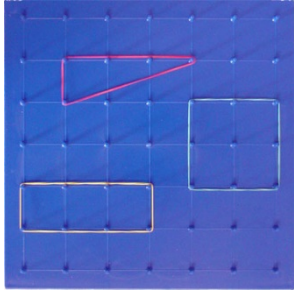
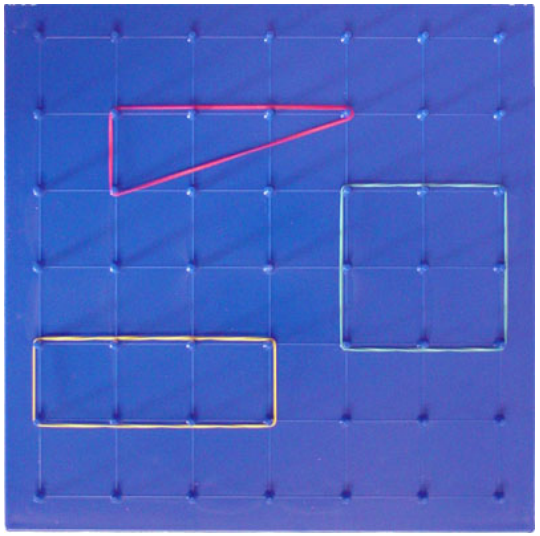
GEOPLOČA

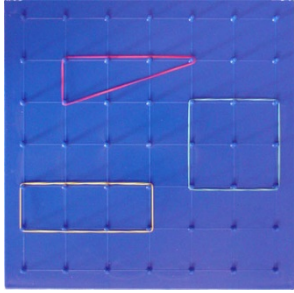
TANJA SOUCIE, IVANA KATALENAC, RENATA SVEDREC



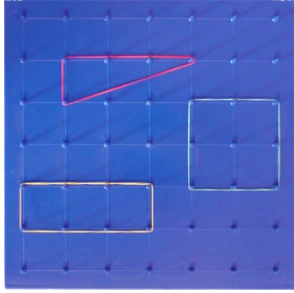
Što je *geoploča*?

Geoploča je (drvena ili plastična) ploča s čavlićima koji su raspoređeni u kvadratnu mrežu oko kojih je moguće rastezati elastične (gumene) vrpce.

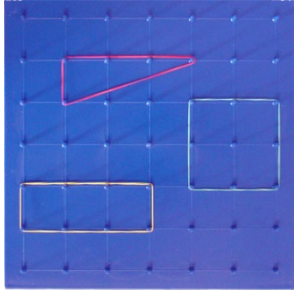




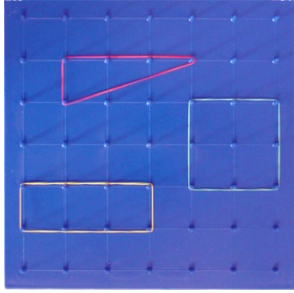
Geoploču je 1952. godine osmislio egipatski matematičar **Caleb Gattegno** (1911. – 1988.), a on je izradio i prve nastavne materijale za primjenu geoploče u nastavi geometrije.



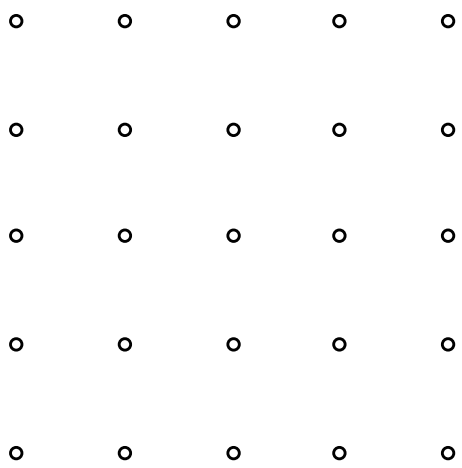
U Europi (i ostatku svijeta) moguće je nabaviti gotove geoploče različitih boja i veličina. Posebno su prikladne prozirne ploče koje je – za potrebe analiziranja problema i/ili diskusije rješenja – moguće staviti na grafoskop.

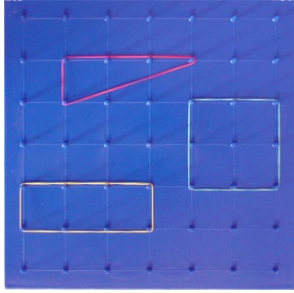


Geoploču je moguće napraviti i u „kućnoj radinosti“ – od drva ili deblje plutene ploče s 25 čavlića raspoređenih u kvadratnu mrežu (5 × 5), na jednakim razmacima (udaljenostima), primjerice 1 cm.

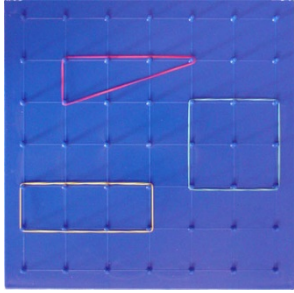


Umjesto rada na konkretnoj (originalnoj ili priručnoj) geoploči, moguće je koristiti i samo *točkasti papir.*

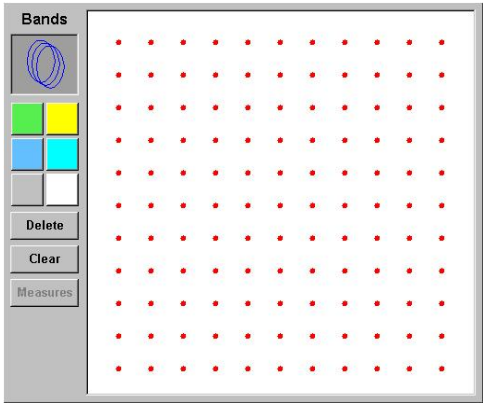


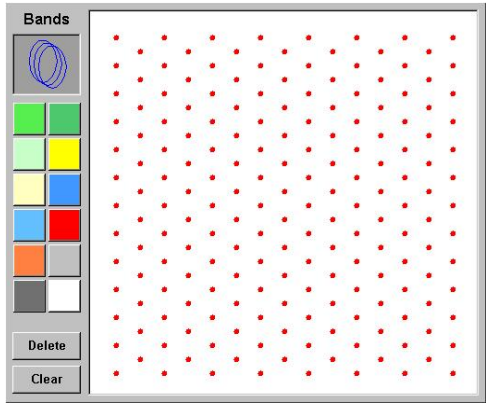


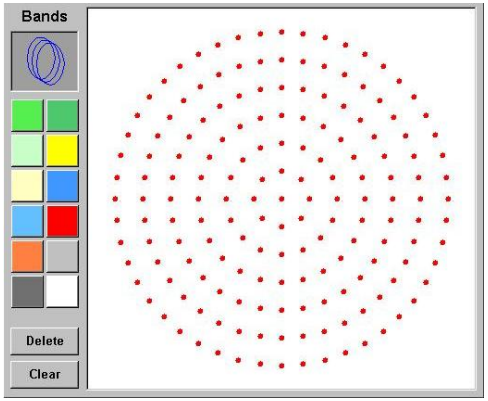
Ukoliko je dostupno, učenici u razredu ili kod kuće mogu koristiti i elektronske aplikacije koje su (besplatno) dostupne na različitim internetskim adresama (kao što su npr. http://nrich.maths.org/5648, http://mste.illinois.edu/users/pavel/java/geoboard/, http://nlvm.usu.edu/en/nav/frames_asid_172_g_2_t_3.html http://www.mathplayground.com/geoboard.html)



