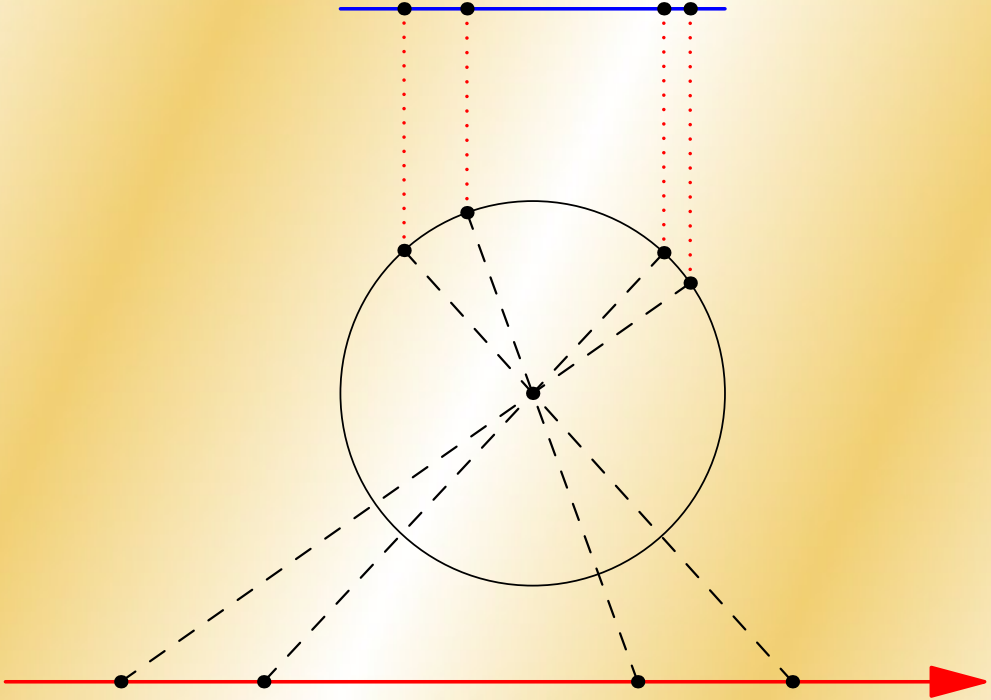
Trisekcija kuta u beskonačno koraka



$$\frac{1}{3} = \frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} + \cdots$$

