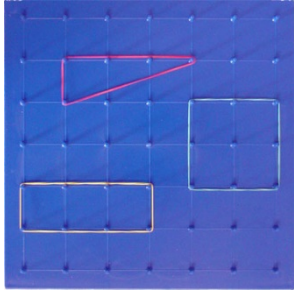
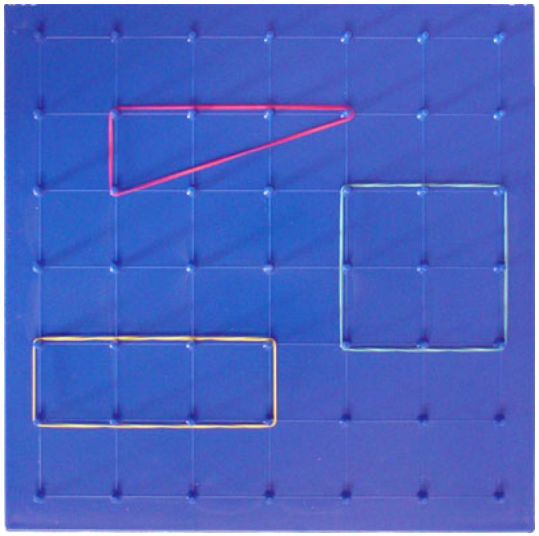
GEOPLOČA

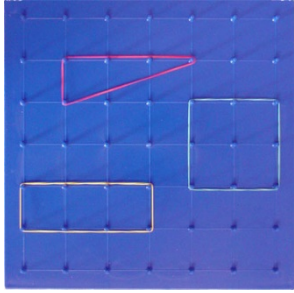
TANJA SOUCIE, IVANA KATALENAC, RENATA SVEDREC



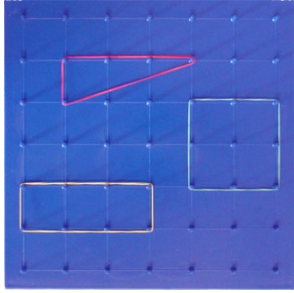
Što je *geoploča*?

Geoploča je (drvena ili plastična) ploča s čavlićima koji su raspoređeni u kvadratnu mrežu oko kojih je moguće rastezati elastične (gumene) vrpce.

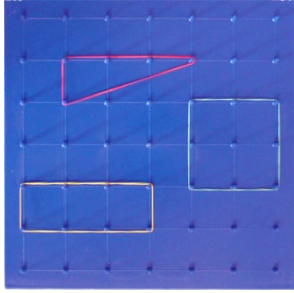




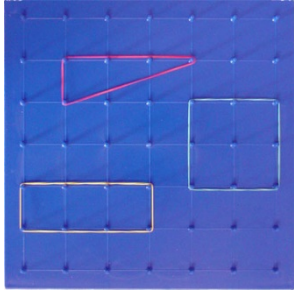
Geoploču je 1952. godine osmislio egipatski matematičar **Caleb Gattegno** (1911. – 1988.), a on je izradio i prve nastavne materijale za primjenu geoploče u nastavi geometrije.



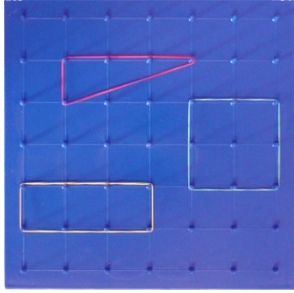
U Europi (i ostatku svijeta) moguće je nabaviti gotove geoploče različitih boja i veličina. Posebno su prikladne prozirne ploče koje je – za potrebe analiziranja problema i/ili diskusije rješenja – moguće staviti na grafoskop.



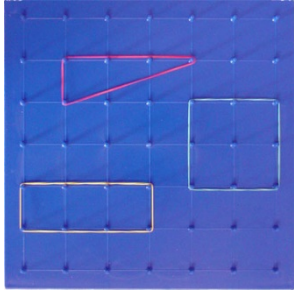
Geoploču je moguće napraviti i u „kućnoj radinosti“ – od drva ili deblje plutene ploče s 25 čavlića raspoređenih u kvadratnu mrežu (5 × 5), na jednakim razmacima (udaljenostima), primjerice 1 cm.



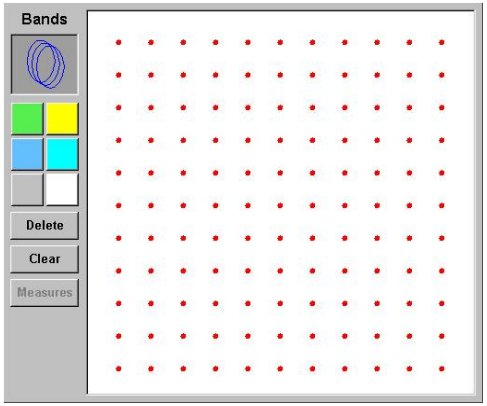
Umjesto rada na konkretnoj (originalnoj ili priručnoj) geoploči, moguće je koristiti i samo *točkasti papir.*

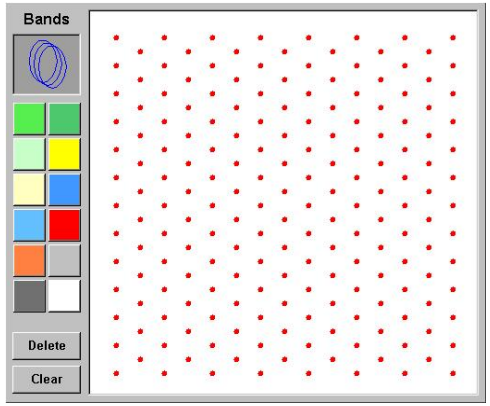


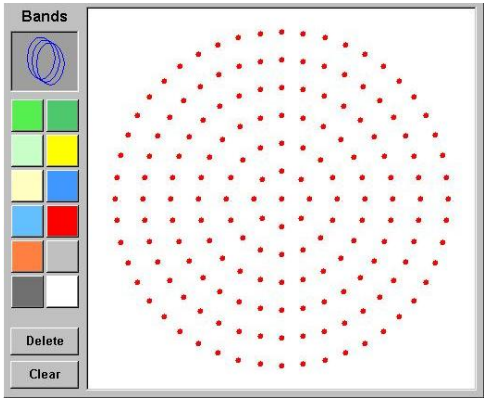
Ukoliko je dostupno, učenici u razredu ili kod kuće mogu koristiti i elektronske aplikacije koje su (besplatno) dostupne na različitim internetskim adresama (kao što su npr. http://nrich.maths.org/5648, http://mste.illinois.edu/users/pavel/java/geoboard/, http://nlvm.usu.edu/en/nav/frames_asid_172_g_2_t_3.html http://www.mathplayground.com/geoboard.html)



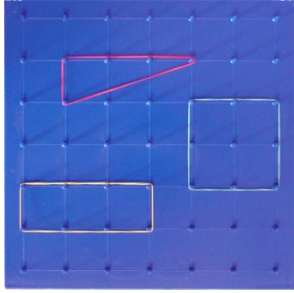
Osim klasičnih *geoploča* koriste se još tzv. *izometričke geoploče* (čavlići raspoređeni u mrežu rombova sa šiljastim kutom veličine 60°, i *kružne geoploče*.



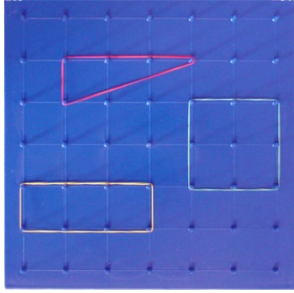




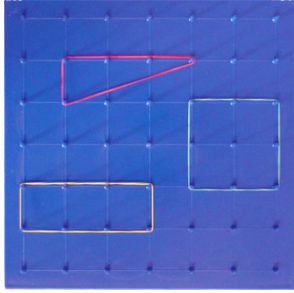
Zašto koristiti *geoploču*?



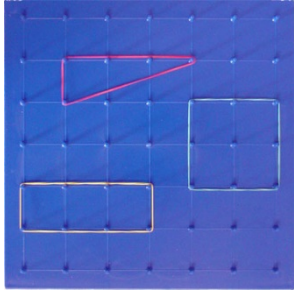
Kao nastavno pomagalo geoploča je vrlo primjenjiva u osnovnoškolskoj nastavi matematike kao pomoć pri istraživanju koncepata kao što su opseg, površina, svojstva geometrijskih likova, preslikavanja ravnine, pojam razlomka i postotka te crtanje likova u koordinatnom sustavu.



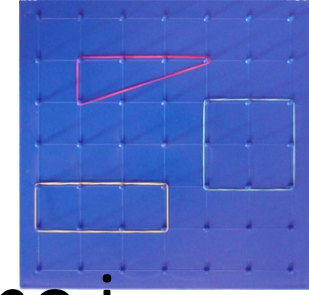
Geoploča učenicima omogućava vizualizaciju te im daje mogućnost da „rade“ geometriju. Pruža im mogućnost da samostalno otkrivaju matematiku te razmjenjuju matematičke ideje čime razvijaju svoje komunikacijske vještine i matematički rječnik.



Korištenje geoploče učenicima omogućuje postavljanje i rješavanje matematičkih problema, potiče ih na istraživanje, sustavnost, kreativnost i ustrajnost u radu, tako da oni postaju aktivni sudionici u procesu učenja.



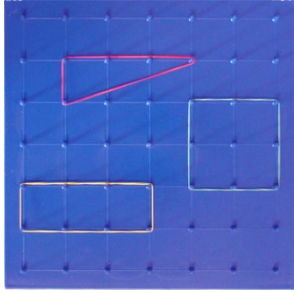
Nadalje, *Nacionalni okvirni kurikulum za matematičko područje* od učenika očekuje da će, između ostaloga, moći stvarati i istraživati pretpostavke o matematičkim objektima, pravilnostima i odnosima; skicirati jednostavne ravninske oblike te ih nacrtati i konstruirati pomoću geometrijskog pribora (i jednostavnoga računalnoga programa za crtanje);



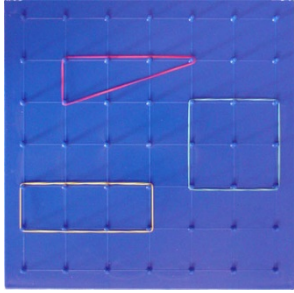
prepoznati sukladne i slične trokute; približno i točno odrediti udaljenost dviju točaka i površinu likova brojanjem jediničnih dužina i jediničnih kvadrata; prepoznati, imenovati, izgraditi i klasificirati ravninske (i prostorne) geometrijske oblike te istražiti, uočiti i (precizno) opisati njihova geometrijska svojstva… [3]

Korištenje geoploče u nastavi matematike pridonijet će ostvarivanju postavljenih ciljeva.

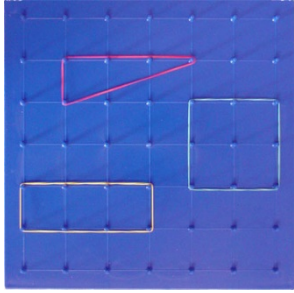
Potrebni materijali



Za rad u paru poželjno je svakom paru učenika osigurati *geoploču*, dovoljan broj elastičnih gumenih vrpci (različitih boja i duljina), a svaki će učenik trebati dovoljno *točkastog papira*, ravnalo ili trokut te olovku i gumicu za brisanje.

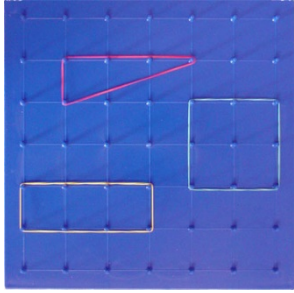


Kad upoznajemo učenike s novim nastavnim pomagalom, svakako im trebamo dati vremena da se s njim upoznaju. Nakon što učencima postavimo pitanja kao štu su *Kojeg je oblika geoploča?, Koliko se čavlića nalazi u svakom retku, a koliko u svakom stupcu?* te *Koliko se ukupno čavlića nalazi na geoploči?* možemo im zadati neke od sljedećih tipova zadataka.



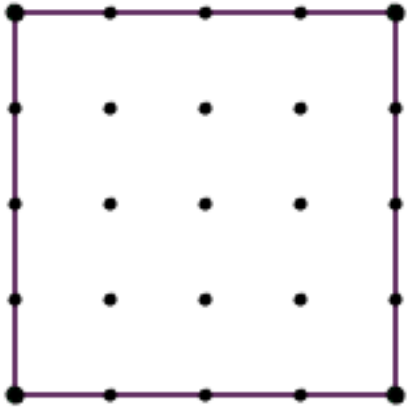
5. I 6. RAZRED

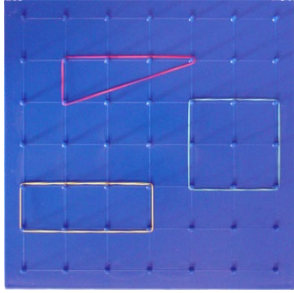
JEDNOSTAVNE IDEJE



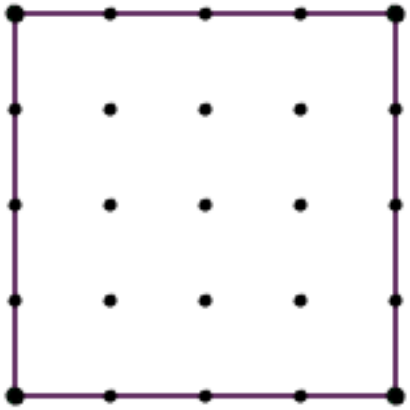
Razlomci na geoploči [1]

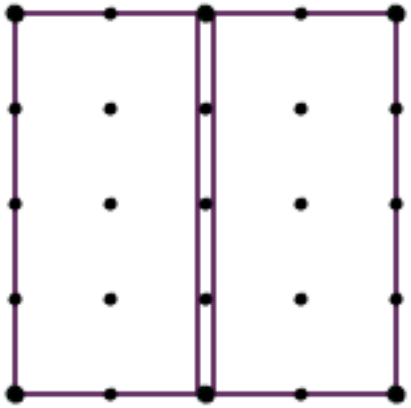
Učitelj prikazuje jedno cijelo, a zatim prikazuje različite razlomke pomoću geoploče, a učenici ih imenuju.