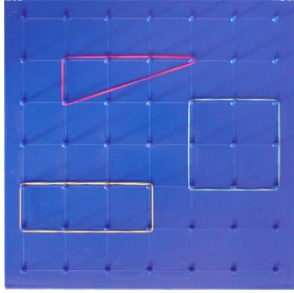
Osim klasičnih *geoploča* koriste se još tzv. *izometričke geoploče* (čavlići raspoređeni u mrežu rombova sa šiljastim kutom veličine 60°, i *kružne geoploče*.



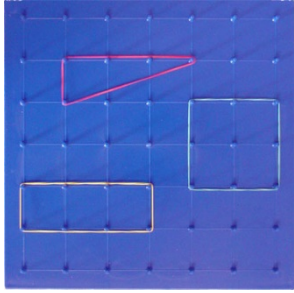




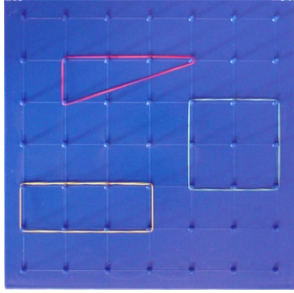
Zašto koristiti *geoploču*?



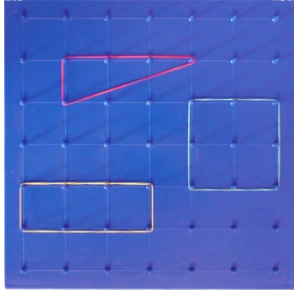
Kao nastavno pomagalo geoploča je vrlo primjenjiva u osnovnoškolskoj nastavi matematike kao pomoć pri istraživanju koncepata kao što su opseg, površina, svojstva geometrijskih likova, preslikavanja ravnine, pojam razlomka i postotka te crtanje likova u koordinatnom sustavu.



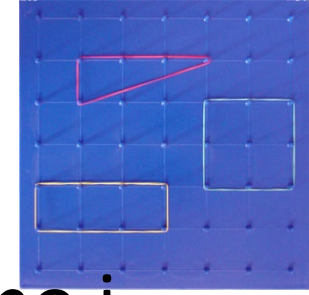
Geoploča učenicima omogućava vizualizaciju te im daje mogućnost da „rade“ geometriju. Pruža im mogućnost da samostalno otkrivaju matematiku te razmjenjuju matematičke ideje čime razvijaju svoje komunikacijske vještine i matematički rječnik.



Korištenje geoploče učenicima omogućuje postavljanje i rješavanje matematičkih problema, potiče ih na istraživanje, sustavnost, kreativnost i ustrajnost u radu, tako da oni postaju aktivni sudionici u procesu učenja.



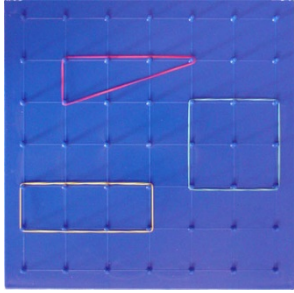
Nadalje, *Nacionalni okvirni kurikulum za matematičko područje* od učenika očekuje da će, između ostaloga, moći stvarati i istraživati pretpostavke o matematičkim objektima, pravilnostima i odnosima; skicirati jednostavne ravninske oblike te ih nacrtati i konstruirati pomoću geometrijskog pribora (i jednostavnoga računalnoga programa za crtanje);



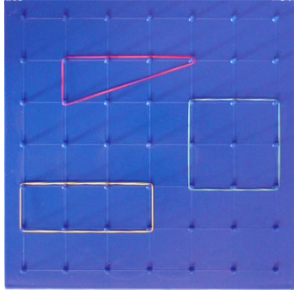
prepoznati sukladne i slične trokute; približno i točno odrediti udaljenost dviju točaka i površinu likova brojanjem jediničnih dužina i jediničnih kvadrata; prepoznati, imenovati, izgraditi i klasificirati ravninske (i prostorne) geometrijske oblike te istražiti, uočiti i (precizno) opisati njihova geometrijska svojstva… [3]

Korištenje geoploče u nastavi matematike pridonijet će ostvarivanju postavljenih ciljeva.

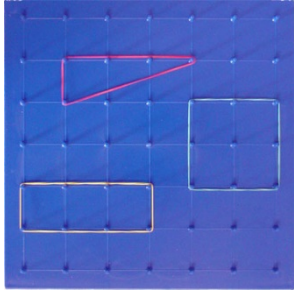
Potrebni materijali



Za rad u paru poželjno je svakom paru učenika osigurati *geoploču*, dovoljan broj elastičnih gumenih vrpci (različitih boja i duljina), a svaki će učenik trebati dovoljno *točkastog papira*, ravnalo ili trokut te olovku i gumicu za brisanje.



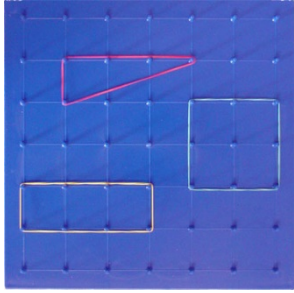
Kad upoznajemo učenike s novim nastavnim pomagalom, svakako im trebamo dati vremena da se s njim upoznaju. Nakon što učencima postavimo pitanja kao štu su *Kojeg je oblika geoploča?, Koliko se čavlića nalazi u svakom retku, a koliko u svakom stupcu?* te *Koliko se ukupno čavlića nalazi na geoploči?* možemo im zadati neke od sljedećih tipova zadataka.