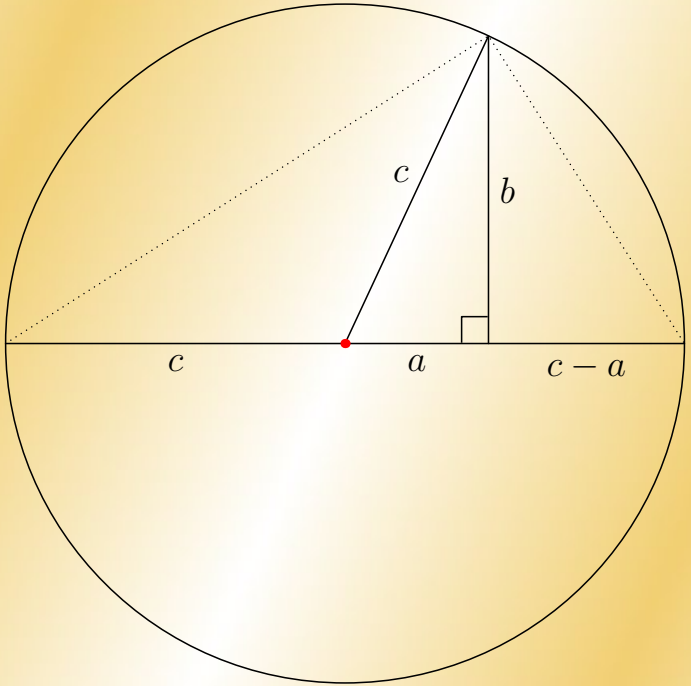
Ekvipotentnost

$$\#(a,b) = \#\mathbb{R}$$



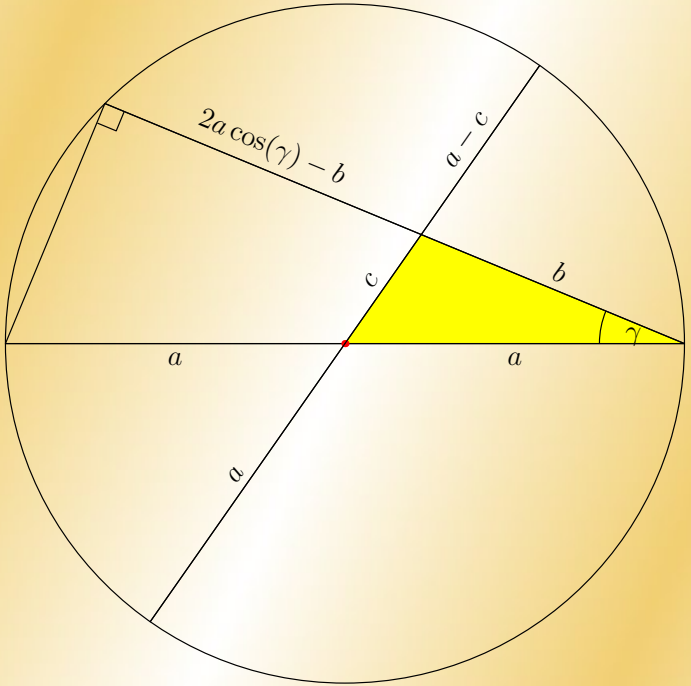
Identiteti za trokut

Pitagorin poučak



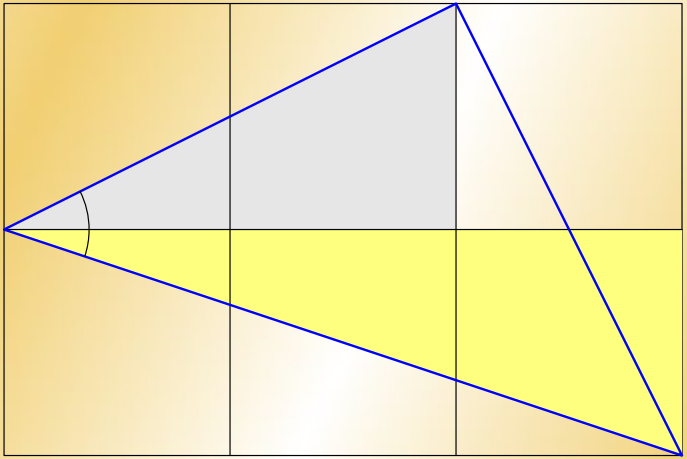
$$\frac{b}{c+a} = \frac{c-a}{b} \Rightarrow c^2 = a^2 + b^2$$

Kosinusov poučak



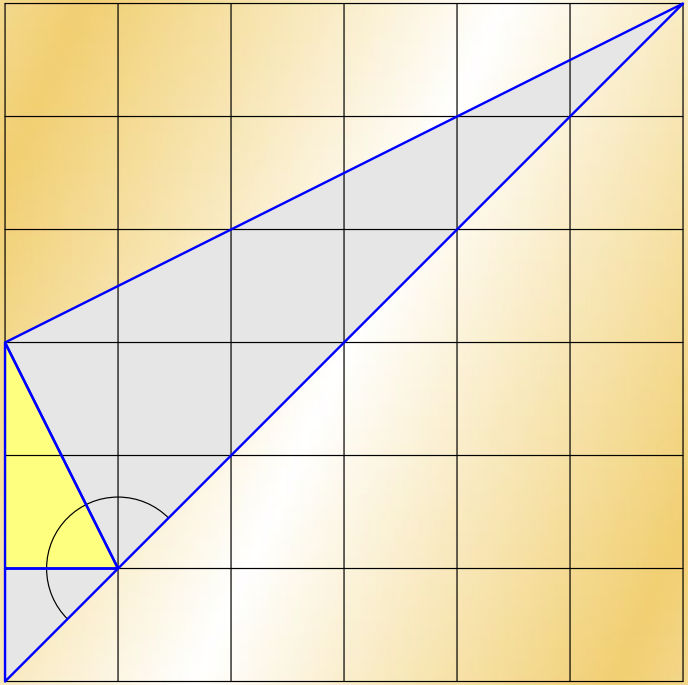
$$(a+c)(a-c) = (2a\cos\gamma - b)b$$
$$\Rightarrow c^2 = a^2 + b^2 - 2ab\cos\gamma$$

Suma arkus tangensa I



$$\operatorname{arctg} \frac{1}{2} + \operatorname{arctg} \frac{1}{3} = \frac{\pi}{4}$$