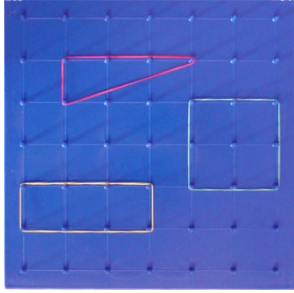


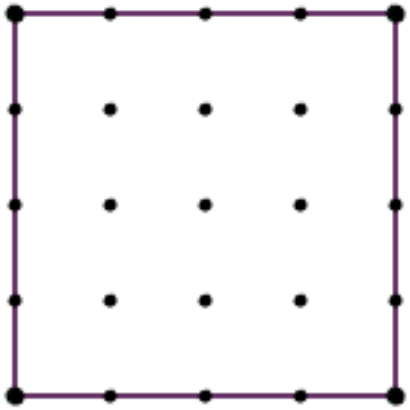
Razlomci na geoploči [2]

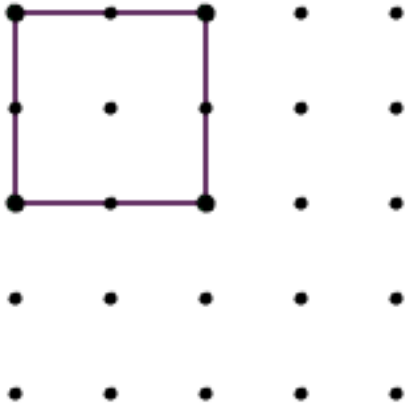
Ako je  jedno cijelo,

koliko je  ?

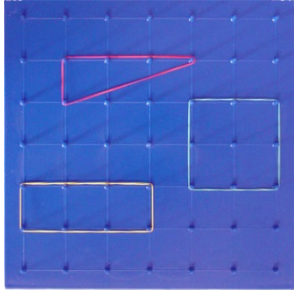
Razlomci na geoploči [3]

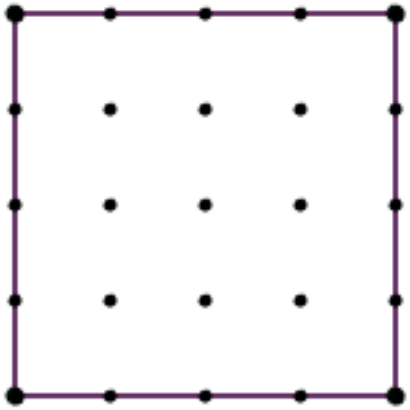


Ako je  jedno cijelo,

koliko je  ?

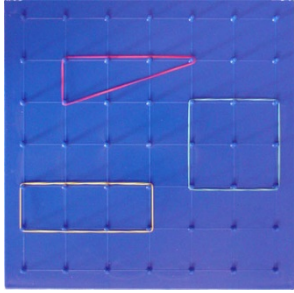
Razlomci na geoploči [4]



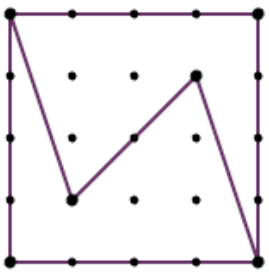
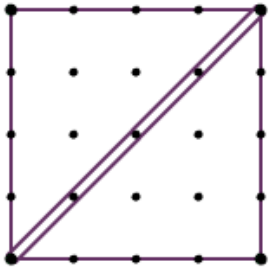
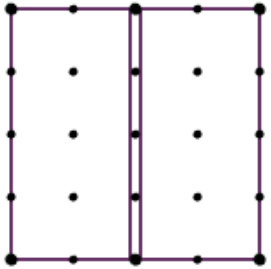
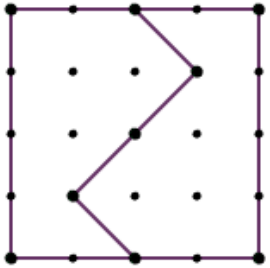
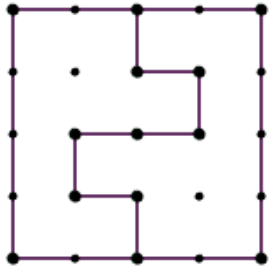
Ako je  jedno cijelo, prikaži jednu

polovinu na što više načina.

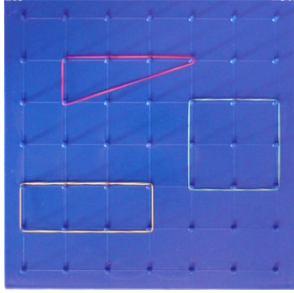
Razlomci na geoploči [5]



Neka od mogućih rješenja:

Razlomci na geoploči [6]



1. Neka je kvadratić dimenzija 4X4 jedno cijelo.

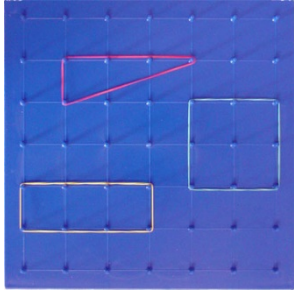
   Označi elastičnim vrpcama.

2. Uspravno prikaži polovine.

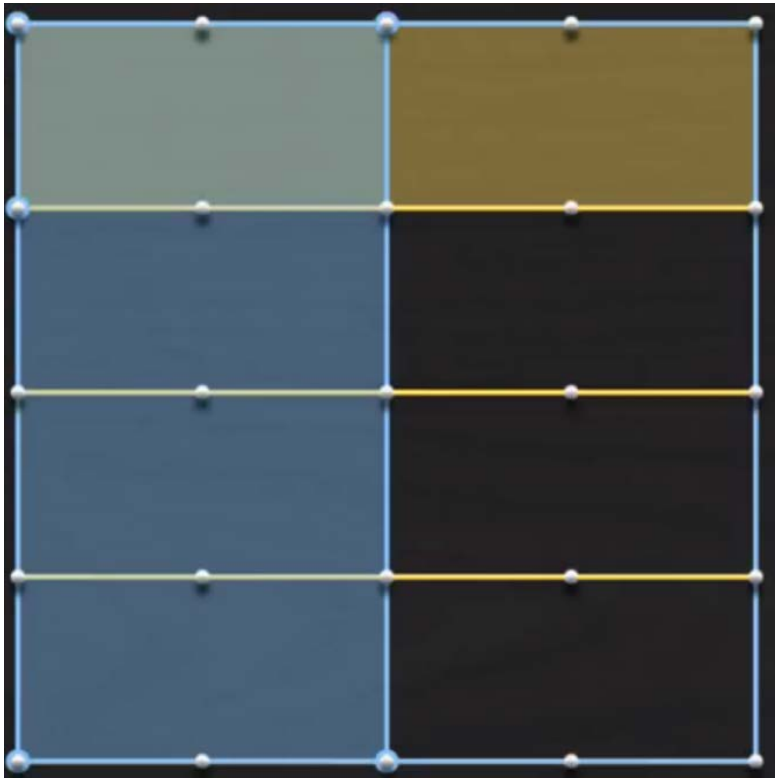
3. Vodoravno prikaži četvrtine.

4. Koliko je $\frac{1}{2} \cdot \frac{1}{4}$?

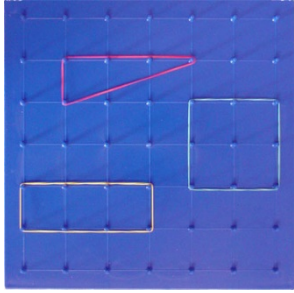
Razlomci na geoploči [7]



Rješenje:



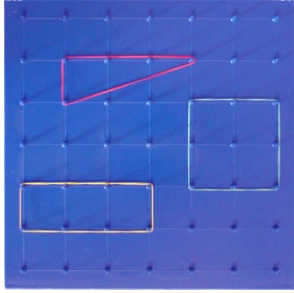
Razlomci na geoploči [8]



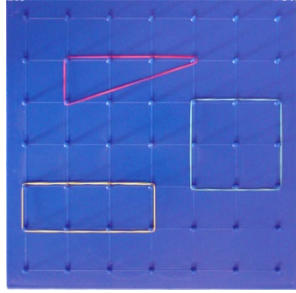
5. Koliko je $\frac{3}{4} \cdot \frac{1}{2}$?

6. Koliko je $\frac{1}{4} \cdot \frac{3}{4}$?

Četverokut [1]

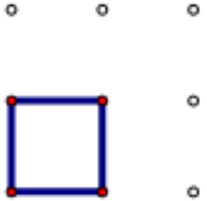


Na priloženom listu 3 × 3 „točkastog papira“ nacrtaj što više (sve) međusobno nesukladnih četverokuta kojima su vrhovi u točkama mreže.

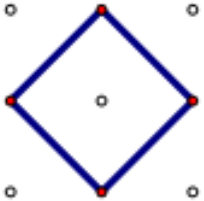


Rješenja

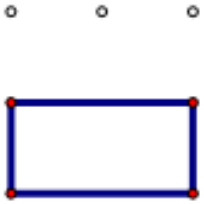
1

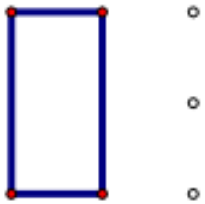


2



3





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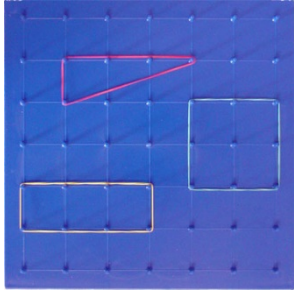

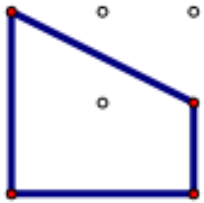

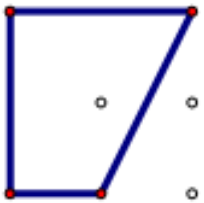

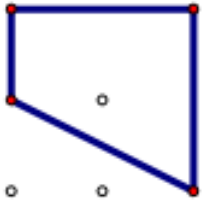

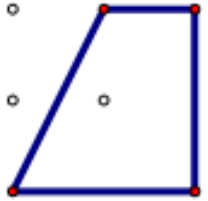

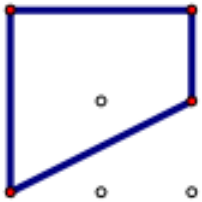

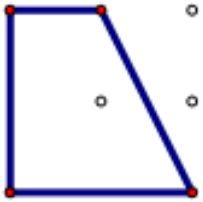

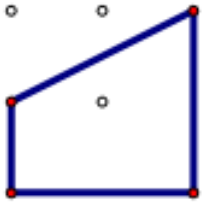

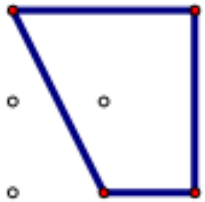

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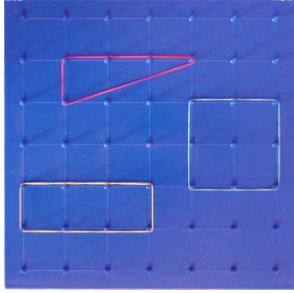

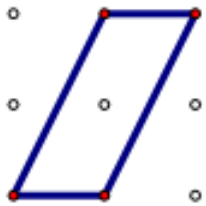

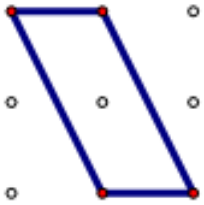

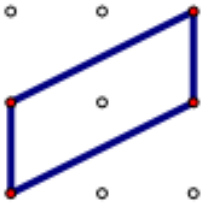

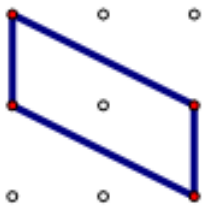

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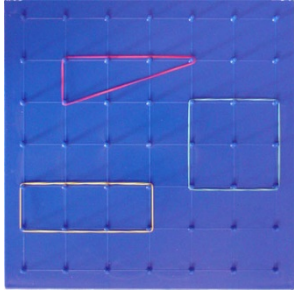