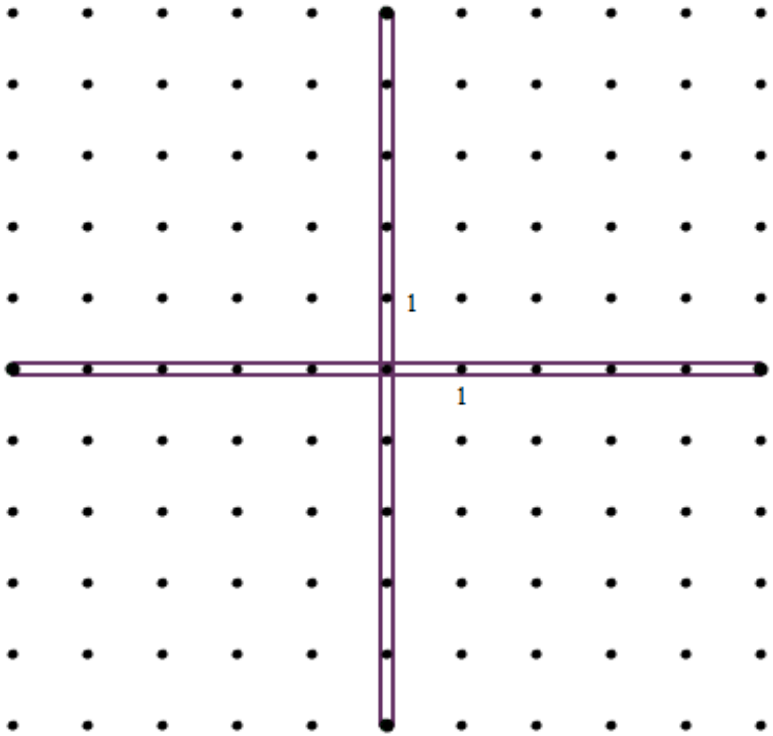
7. I 8. RAZRED

JEDNOSTAVNE IDEJE

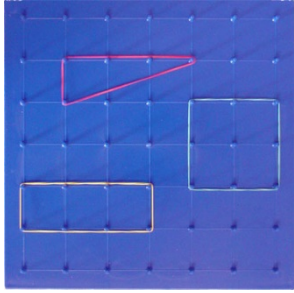
Koordinatni sustav [1]



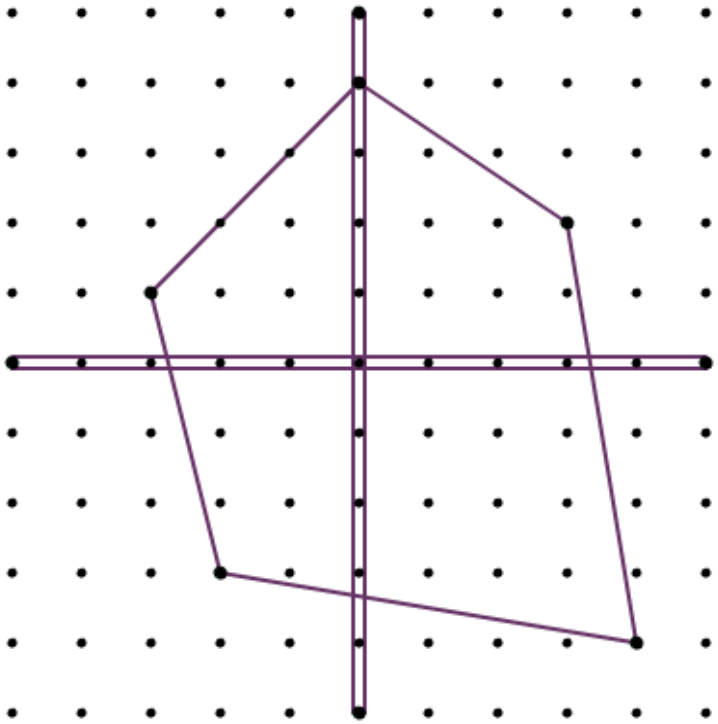
Koristeći dvije elastične vrpce na geoploči učenici definiraju koordinatni sustav u ravnini.

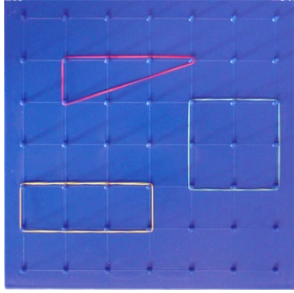


Koordinatni sustav [2]



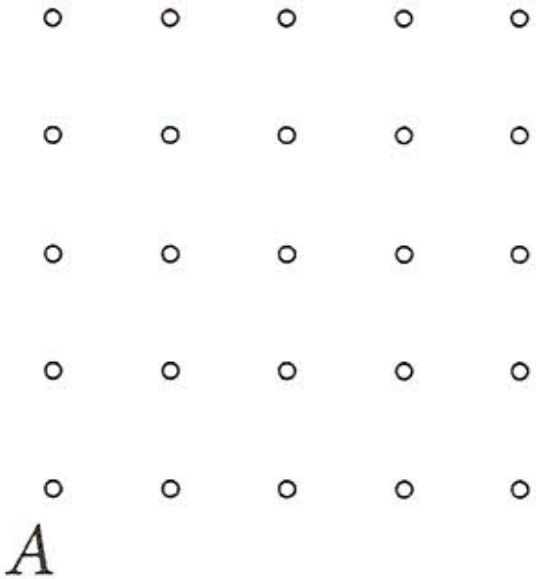
- U koordinatnom sustavu napravljenom na geoploči prikažite lik čiji su vrhovi zadani koordinatama (-2, -3), (-3, 1), (0, 4), (3, 2), (4, -4).



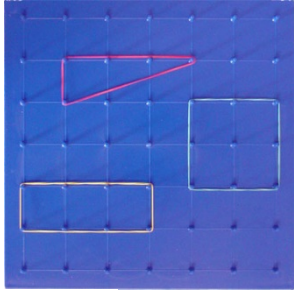


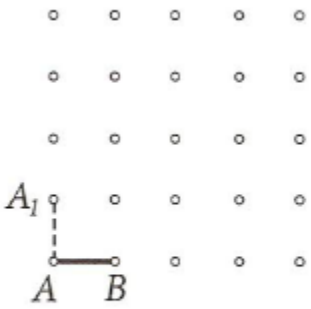
Duljina dužine [1]

1. Na geoploči dimenzija 5x5 prikaži sve dužine međusobno različitih duljina kojima je jedna rubna točka na slici označena slovom A. Kolike su duljine tih dužina ako je udaljenost dviju susjednih točaka u retku (stupcu) 1 cm?