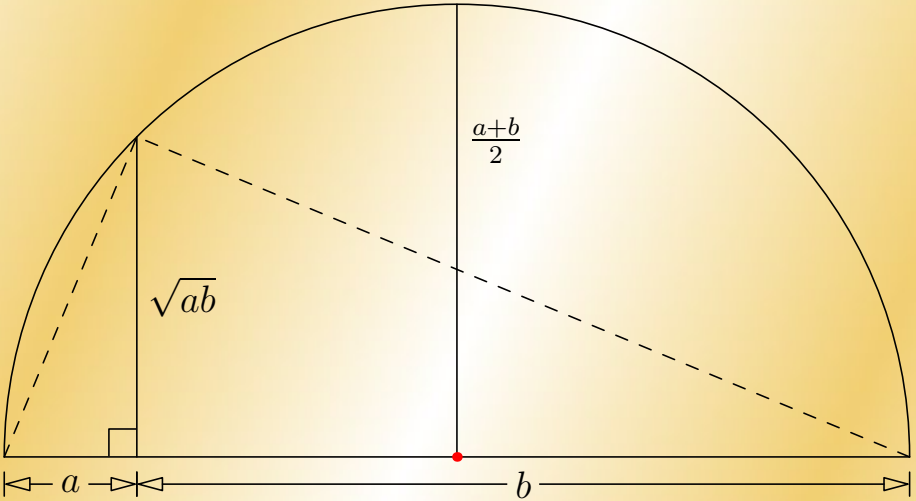
Suma arkus tangensa II



$$\operatorname{arctg} 1 + \operatorname{arctg} 2 + \operatorname{arctg} 3 = \pi$$