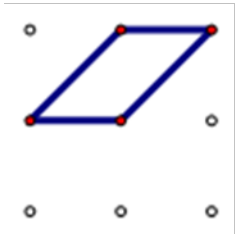

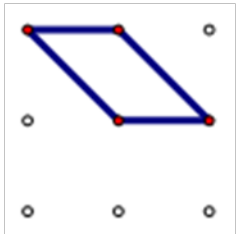

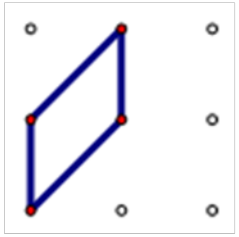

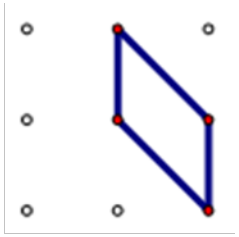

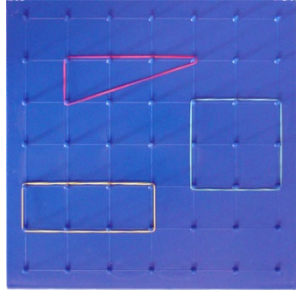

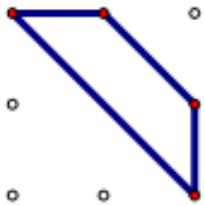

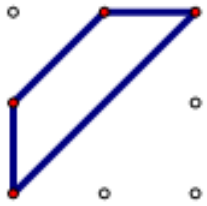

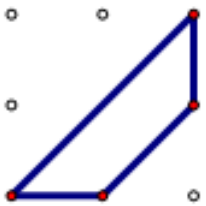

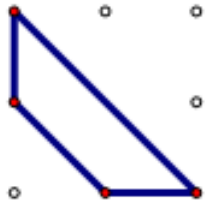

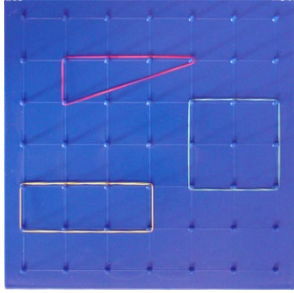

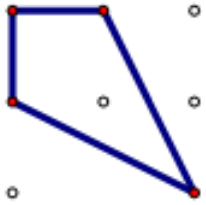

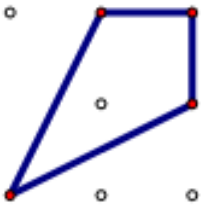

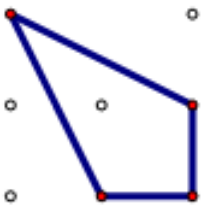

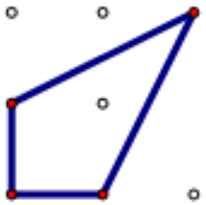

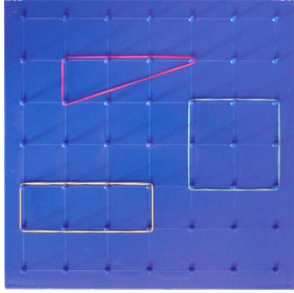

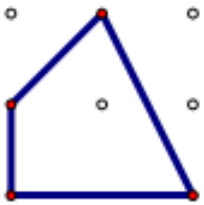

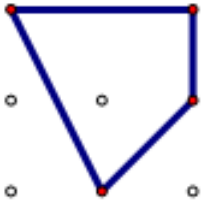

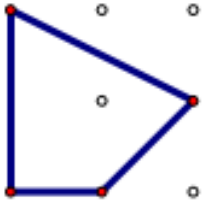

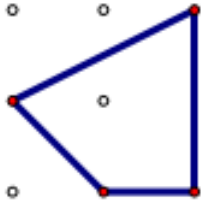

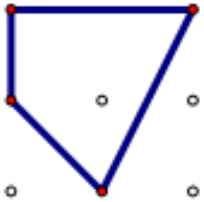

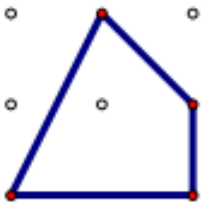

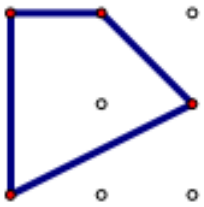

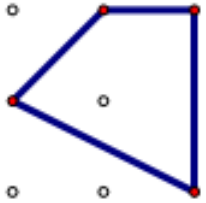

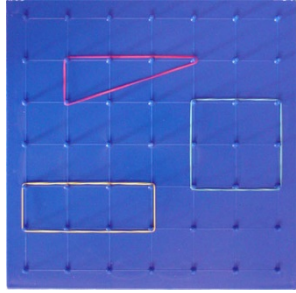

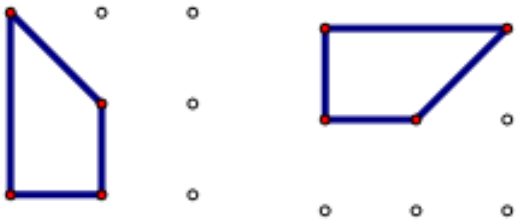

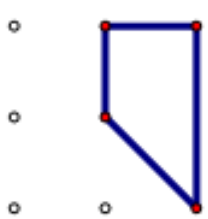

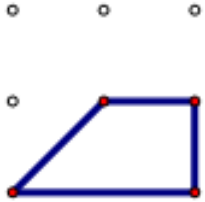

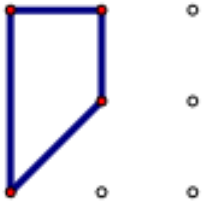

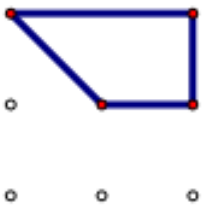

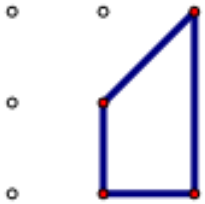

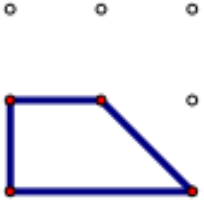

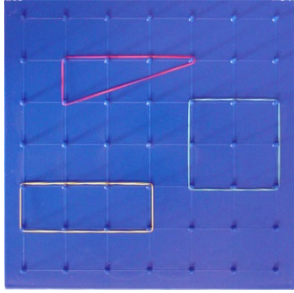

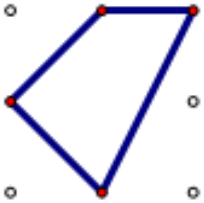

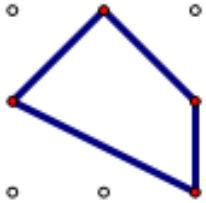

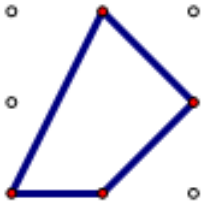

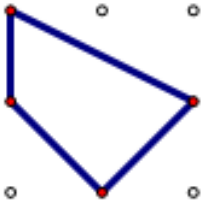

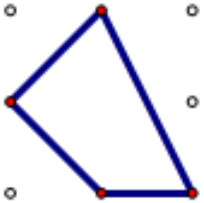

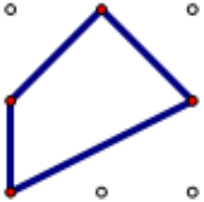

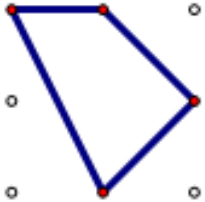

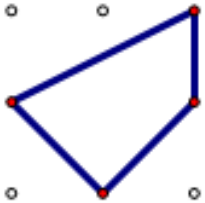

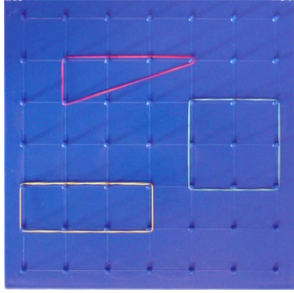

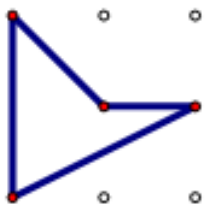

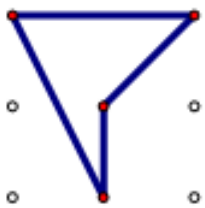

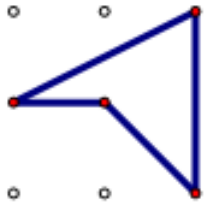

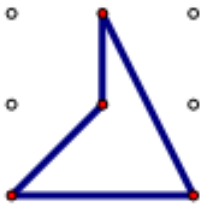

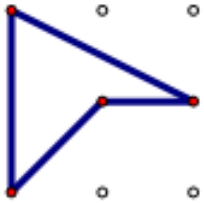

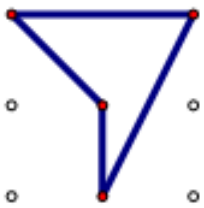

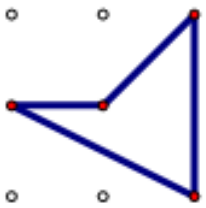

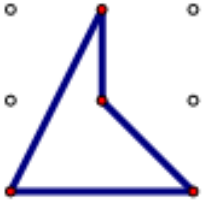

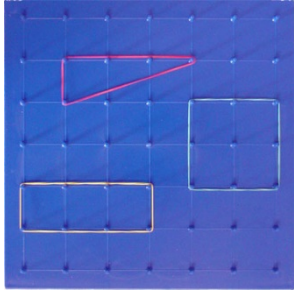

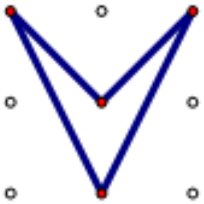

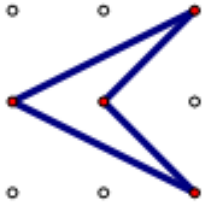

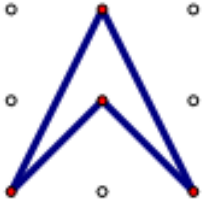

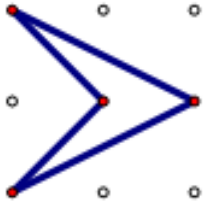

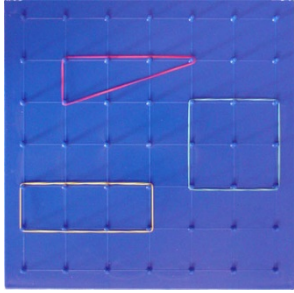

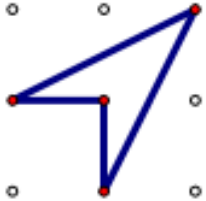

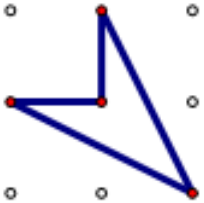

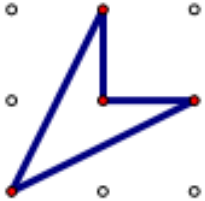

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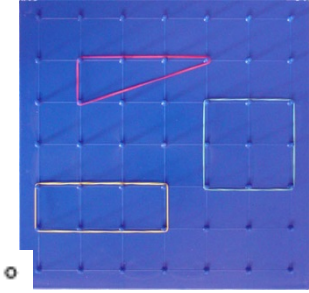
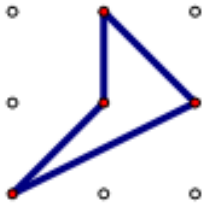
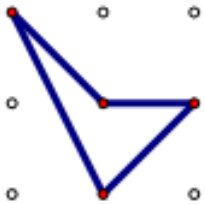
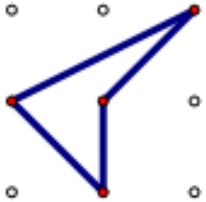
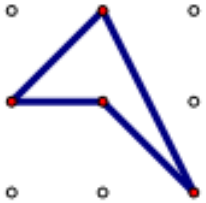
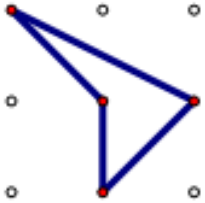


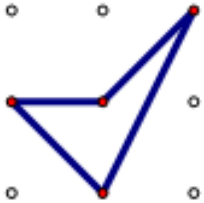


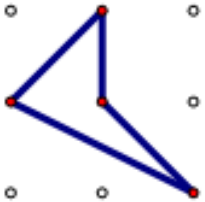


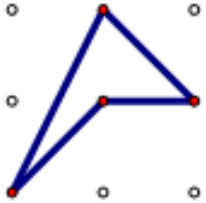




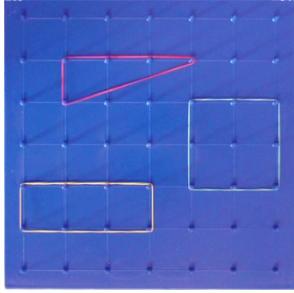






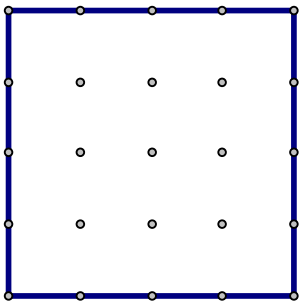


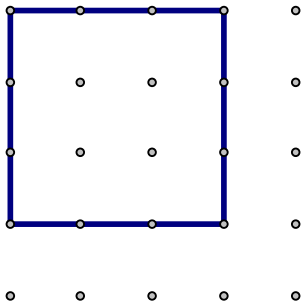
Kvadrati i pravokutnici [1]

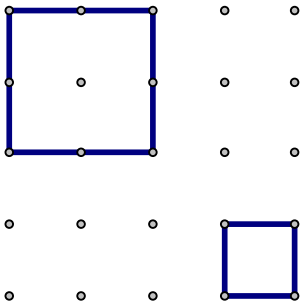


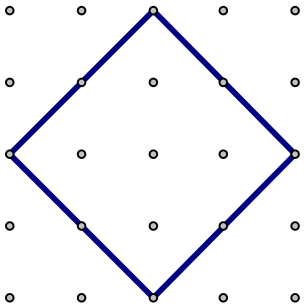
Nacrtaj sve moguće kvadrate i pravokutnike kojima su vrhovi u točkama mreže 5 × 5.

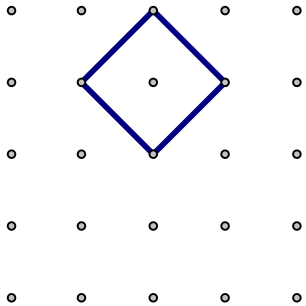
Kvadrati i pravokutnici – Rješenja [1]

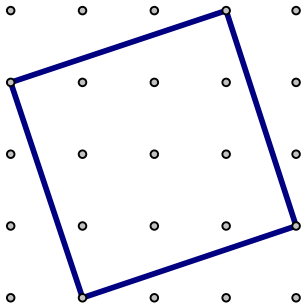


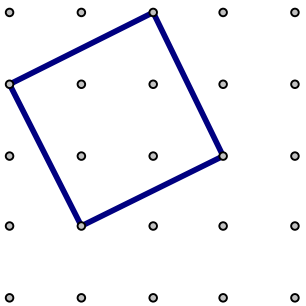




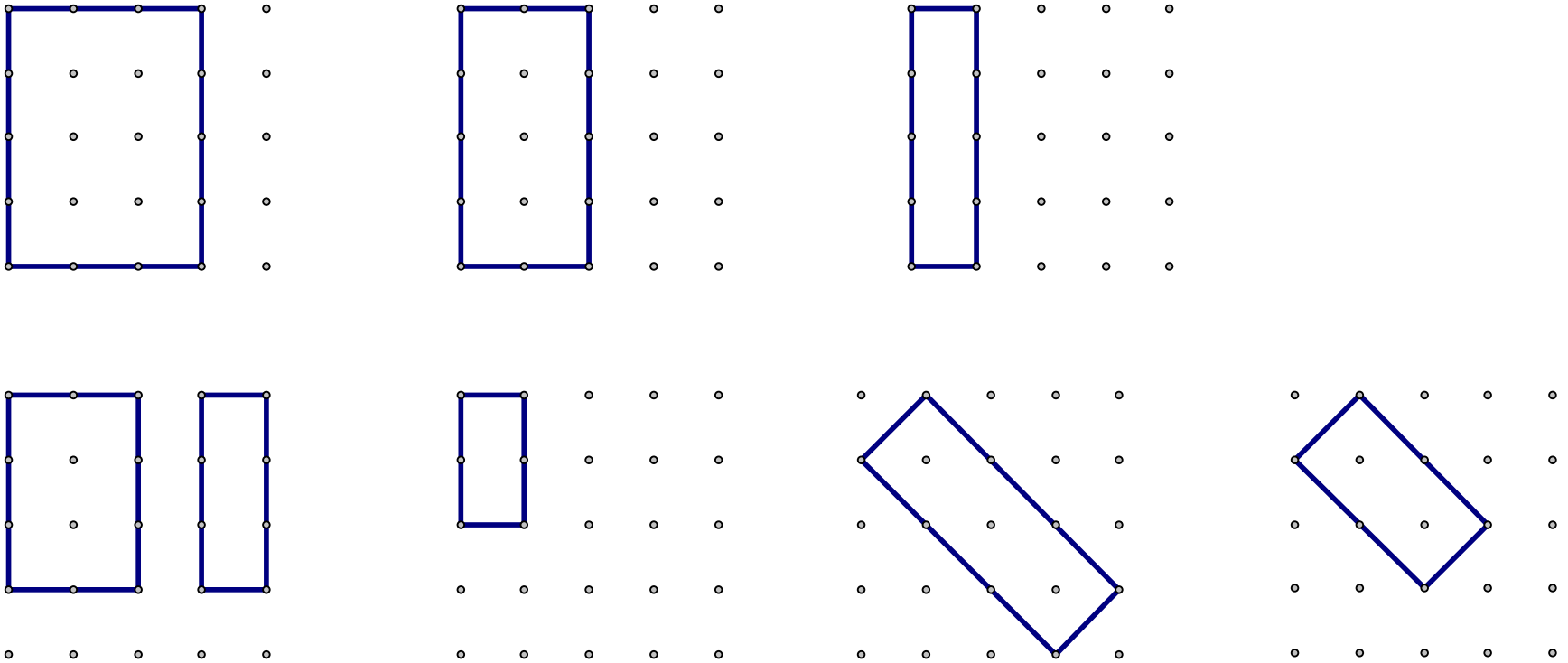




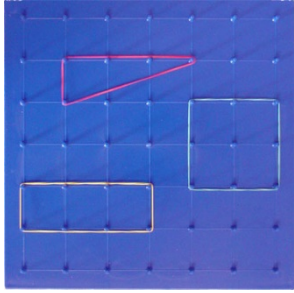




Kvadrati i pravokutnici – Rješenja [2]