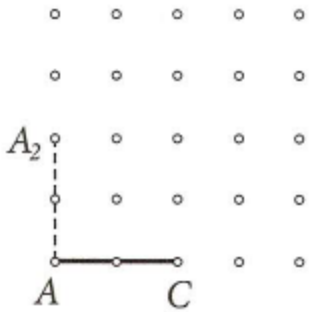


Duljina dužine [2]

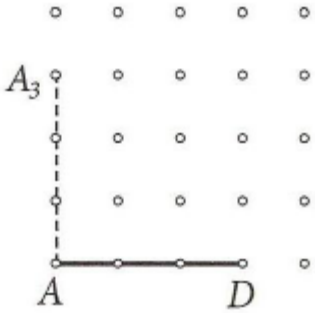




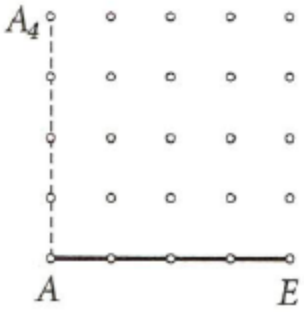
$|AB| = |AA_1| = 1$ cm



$|AC| = |AA_2| = 2$ cm

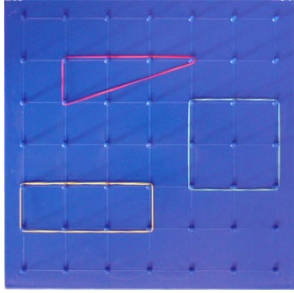


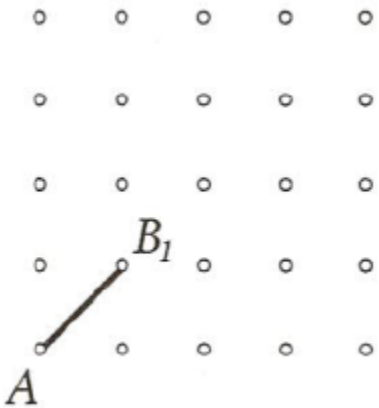
$|AD| = |AA_3| = 3$ cm



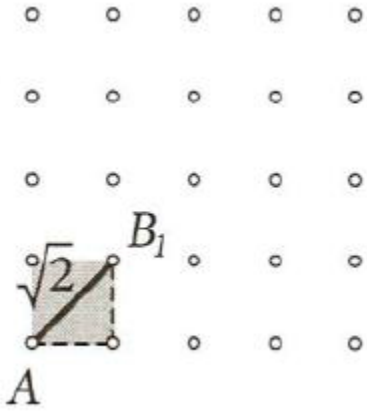
$|AE| = |AA_4| = 4$ cm

Duljina dužine [3]

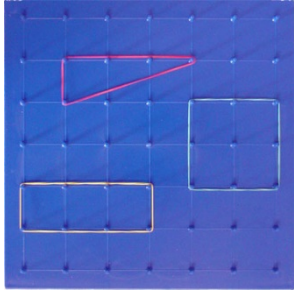


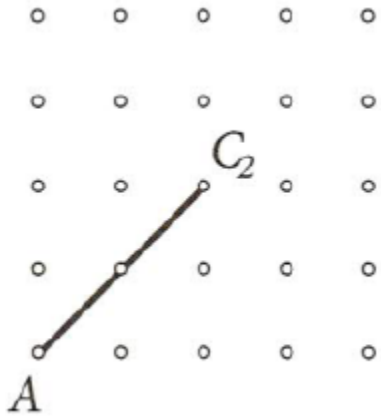


$$|AB1|^2 = 1^2 + 1^2 \Rightarrow |AB1| = \sqrt{2}$$

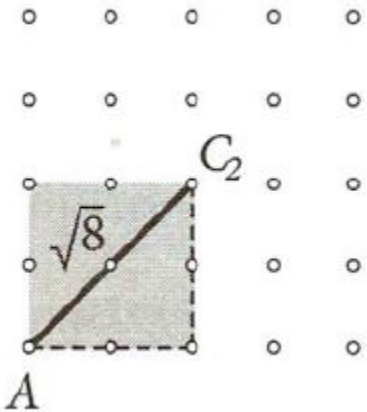


Duljina dužine [4]

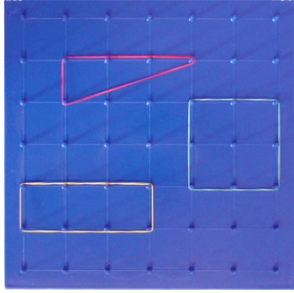


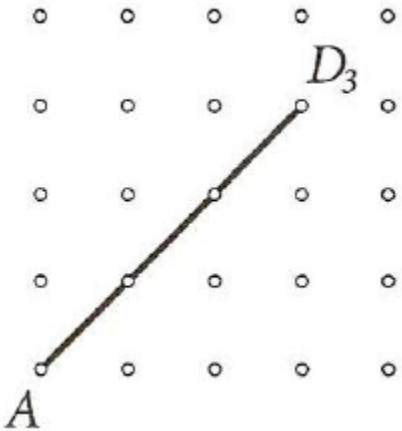


$$|AC2|^2 = 2^2 + 2^2 => |AC2| = \sqrt{8}$$

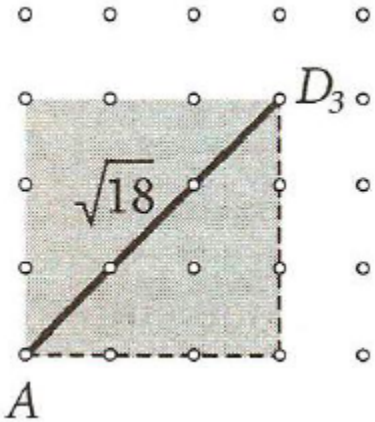


Duljina dužine [5]

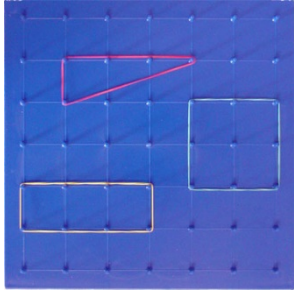


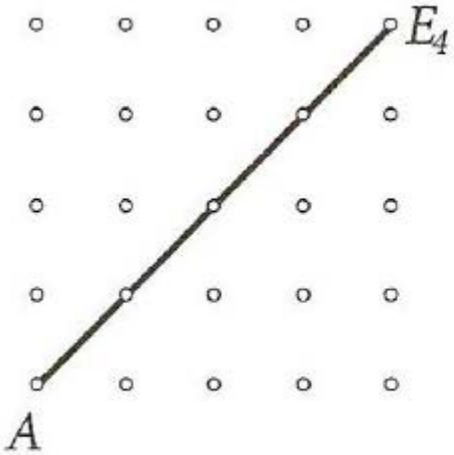


$$|AD3|^2 = 3^2 + 3^2 \Rightarrow |AD3| = \sqrt{18}$$

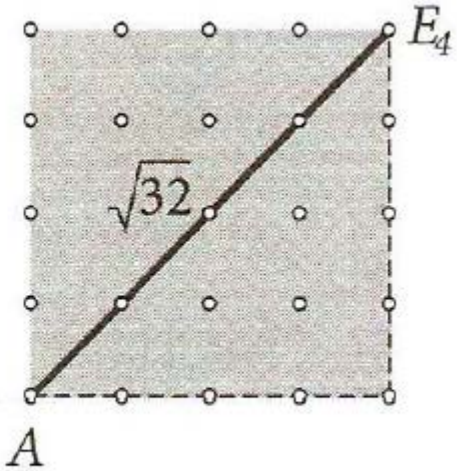


Duljina dužine [6]