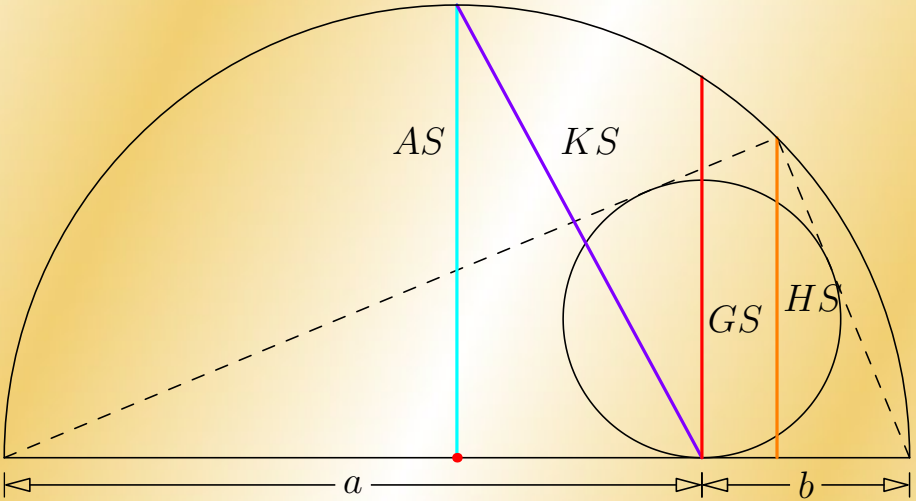
Nejednakosti

A-G nejednakost I



$$\sqrt{ab} \leq \frac{a+b}{2}$$

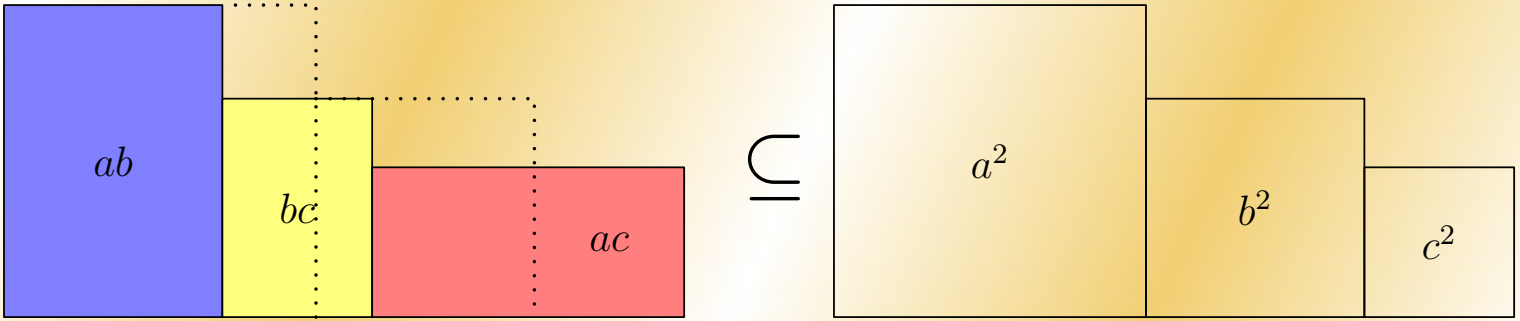
Nejednakost sredina



$$\frac{2}{\frac{1}{a}+\frac{1}{b}} \leq \sqrt{ab} \leq \frac{a+b}{2} \leq \sqrt{\frac{a^2+b^2}{2}}$$