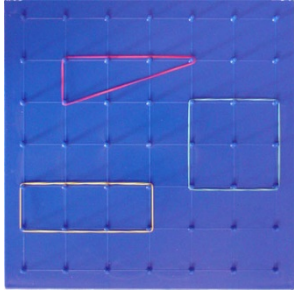
Opseg i površina [1]

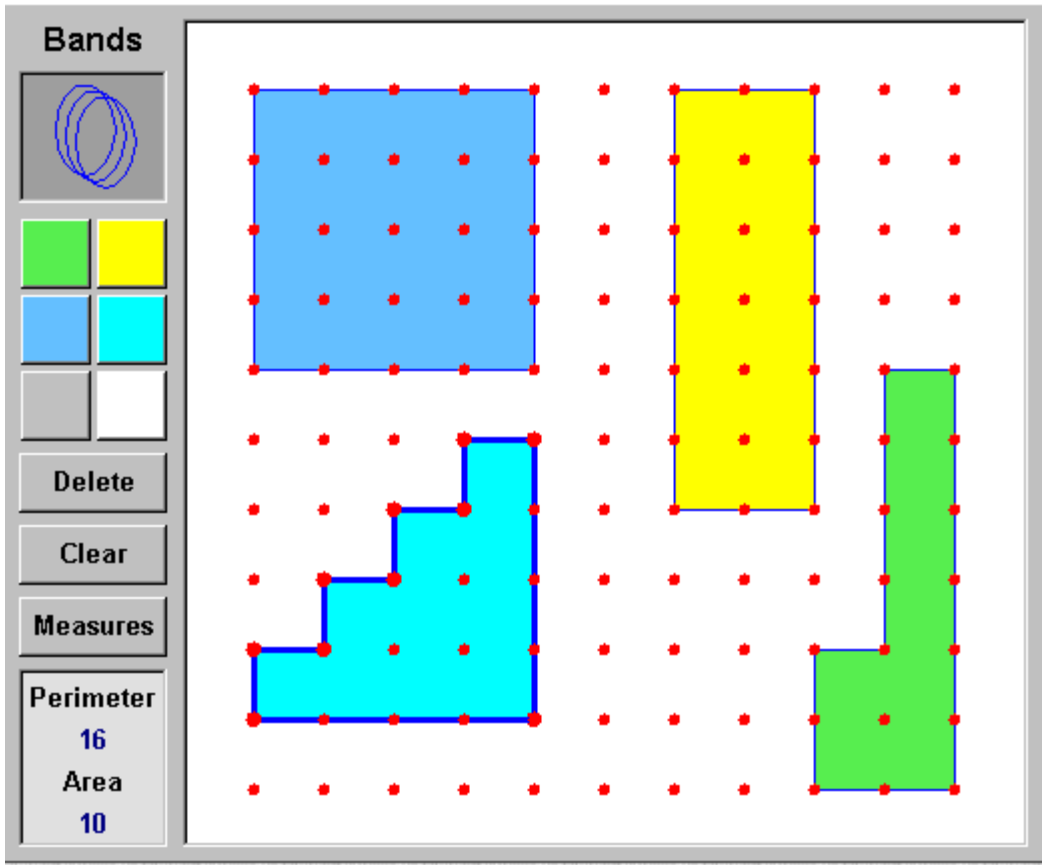


Primjeri zadataka koji mogu učenicima pomoći u razumijevanju pojmova opsega i površine:

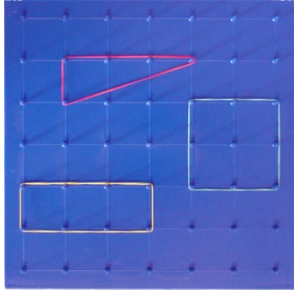
a) Pomoću geoploče prikaži te na točkastom papiru skiciraj što više likova čiji je opseg 16 cm. Odredi površinu svakog od tih likova.

Opseg i površina [2]



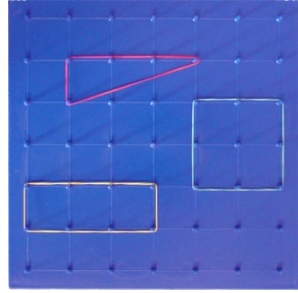


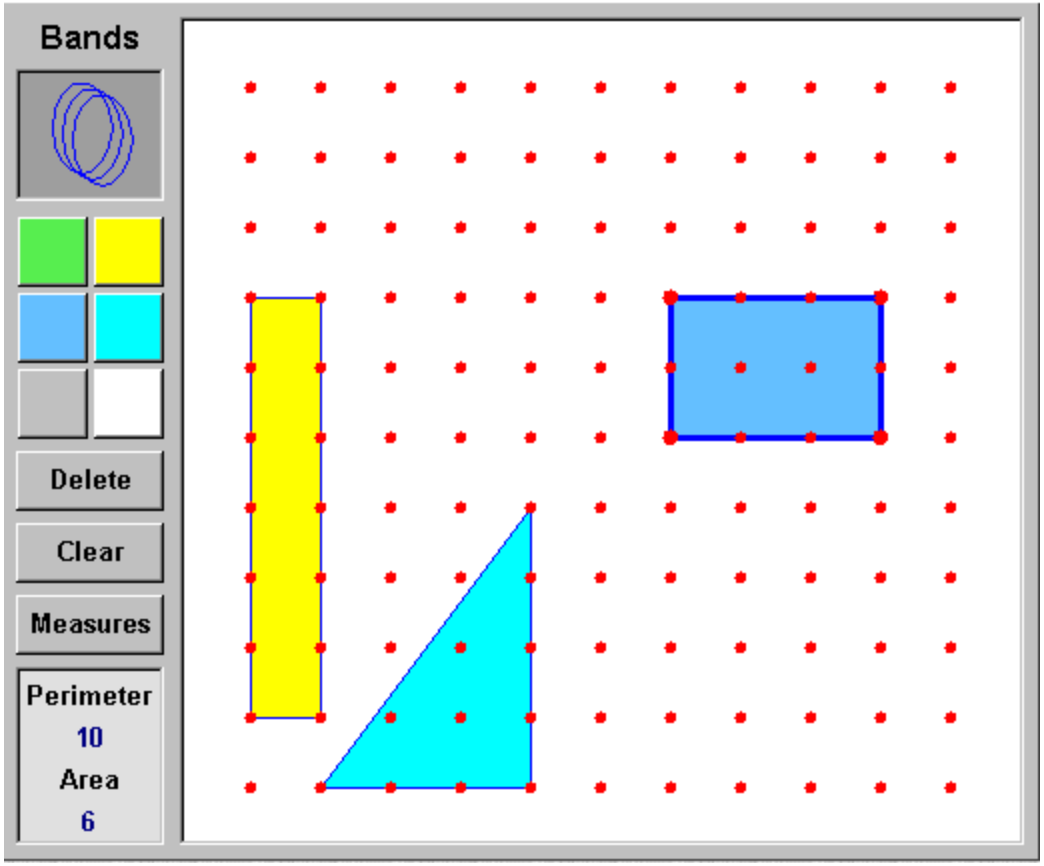
Opseg i površina [3]



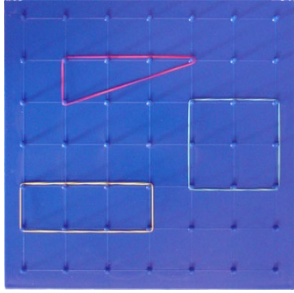
b) Pomoću geoploče prikaži te na točkastom papiru skiciraj što više likova čija je površina 6 cm^2. Odredi opseg svakog od tih likova.

Opseg i površina [4]

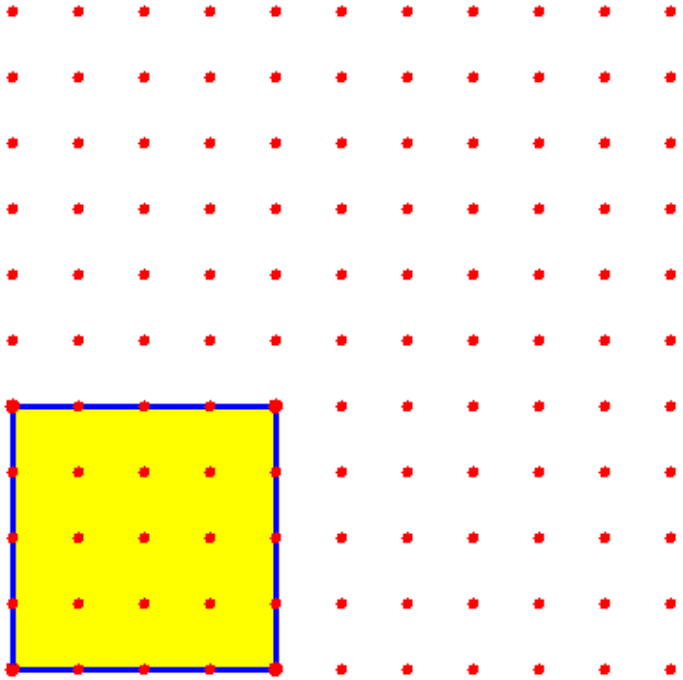




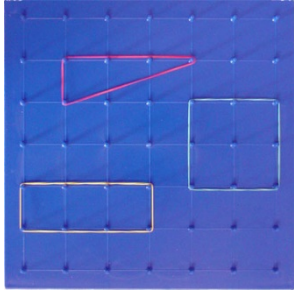
Opseg i površina [5]

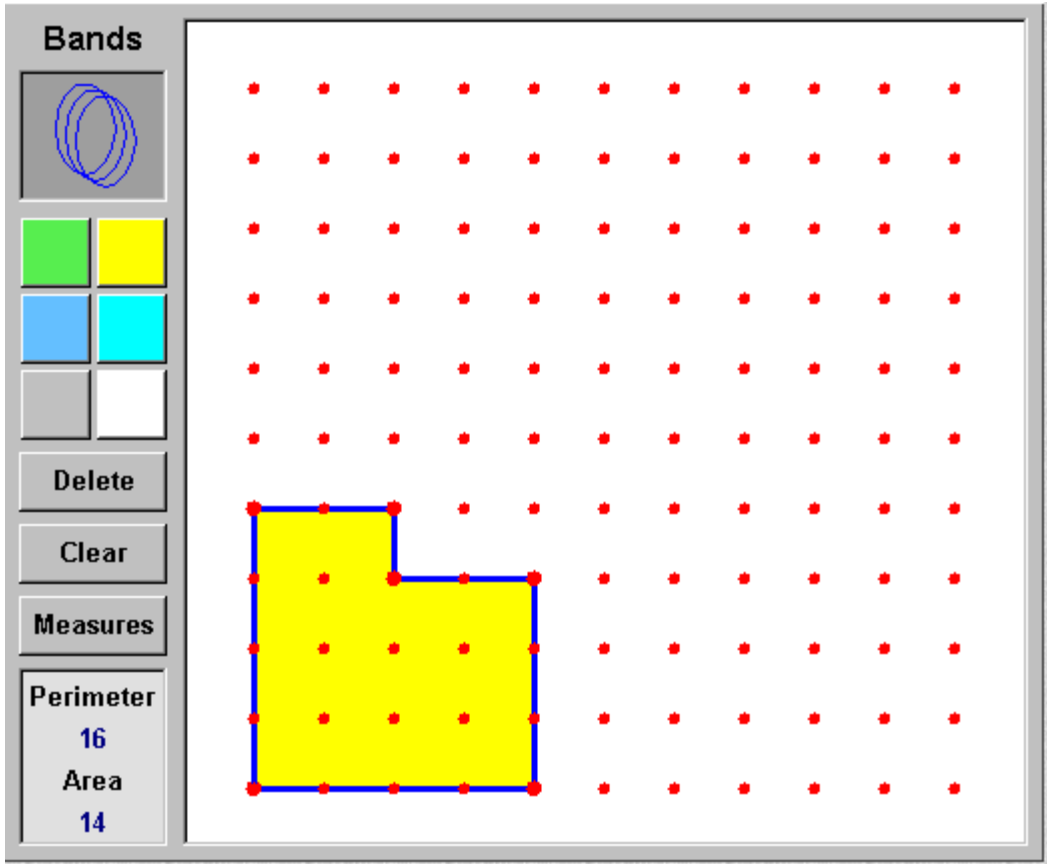


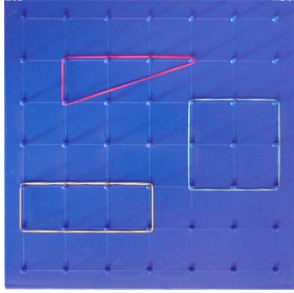
c) Na geoploči je prikazan lik čiji je opseg 16 cm
Pomakni elastičnu vrpcu tako da novi lik ima isti opseg kao zadani, ali manju površinu.



Opseg i površina [6]

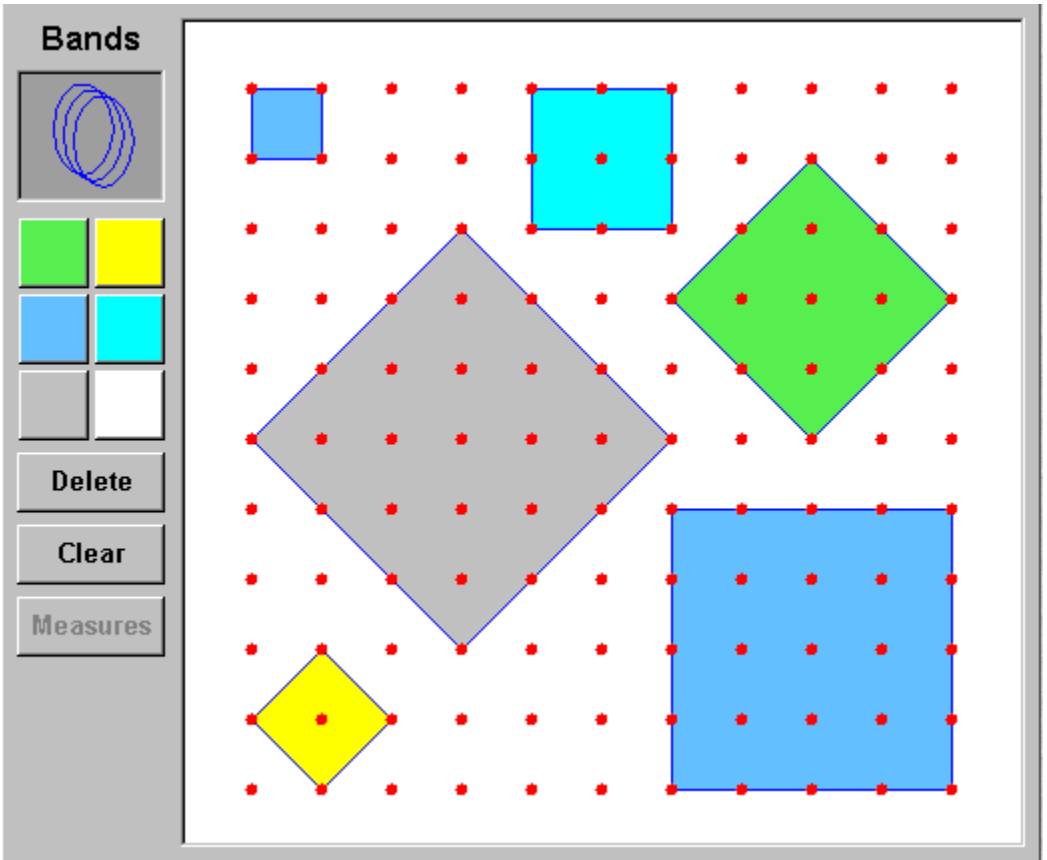




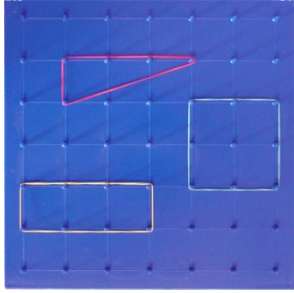


Opseg i površina [7]

d) Pomoću geoploče prikaži te na točkastom papiru skiciraj što više kvadrata različitih dimenzija.