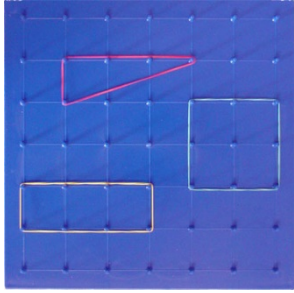


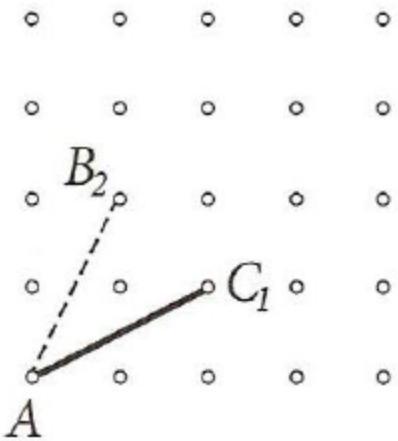


$$|AE4|^2 = 4^2 + 4^2 => |AE4| = \sqrt{32}$$

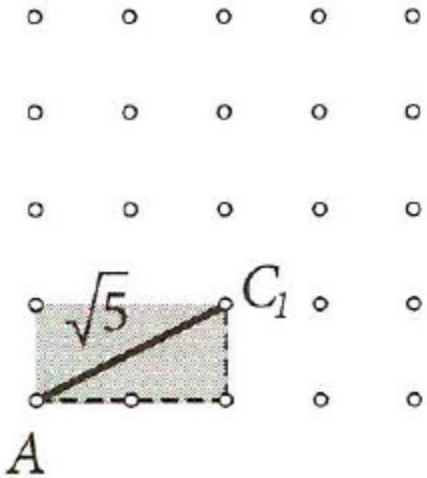


Duljina dužine [7]

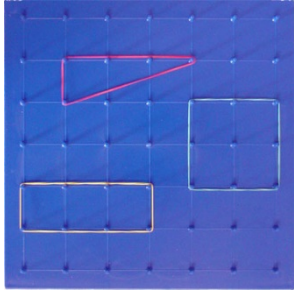


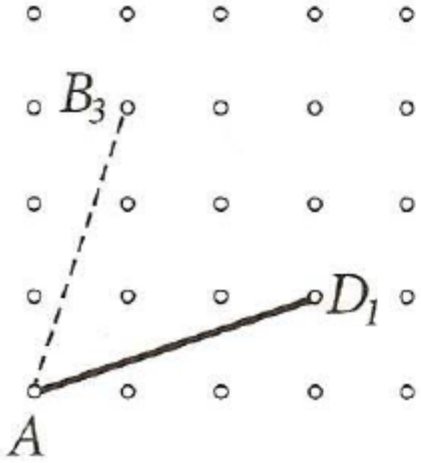


$$|AC1|^2 = 2^2 + 1^2 => |AC1| = \sqrt{5}$$

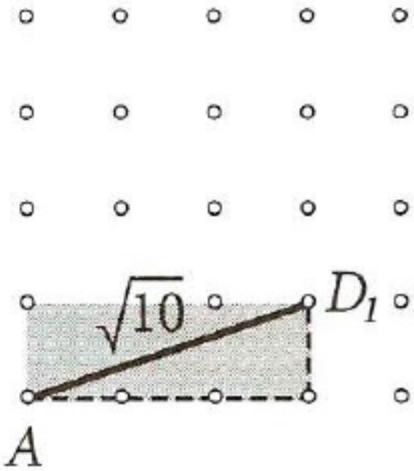


Duljina dužine [8]

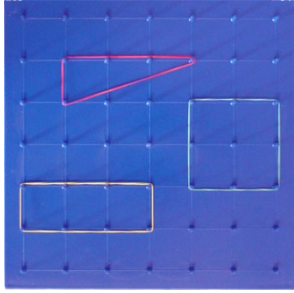


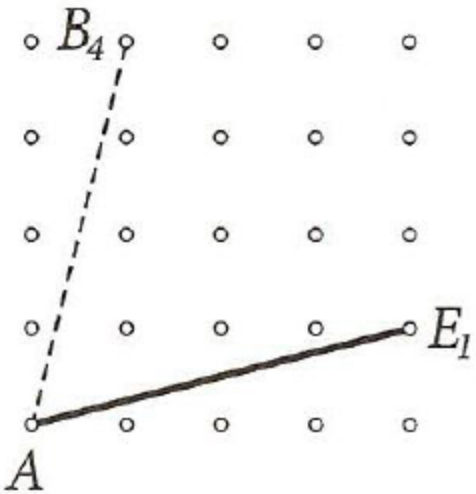


$$|AD1|^2 = 3^2 + 1^2 => |AD1| = \sqrt{10}$$

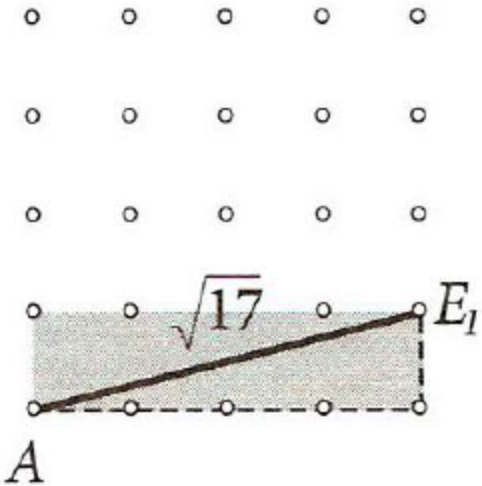


Duljina dužine [9]

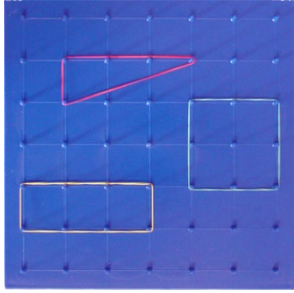


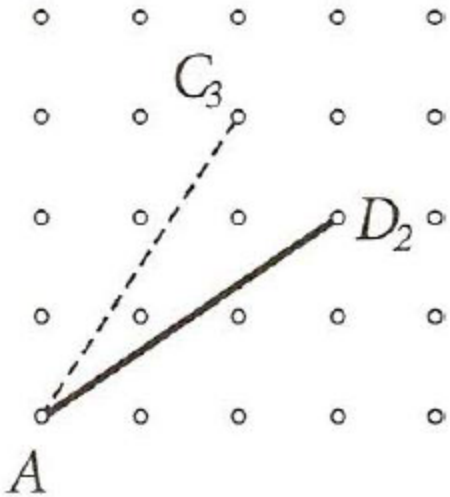


$$|AE1|^2 = 4^2 + 1^2 => |AE1| = \sqrt{17}$$

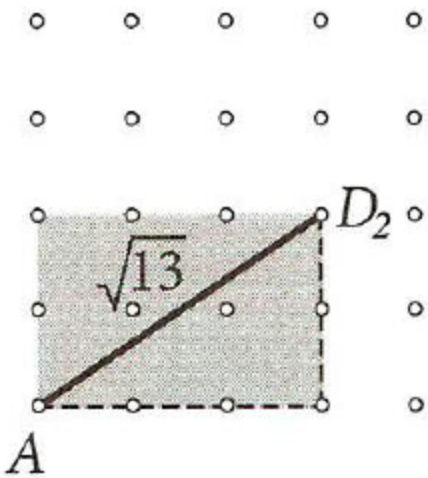


Duljina dužine [10]