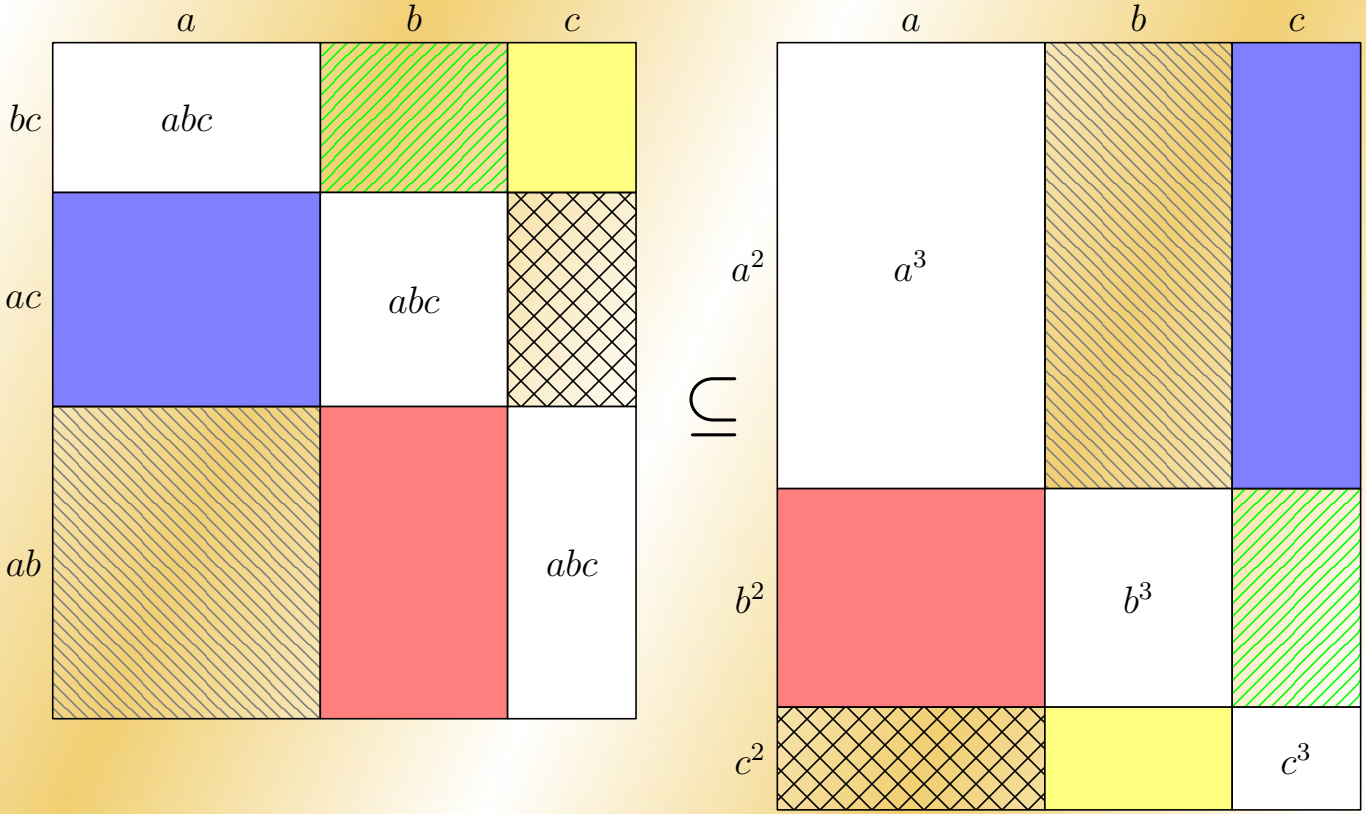
Površina trokuta $= ab$

A-G nejednakost II-Lema



$$ab + bc + ac \leq a^2 + b^2 + c^2$$

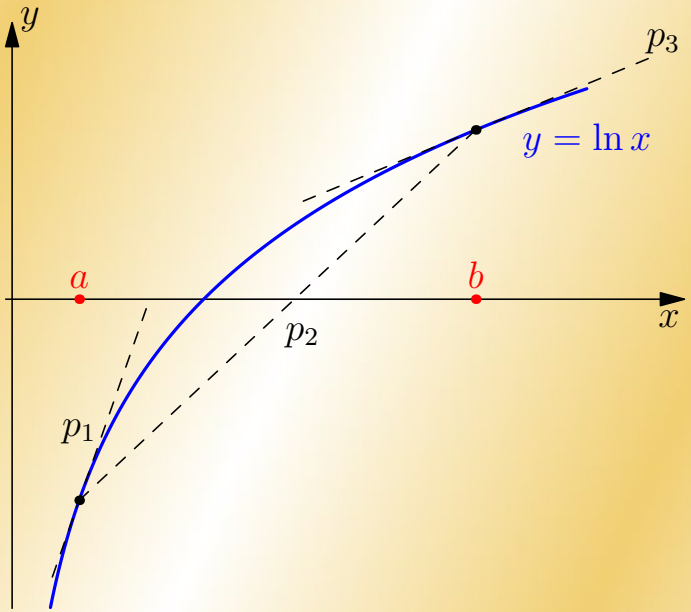
A-G nejednakost II



$$3abc \leq a^3 + b^3 + c^3$$

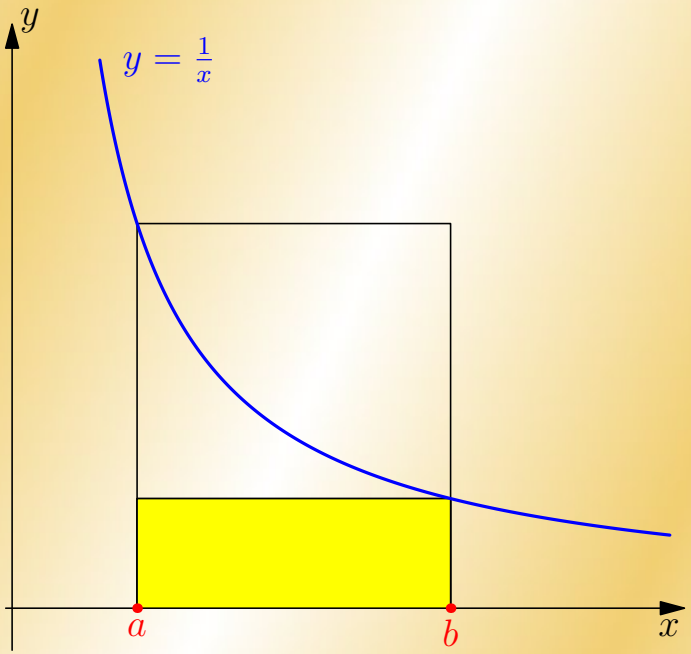
Napierova nejednakost-prvi dokaz

$$0 < a < b \Rightarrow \frac{1}{b} < \frac{\ln b - \ln a}{b - a} < \frac{1}{a}$$