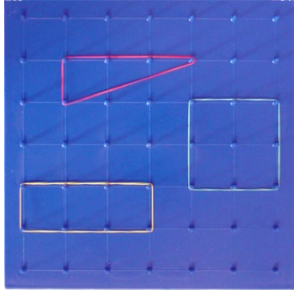
Paralelogrami [1]



Ciljevi:

- razvoj prostornog zora i podizanje razine geometrijskog mišljenja s razine vizualizacije na razinu neformalne dedukcije [5]
- prepoznavanje svojstava paralelograma kao karakteristike klase likova te uočavanje i formuliranje logičkih odnosa među svojstvima
- razvoj osjećaja za površinu paralelograma s posebnim naglaskom na vezu površine paralelograma i površine njemu odgovarajućeg pravokutnika
- primjena koncepta sukladnosti i sličnosti na paralelograme
- eksperimentiranje s geometrijskim uzorcima

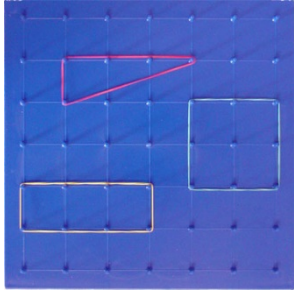
Paralelogrami [2]



Potrebni materijali:

- za svaki par učenika geoploča 5 × 5 i dovoljan broj elastičnih vrpca
- za svakog učenika nastavni listić za zadacima
- za svakog učenika točkasti papir 5 × 5 točaka i točkasti papir 11 × 11 točaka
- za učitelja interaktivna geoploča za analizu rješenja

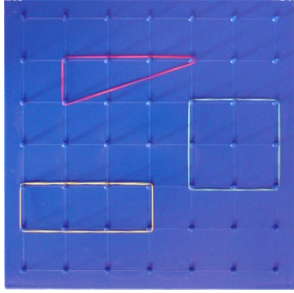
Paralelogrami [3]



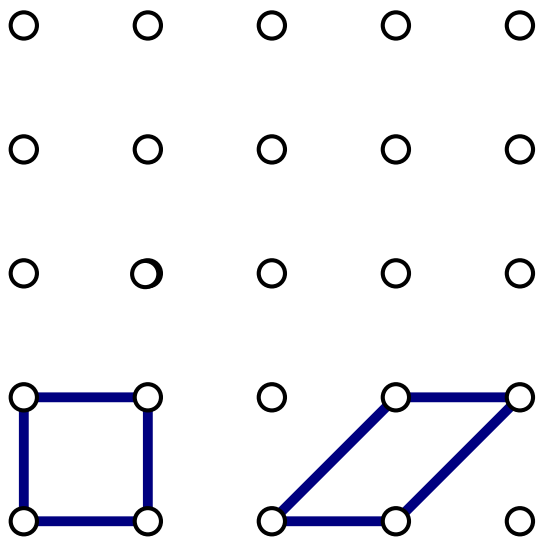
Uvodni zadaci:

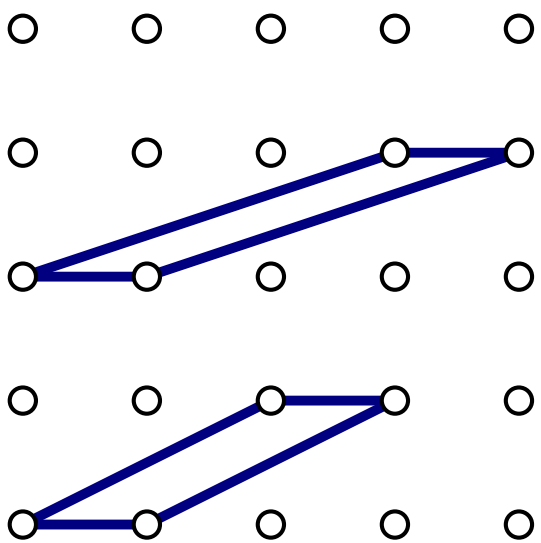
1. Na geoploči dimenzija 5 × 5 napravite paralelogram s osnovicom duljine 1.

Paralelogrami [4]

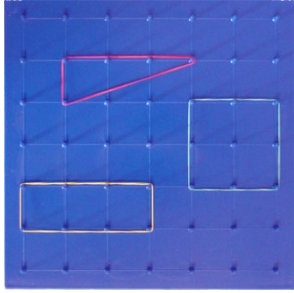


2. Na geoploči dimenzija 5 × 5 napravite još tri paralelograma s osnovicom duljine 1 koji imaju jednaku površinu kao paralelogram iz zadatka 1.

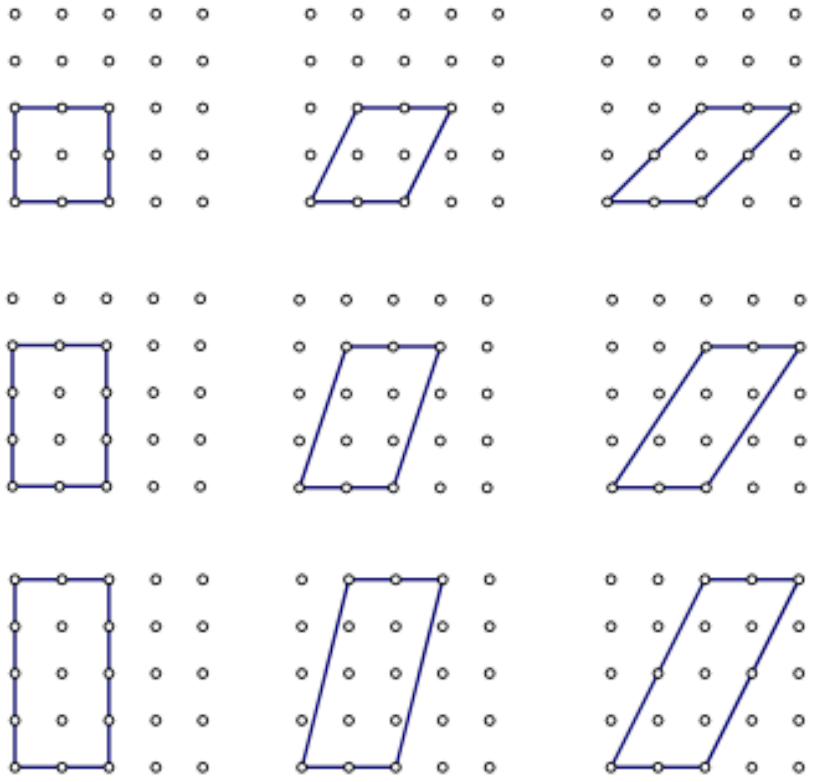




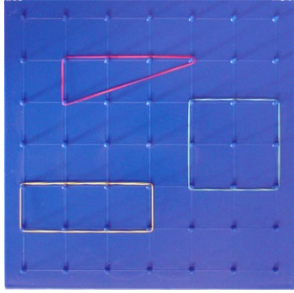
Paralelogrami [5]



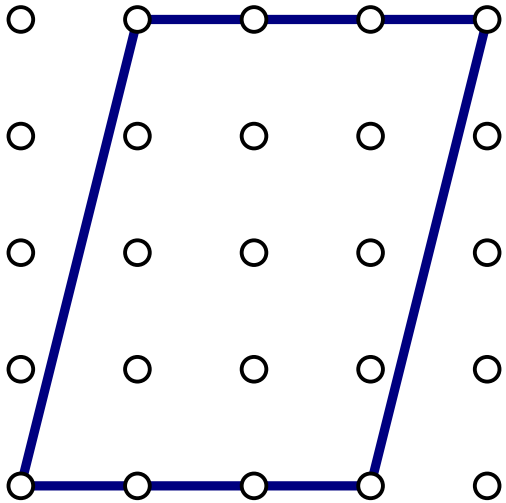
3. Na geoploči dimenzija 5 × 5 napravite sve međusobno nesukladne paralelograme s osnovicom duljine 2. Koliko ih ima?



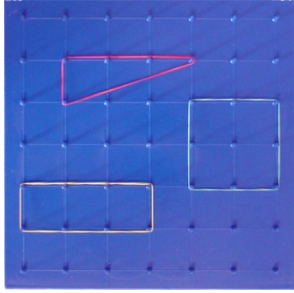
Paralelogrami [6]



4. Na geoploči dimenzija 5 × 5 napravite nepravokutni paralelogram najveće moguće površine. Kolika je ta površina?



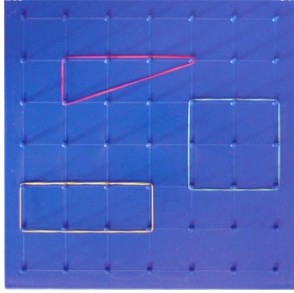
Paralelogrami [7]

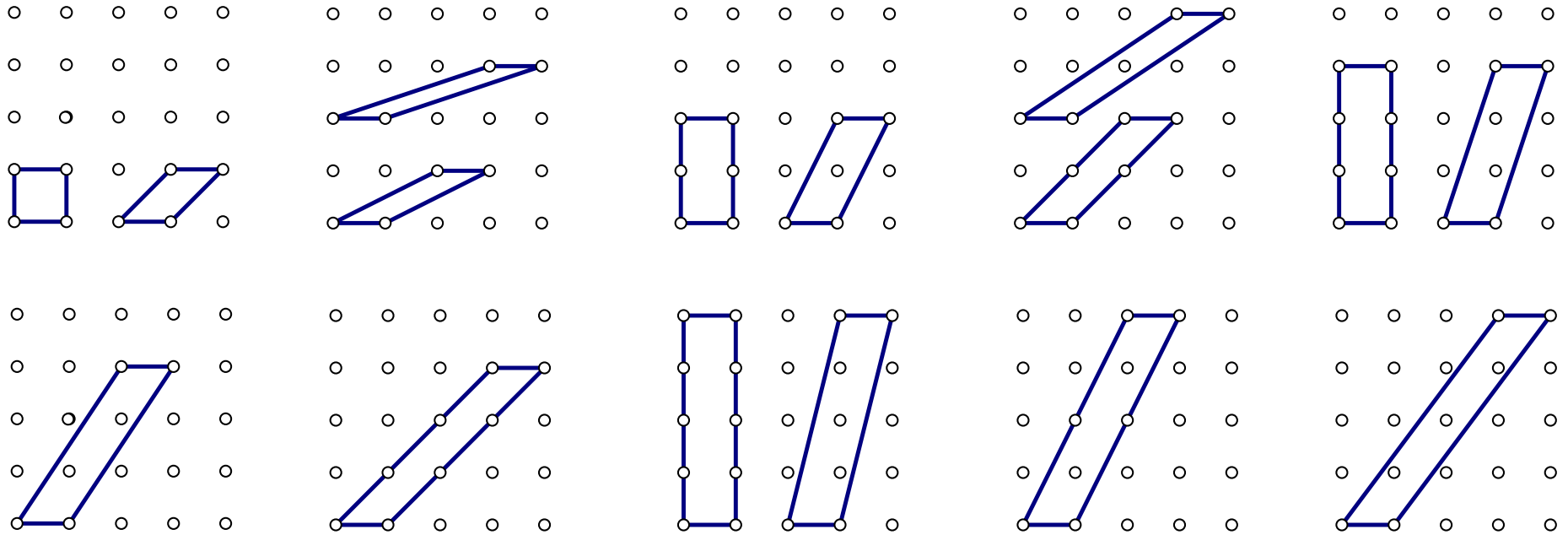


Glavni zadatak:

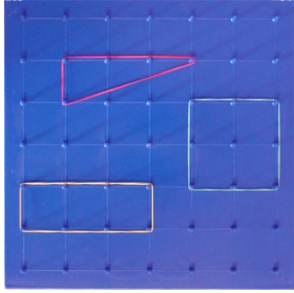
Na geoploči dimenzija 5 × 5 napravite sve međusobno nesukladne paralelograme. Nacrtajte odgovarajuće slike na točkastom papiru.