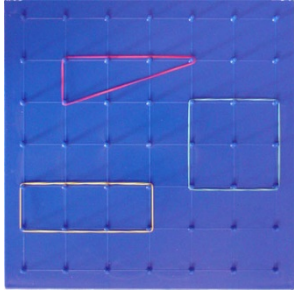


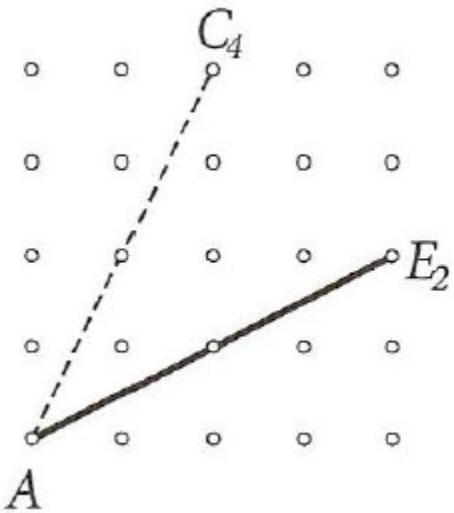


$$|AD2|^2 = 3^2 + 2^2 => |AD2| = \sqrt{13}$$

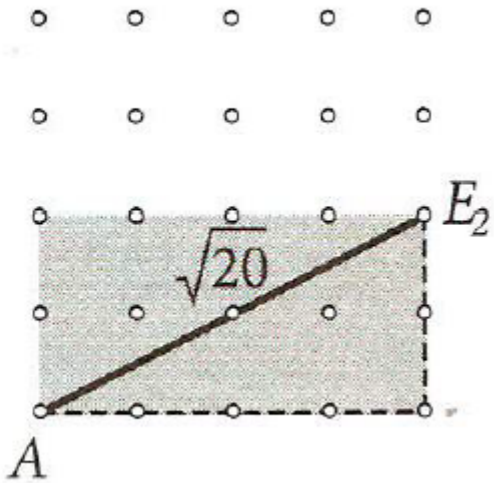


Duljina dužine [11]

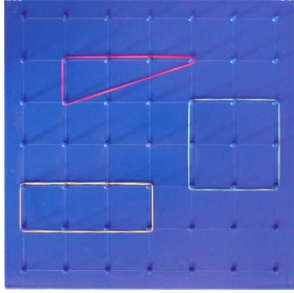


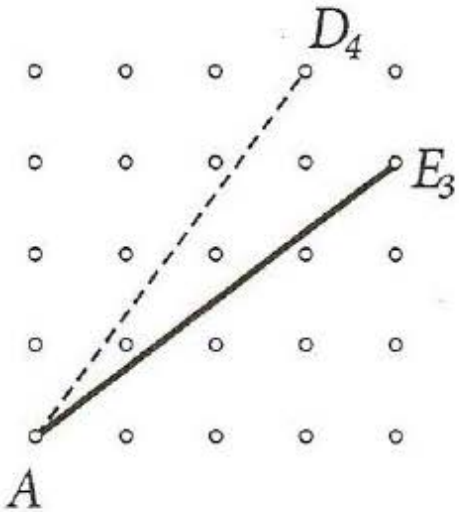


$$|AE2|^2 = 4^2 + 2^2 => |AE2| = \sqrt{20}$$

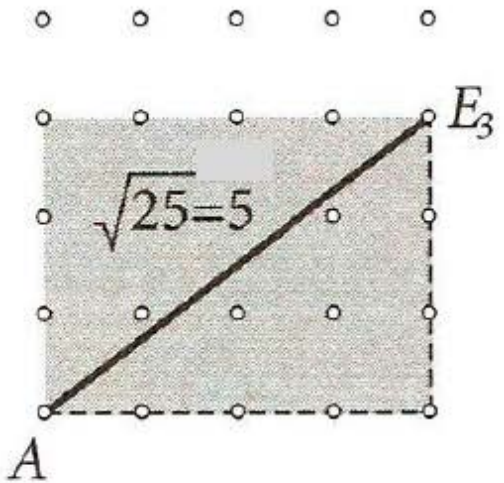


Duljina dužine [12]



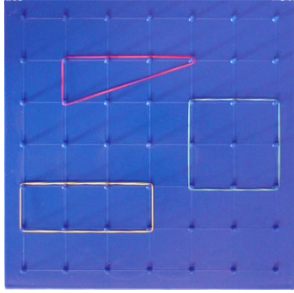


$$|AE3|^2 = 4^2 + 3^2 => |AE3| = \sqrt{25} = 5$$

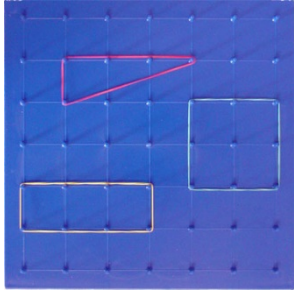


Egipatski trokut (3, 4, 5)

Duljina dužine [13]

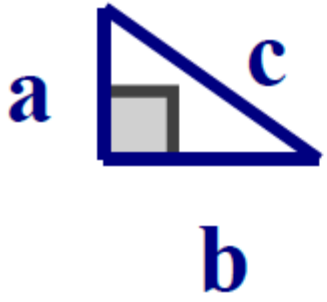


2. Kako bismo konstruirali dužinu duljine $\sqrt{3}$?



Duljina dužine [14]

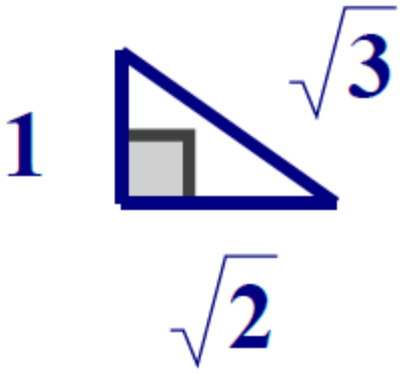
1. način: hipotenuza je duljine $\sqrt{3}$