$$n(p_3) < n(p_2) < n(p_1)$$

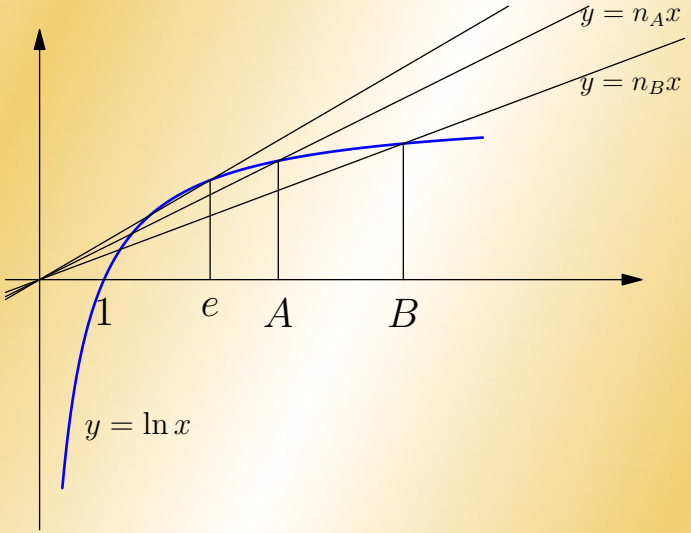
Napierova nejednakost-drugi dokaz



$$\frac{1}{b}(b-a) < \int_a^b \frac{1}{x}dx < \frac{1}{a}(b-a)$$

Eksponencijalna nejednakost

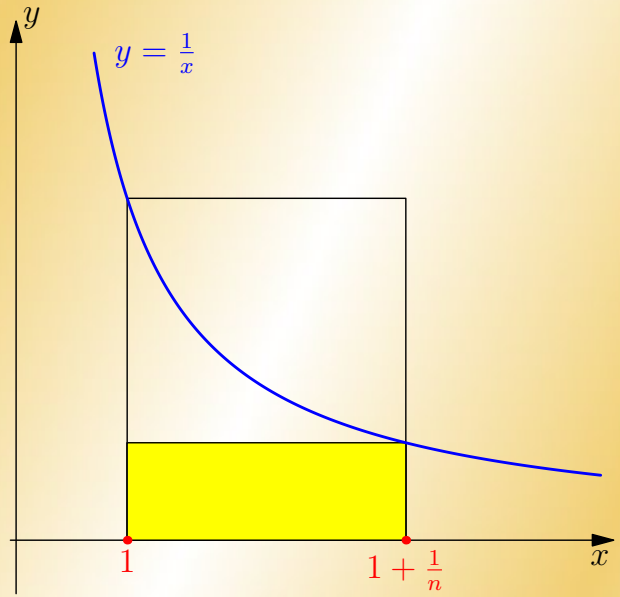
$A^B > B^A$ za $e \leq A < B$



$$n_A > n_B \Rightarrow \frac{\ln A}{A} > \frac{\ln B}{B} \Rightarrow A^B > B^A$$

Limesi

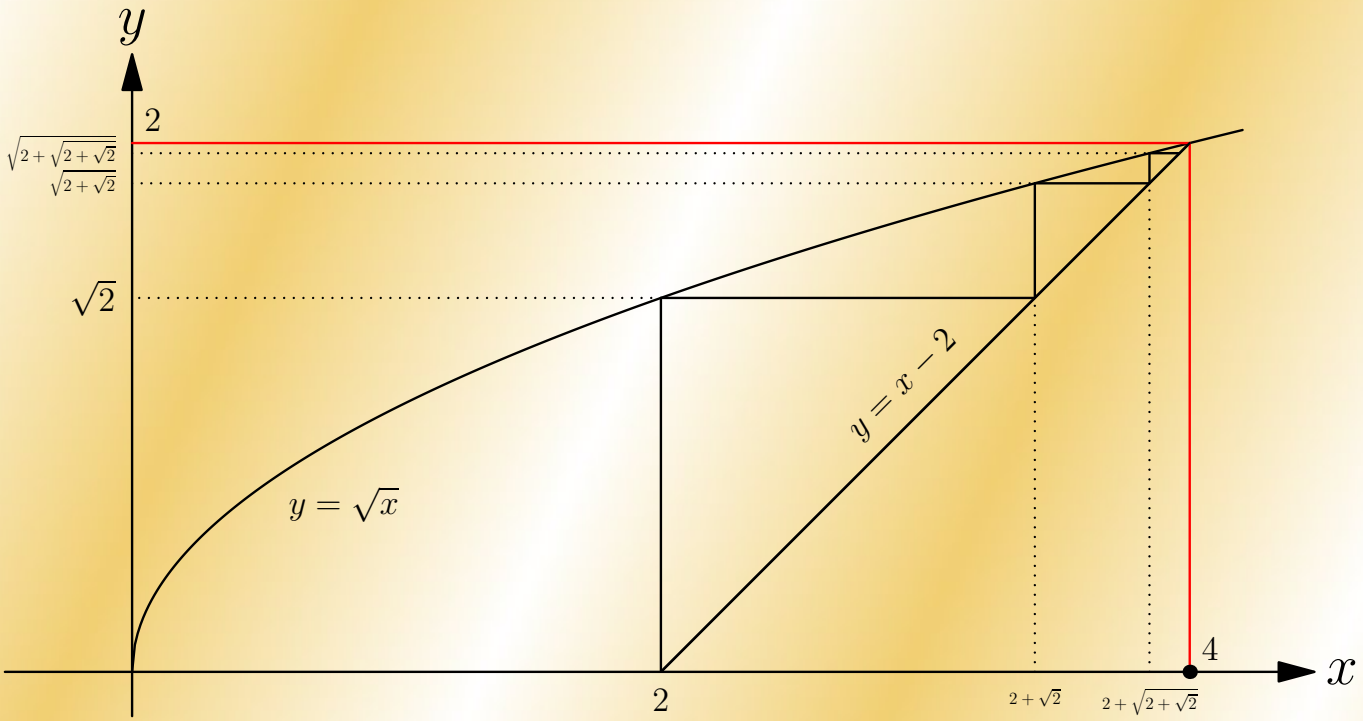
$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$$