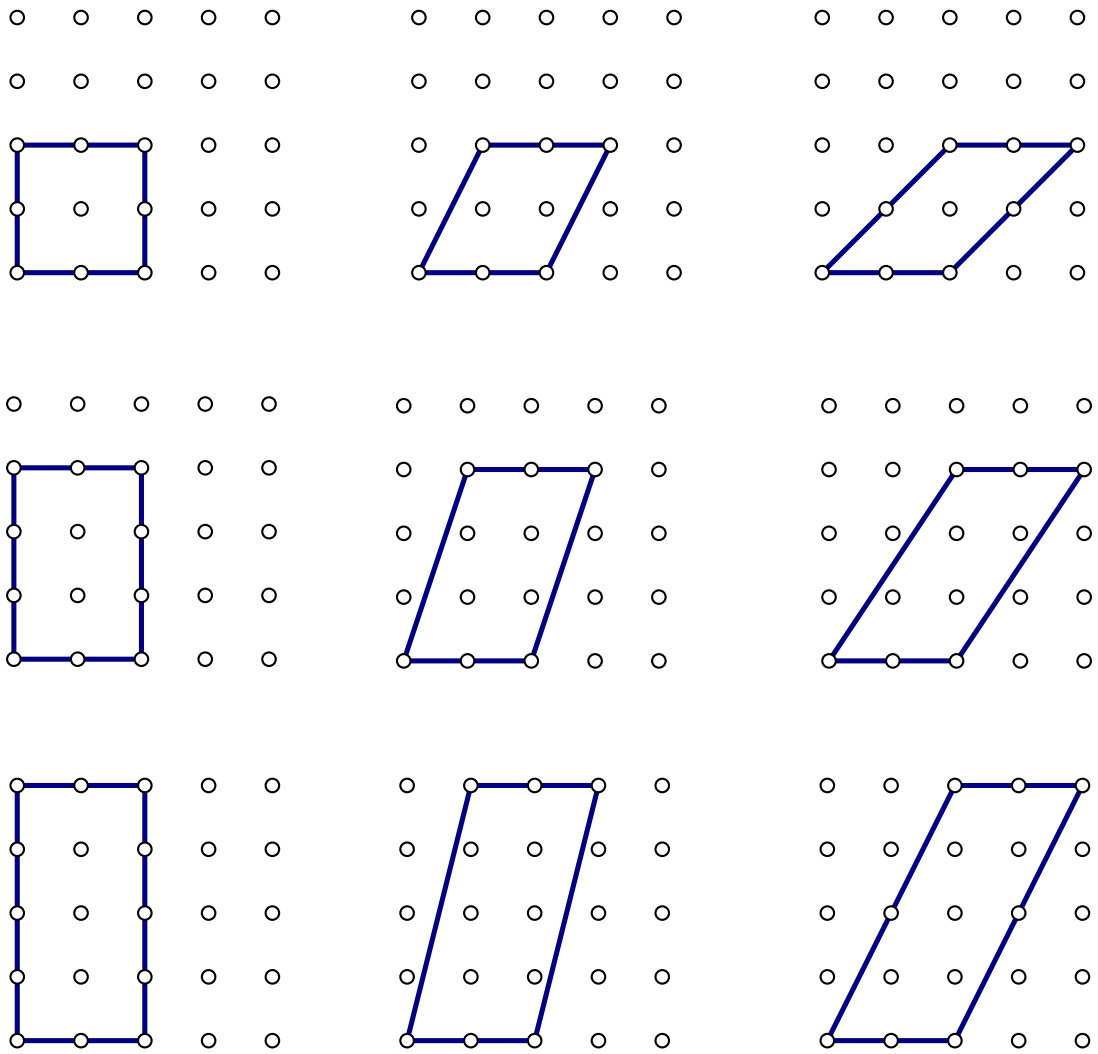
Paralelogrami [8]



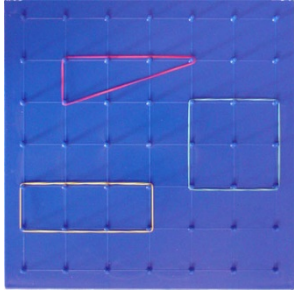


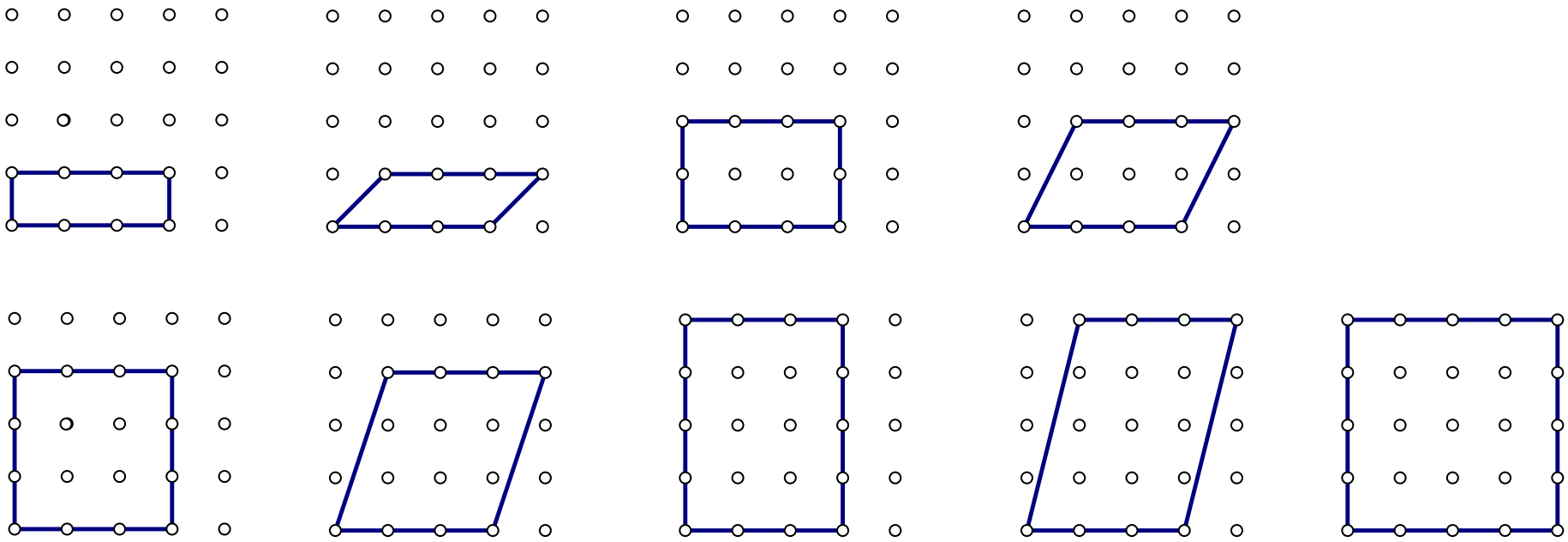
Paralelogrami [9]



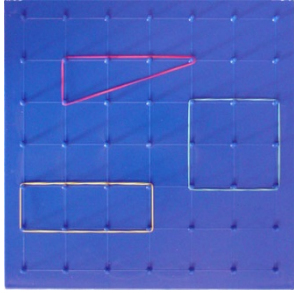


Paralelogrami [10]



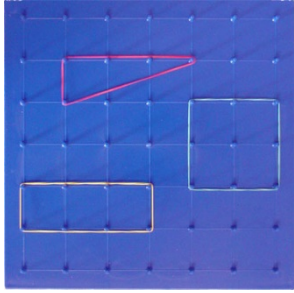


Paralelogrami [11]



Zadatak za domaću zadaću:

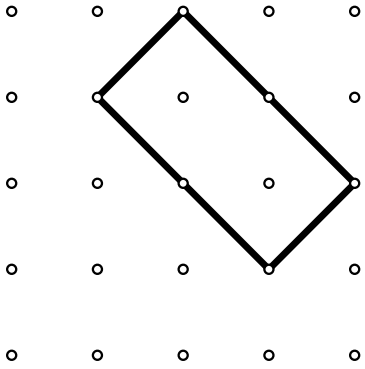
Izračunajte površinu svakoga od paralelograma iz prethodnog zadatka ako je udaljenost dviju susjednih točaka u retku/stupcu jednaka 1 cm.



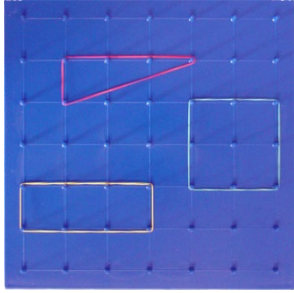
Paralelogrami [12]

Dodatni zadaci:

1. Na geoploči dimenzija 5 × 5 prikažite paralelogram s površinom jednakom kao pravokutnik na slici:



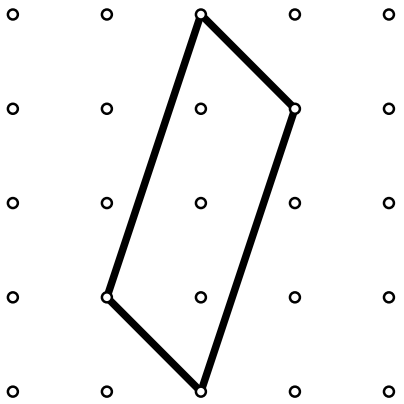
Nacrtajte odgovarajuću sliku na točkastom papiru.



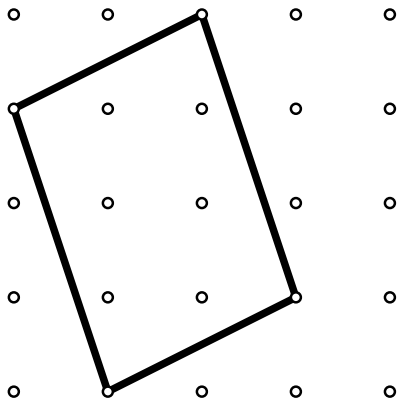
Paralelogrami [13]

2. Na geoploči dimenzija 5 × 5 prikažite paralelogram sukladan zadanom:

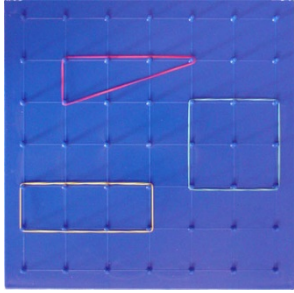
a)



b)



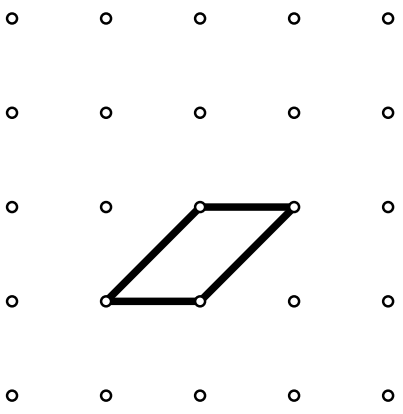
Nacrtajte odgovarajuće slike na točkastom papiru.



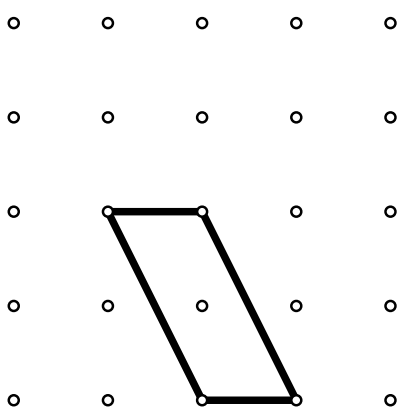
Paralelogrami [14]

3. Na geoploči dimenzija 5 × 5 prikažite paralelogram sukladan zadanom:

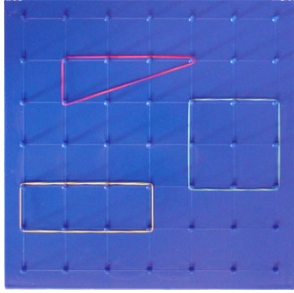
a)



b)

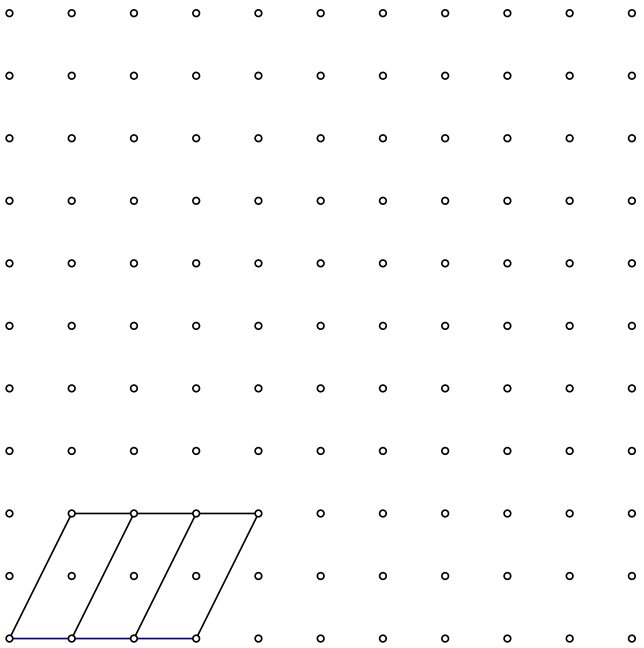


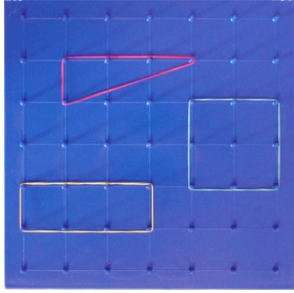
Nacrtajte odgovarajuće slike na točkastom papiru.



Paralelogrami [15]

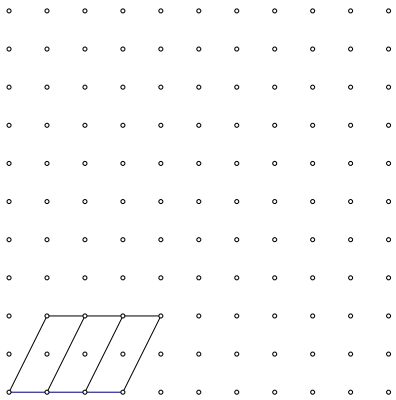
4. Nastavite niz paralelograma na geoploči dimenzija 11 × 11.





Paralelogrami [16]

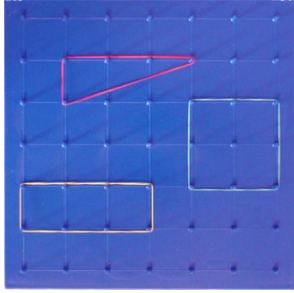
4. Nastavite niz paralelograma na geoploči dimenzija 11 × 11.



Nacrtajte odgovarajuću sliku na točkastom papiru i ispunite tablicu:

Duljina stranice	1	2	3	4	5	6	7	8	9	10
Površina	2	4	6							
Broj čavlića (točaka) na rubu	4	6	8							
Broj čavlića (točaka) u unutrašnjosti	1	2	3							

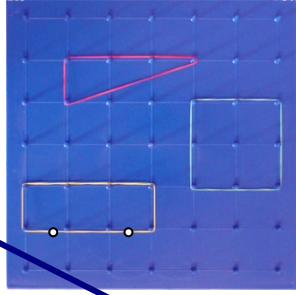
Četverokuti s okomitim dijagonalama [1]

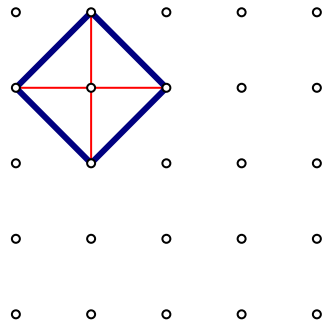


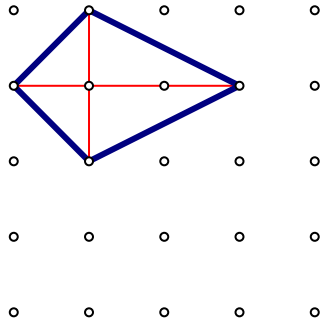
Na geopapiru dimenzija 5×5 nacrtajte sve moguće četverokute s međusobno okomitim dijagonalama tako da su im vrhovi u čvorovima, a dijagonale duž čvorova mreže.

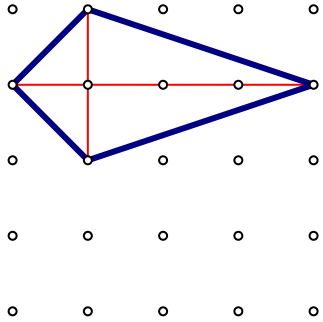
Izračunajte površine nacrtanih četverokuta.