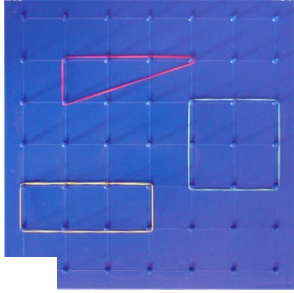


$$c^2 = a^2 + b^2$$

$$\sqrt{3}^2 = \_^2 + \_^2$$

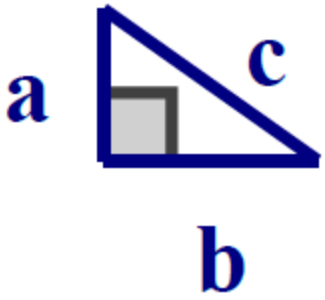
$$=> \sqrt{3}^2 = 1^2 + \sqrt{2}^2$$





Duljina dužine [15]

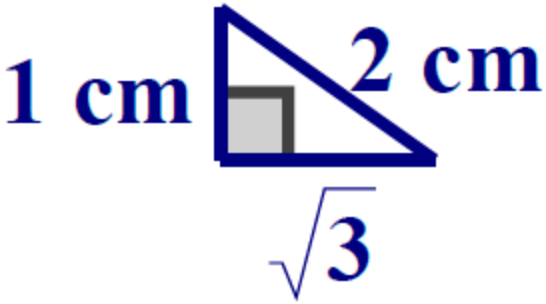
2. način: kateta je duljine $\sqrt{3}$



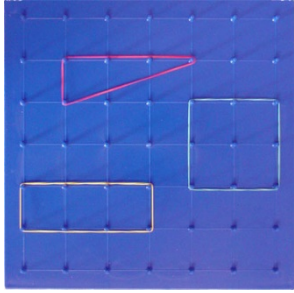
$$c^2 = a^2 + b^2$$

$$\_^2 = \sqrt{3}^2 + \_^2$$

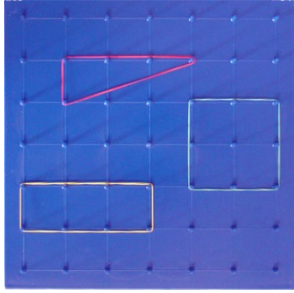
$$=> 2^2 = \sqrt{3}^2 + 1^2$$



Duljina dužine [16]



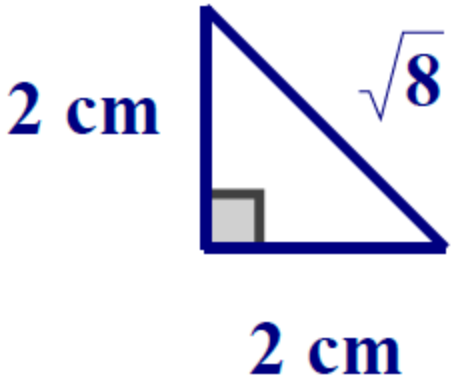
3. Promotrite sliku na kojoj se nalazi dužina duljine $\sqrt{8}$. Kako bismo konstruirali dužinu duljine $\sqrt{8}$ na neki drugi način?



Duljina dužine [17]

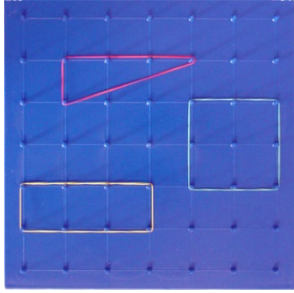
Ponovimo 1. način:

katete duljine 2 cm→hipotenuza duljine $\sqrt{8}$ cm



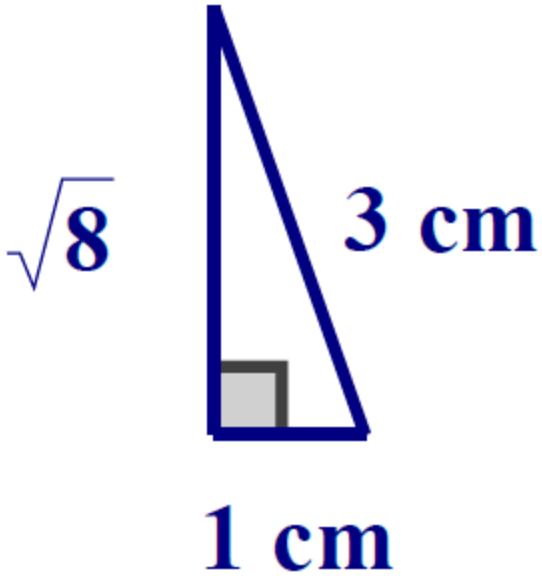
$$\sqrt{8}^2 = 2^2 + 2^2$$

Duljina dužine [18]



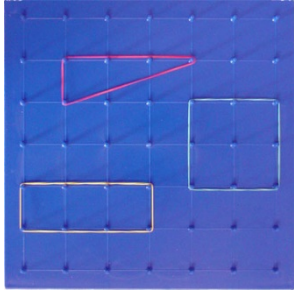
2. način: kateta duljine 1 cm i hipotenuza duljine 3 cm

→ druga kateta duljine $\sqrt{8}$ cm



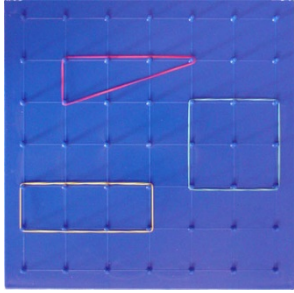
$$3^2 = 1^2 + \sqrt{8}^2$$

Duljina dužine [19]



4. Kako bismo na dva različita načina konstruirali dužinu duljine $\sqrt{12}$?

Duljina dužine [20]



4. Kako bismo na dva različita načina konstruirali dužinu duljine $\sqrt{12}$?

Rješenje:

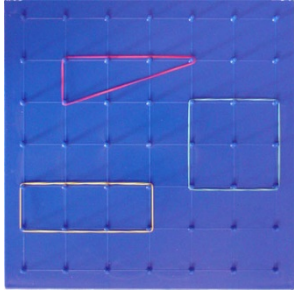
1. način:

$$4^2 = 2^2 + \sqrt{12}^2$$

$$\sqrt{12}^2 = 3^2 + \sqrt{3}^2$$

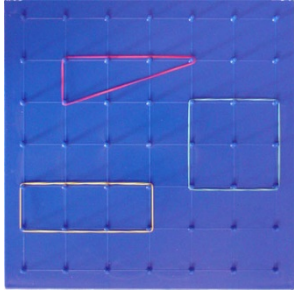
2. način:

Četverokut [1]



Na priloženom listu 3 × 3 „točkastog papira“ nacrtajte što više (sve) međusobno nesukladnih četverokuta kojima su vrhovi u točkama mreže.

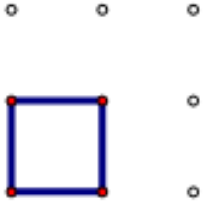
Izračunajte opseg i površinu svakoga četverokuta iz prethodnog zadatka ako je udaljenost dviju susjednih točaka u retku/stupcu jednaka 1 cm.