$$\frac{n}{n+1}\frac{1}{n} < \ln\left(1+\frac{1}{n}\right) < \frac{1}{n}$$

$$\frac{n}{n+1} < n\ln\left(1+\frac{1}{n}\right) < 1$$

$$\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{\cdots}}}} = 2$$



Parcijalna integracija

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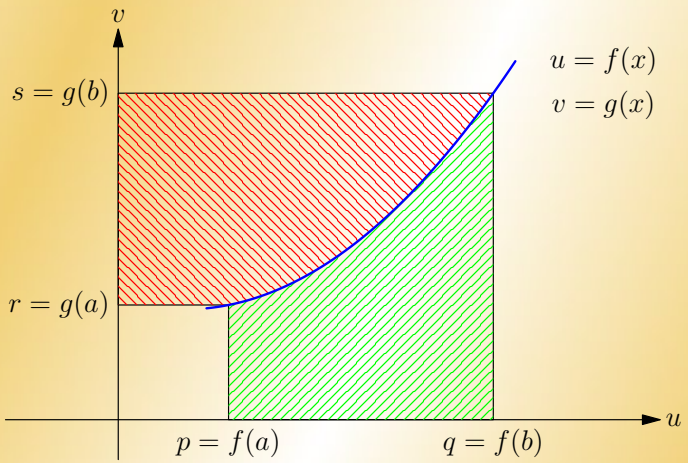
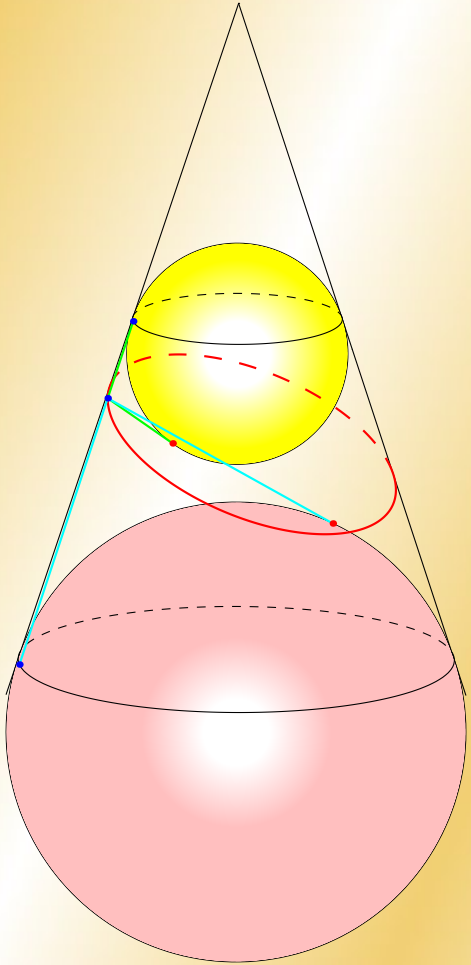


$$\int_r^s u dv + \int_p^q v du = uv\Big|_{(p,r)}^{(q,s)}$$

$$\int_a^b f(x)g'(x)dx = f(x)g(x)\Big|_a^b - \int_a^b g(x)f'(x)dx$$

Dandelinove kugle

Dandelinove kugle



Hvala na pažnji!

$$\log 2 \approx 0.$$

$$\mathrm{O}$$