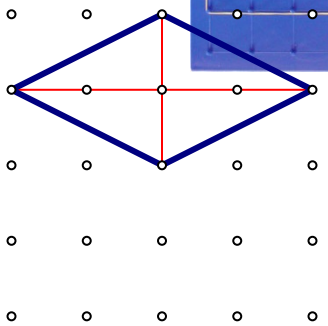
Četverokuti s okomitim dijagonalama [2]

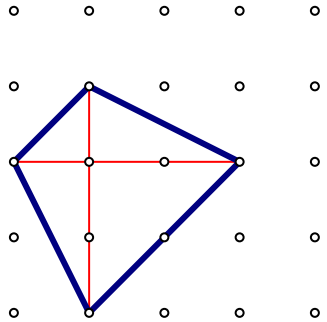


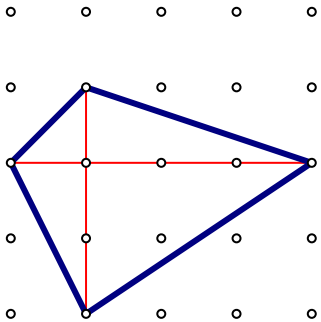


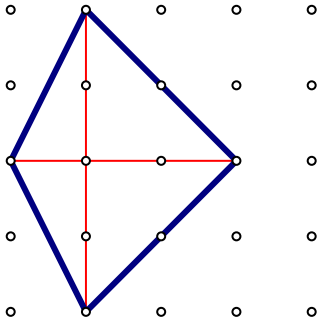


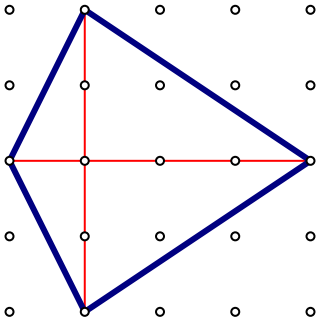


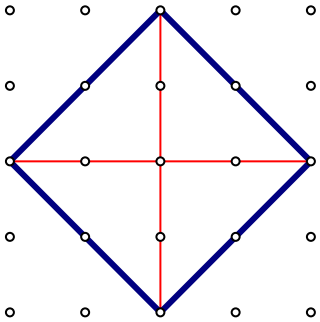


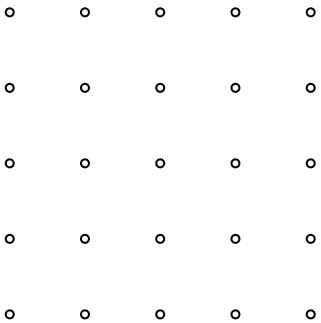


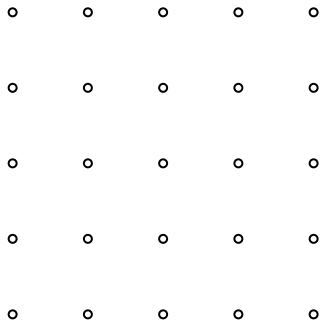


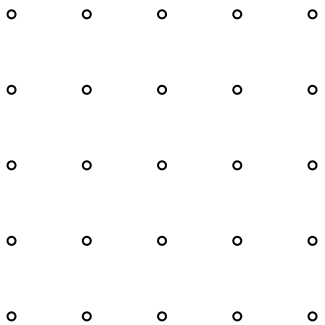




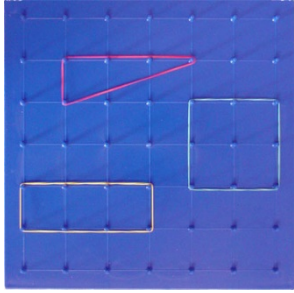






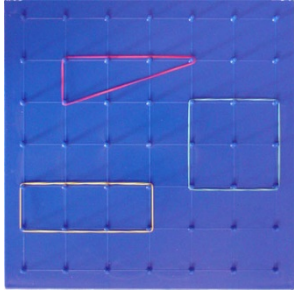


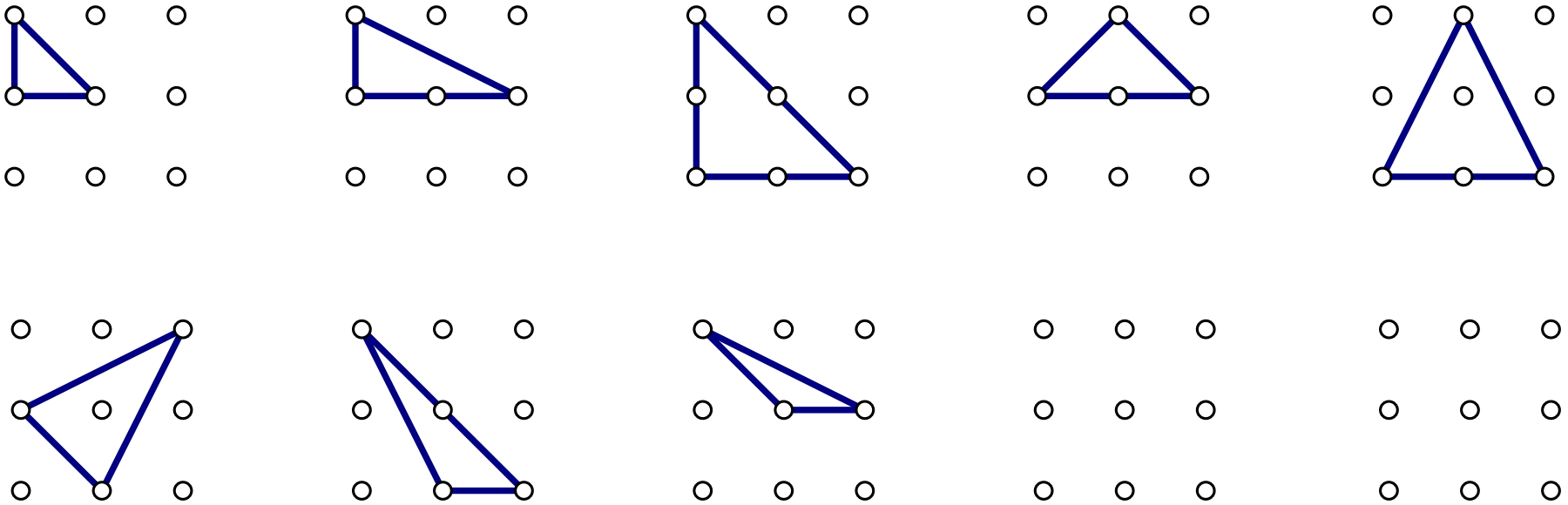
Trokuti [1]



1. U kvadratnoj mreži točaka dimenzija 3 × 3 prikaži sve međusobno različite trokute. Koliko je među njima pravokutnih? Šiljastokutnih? Tupokutnih? Jednakokračnih? Je li moguće prikazati jednakostranični trokut?

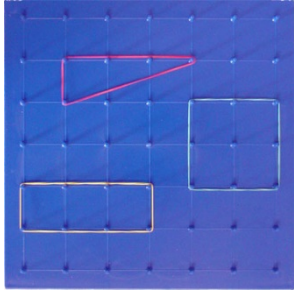
Trokuti [2]





Moguće je prikazati 4 pravokutna trokuta, te po 2 šiljastokutna i tupokutna trokuta. Pri tome je 5 trokuta jednakokračno, a jednakostraničnih trokuta nema.

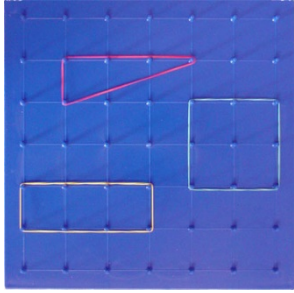
Trokuti [3]

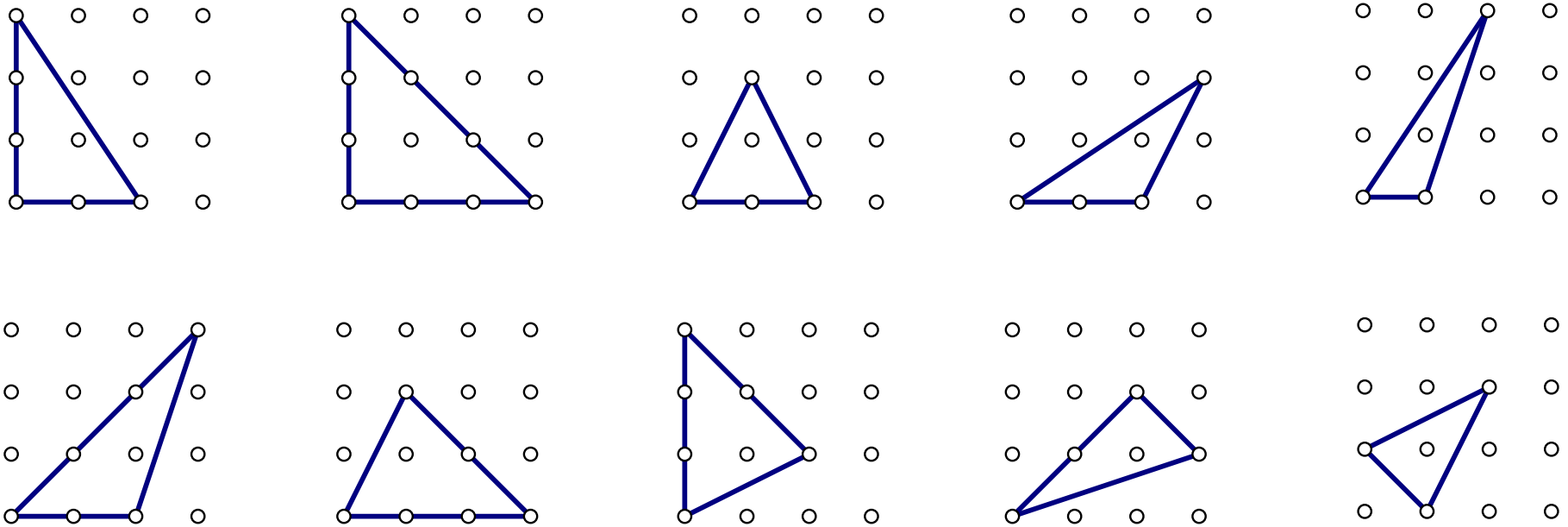


2. U kvadratnoj mreži točaka dimenzija 4 × 4 prikaži sve međusobno nesukladne trokute u čijoj se unutrašnjosti nalazi točno jedan čavlić.

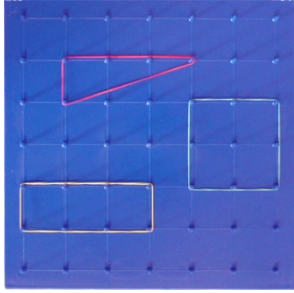
   Koliko ima takvih trokuta?

Trokuti [4]



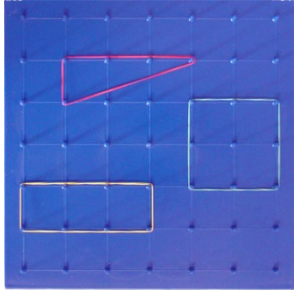


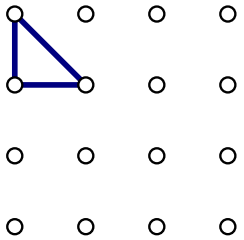
Trokuti [5]

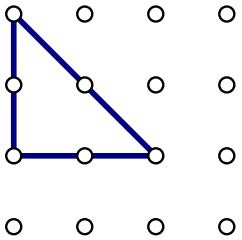


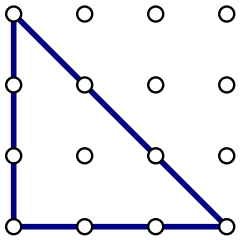
3. U kvadratnoj mreži točaka dimenzija 4 × 4 prikaži sve međusobno nesukladne pravokutne trokute.

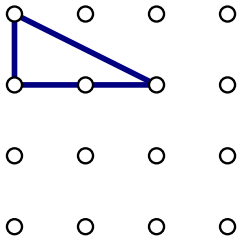
Trokuti [6]

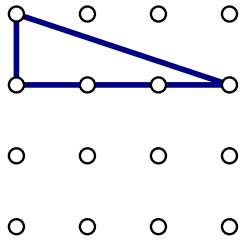


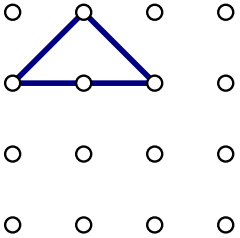


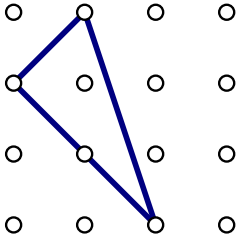


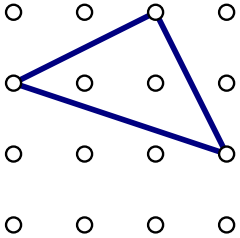


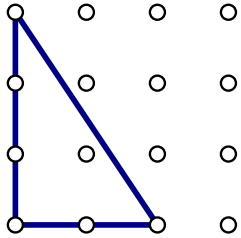


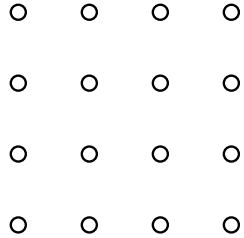




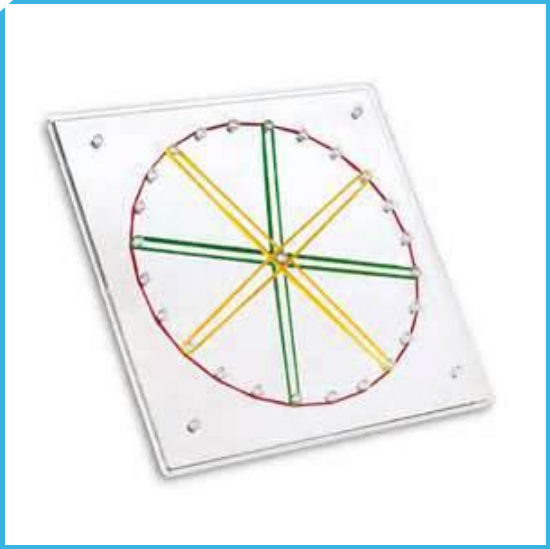


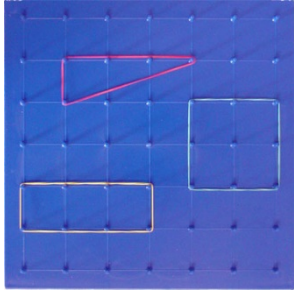






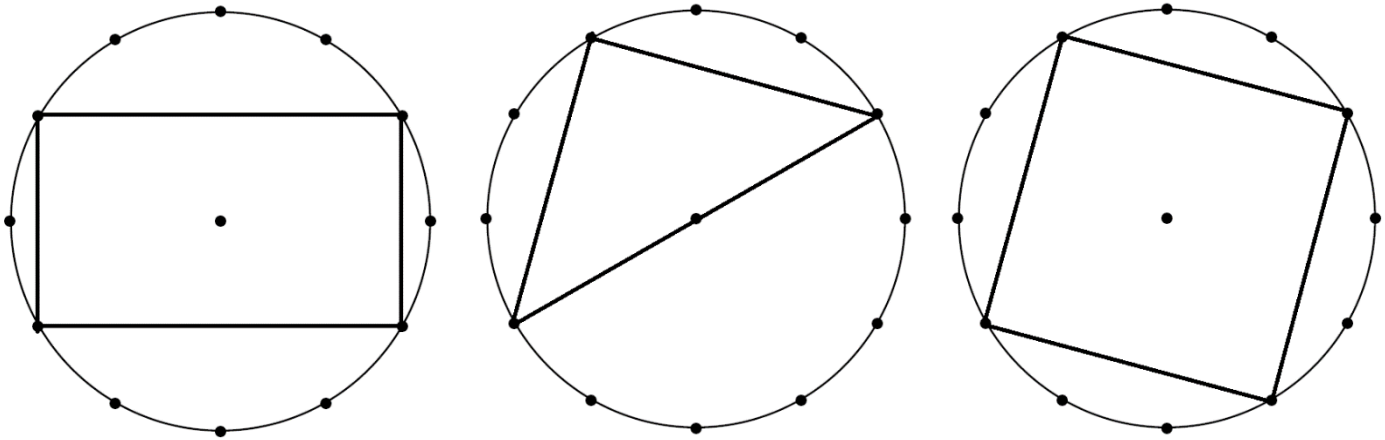
KRUŽNA GEOPLOČA

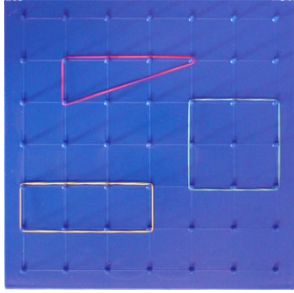




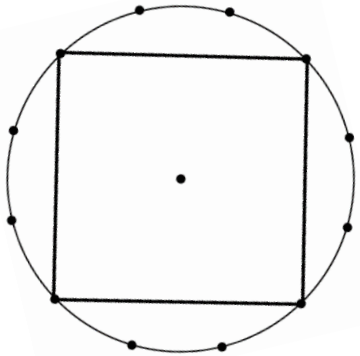
Primjer 1.

Koristeći kružnu geoploču prikaži pravokutnik, trokut i kvadrat.

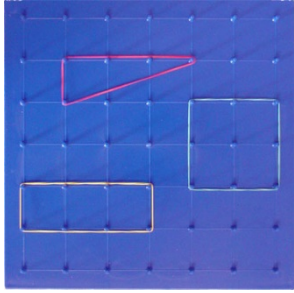




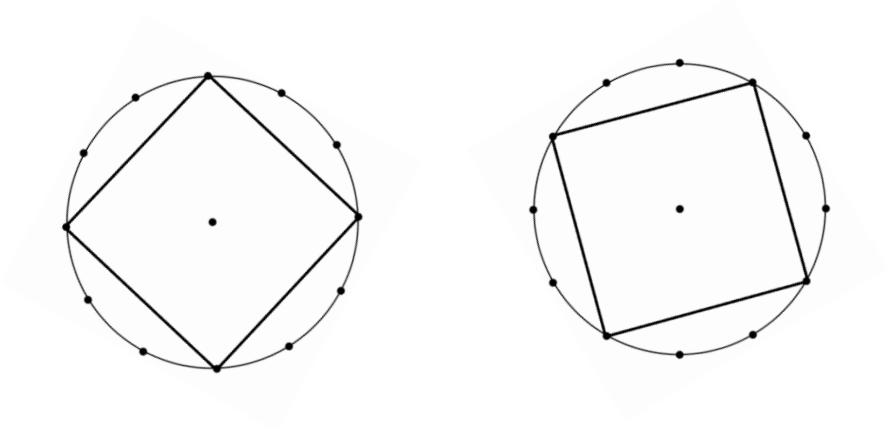
Učenici često kvadrat prepoznaju samo u njima poznatom položaju.

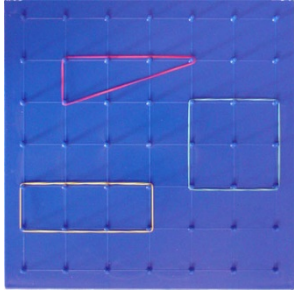


Iz same činjenice da učenici mogu pomicati kružnu geoploču, oni će se susreti i s kvadratima u različitim „položajima“ u ravnini. Jednostavnim zakretanjem ploče, učenici se mogu uvjeriti da su i ti likovi kvadrati.



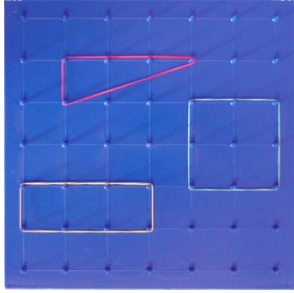
Vrlo brzo učenici će i likove prikazane slikom prepoznati kao kvadrat. Samim time učenicima pomažemo razvijati geometrijsko mišljenje te prelaziti s razine vizualizacije na razinu analize (vidi Van Hieleovu teoriju geometrijskog mišljenja).

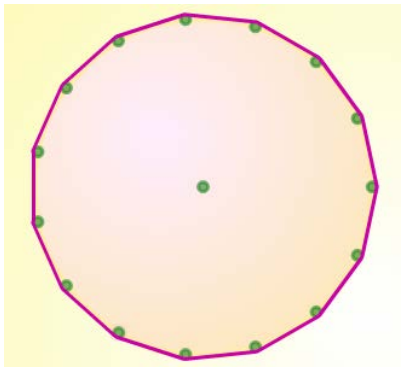


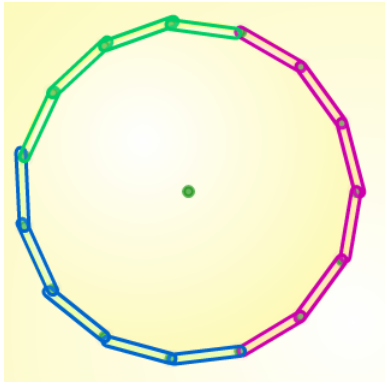


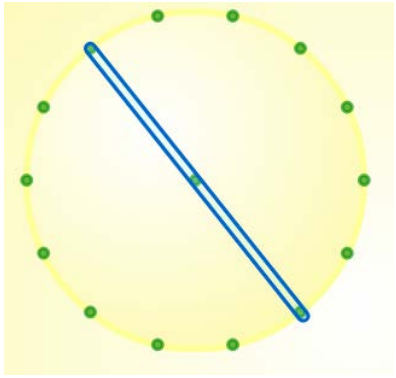
Primjer 2.

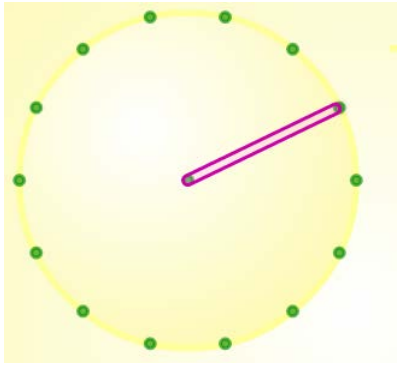
Na kružnoj geoploči prikaži krug, kružnicu, polumjer i promjer Što je dulje, polumjer ili promjer? Koliko puta? Objasni.

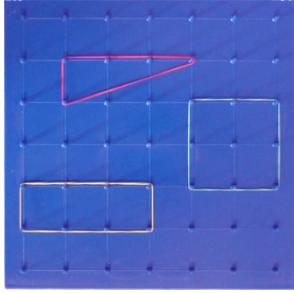












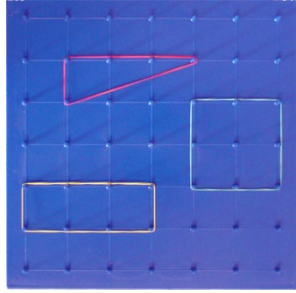
Primjer 3.

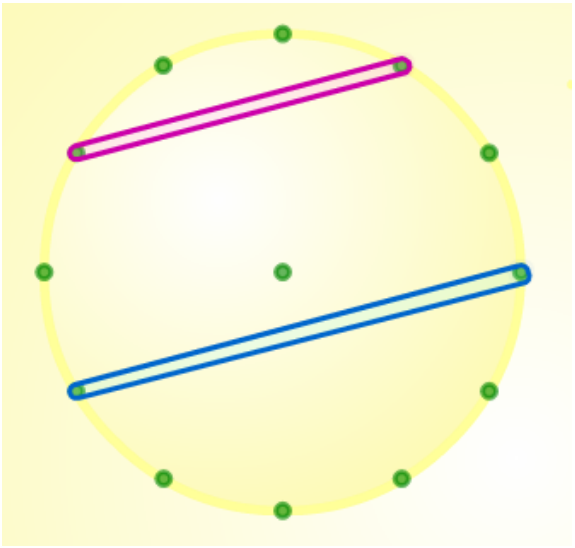
Na kružnoj ploči prikaži dvije usporedne dužine:

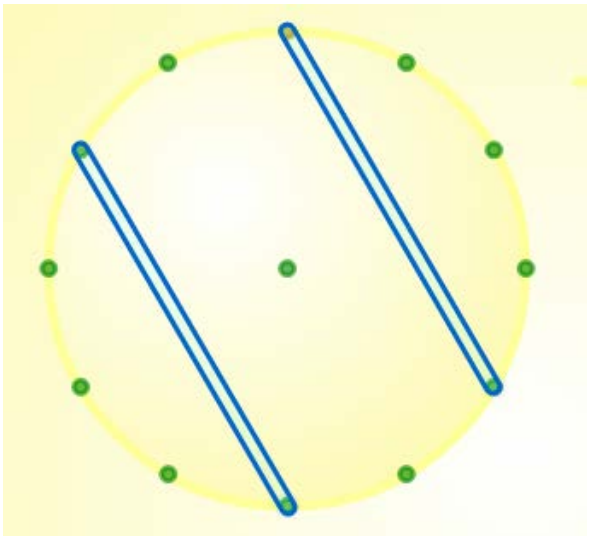
a) različitih duljina,

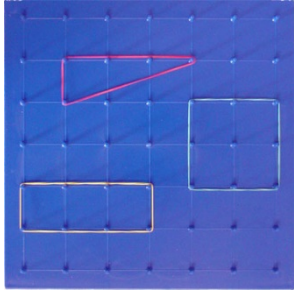
b) istih duljina.

Primjeri zadataka za 3. razred



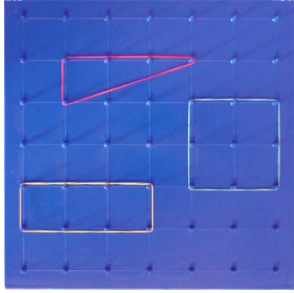




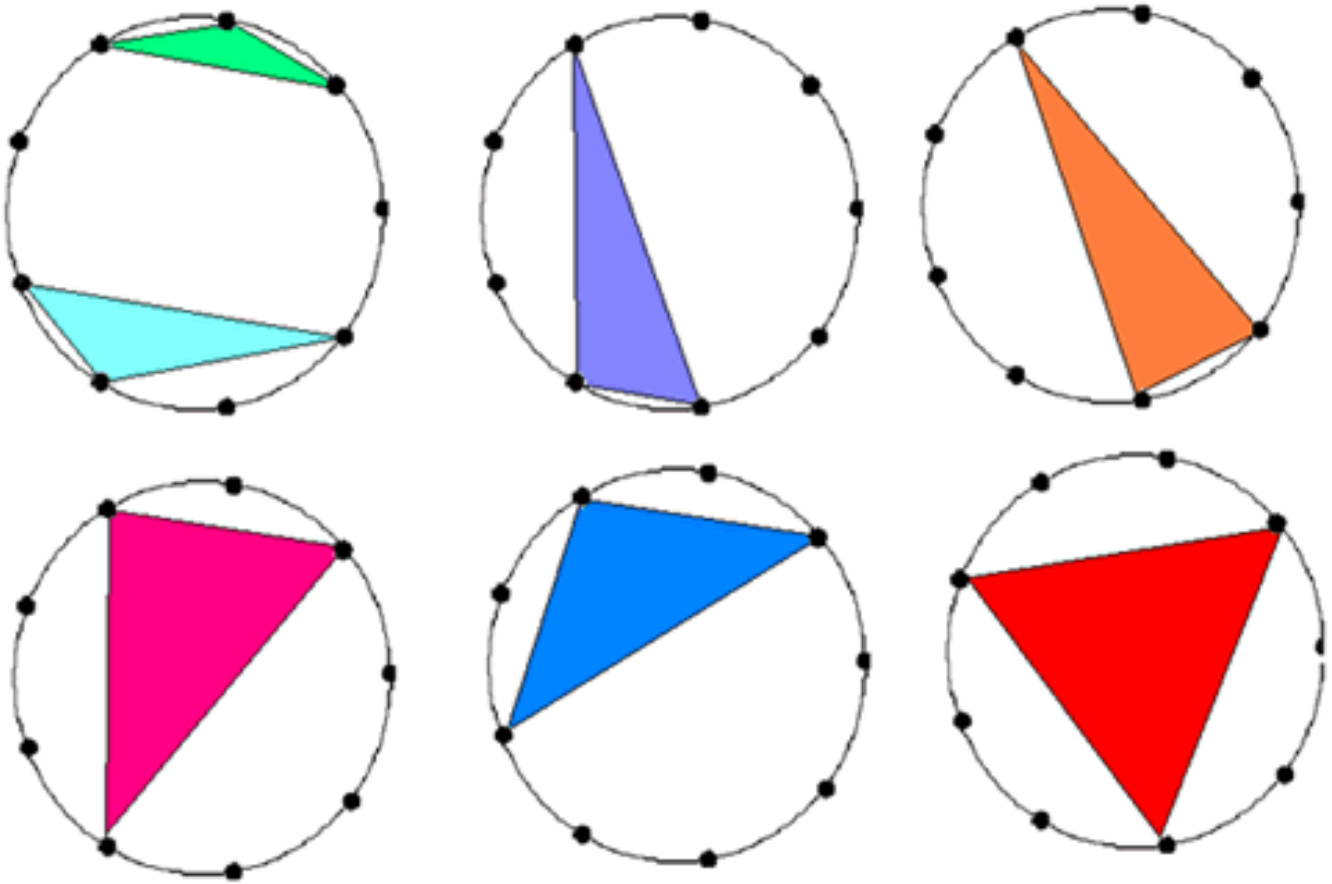


Primjer 4.

Koliko različitih trokuta možemo prikazati na kružnoj geoploči s devet čavlića?



Rješenje:



Final Egad