Rješenja

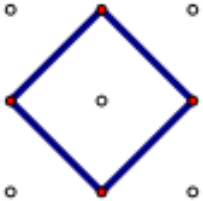
$o = 8$ cm
$p = 4$ cm^2

1



$o = 4$ cm
$p = 1$ cm^2

2

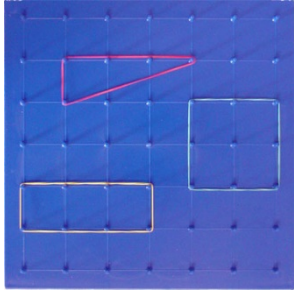


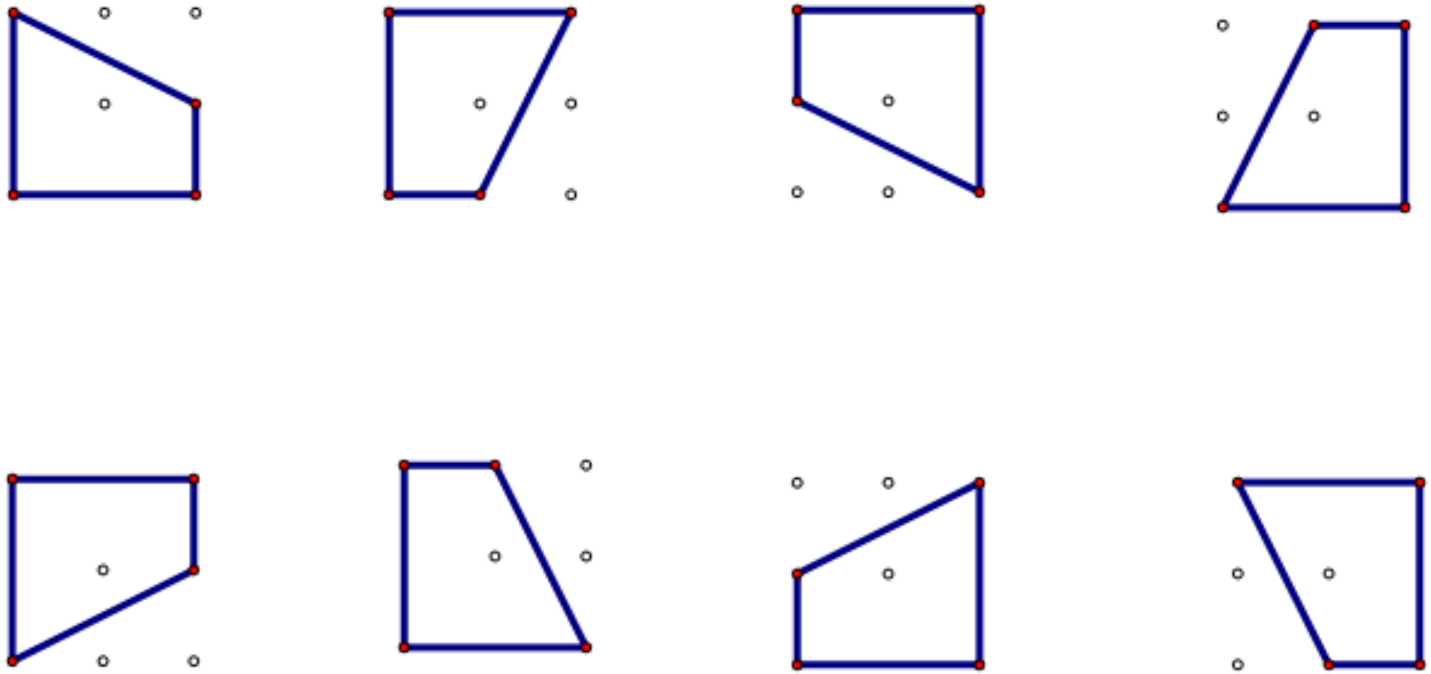
$o = 4\sqrt{2}$ cm
$p = 2$ cm^2

3

$o = 6$ cm
$p = 2$ cm^2

4

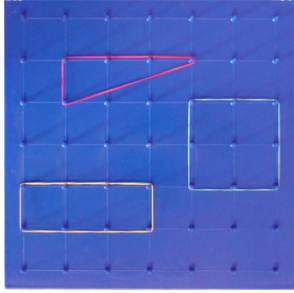




5

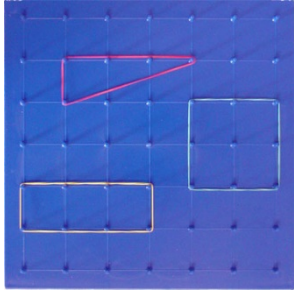
$o = 5 + \sqrt{5}$ cm

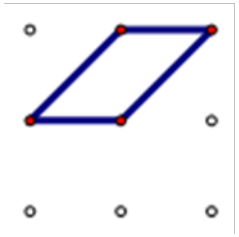
$p = 3$ cm^2

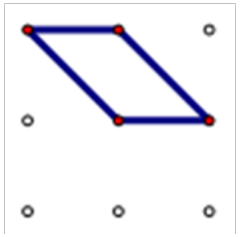


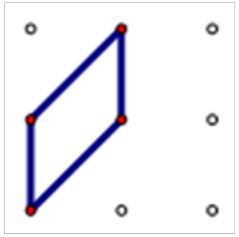
$o = 2(1+\sqrt{3})$ cm

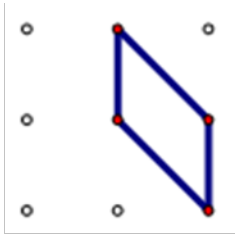
$p = 2$ cm^2





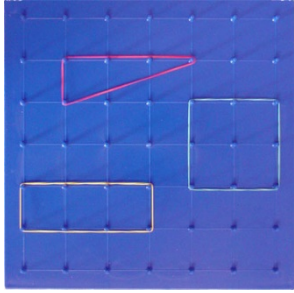






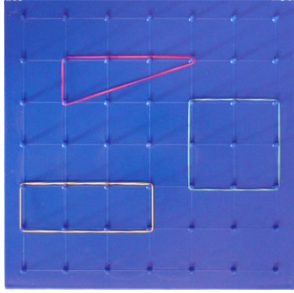
$o = 2(1+\sqrt{2})\,\text{cm}$

$p = 1\,\text{cm}^2$



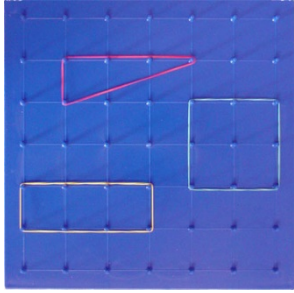
$$o = 3\sqrt{2} + 2\ \text{cm}$$

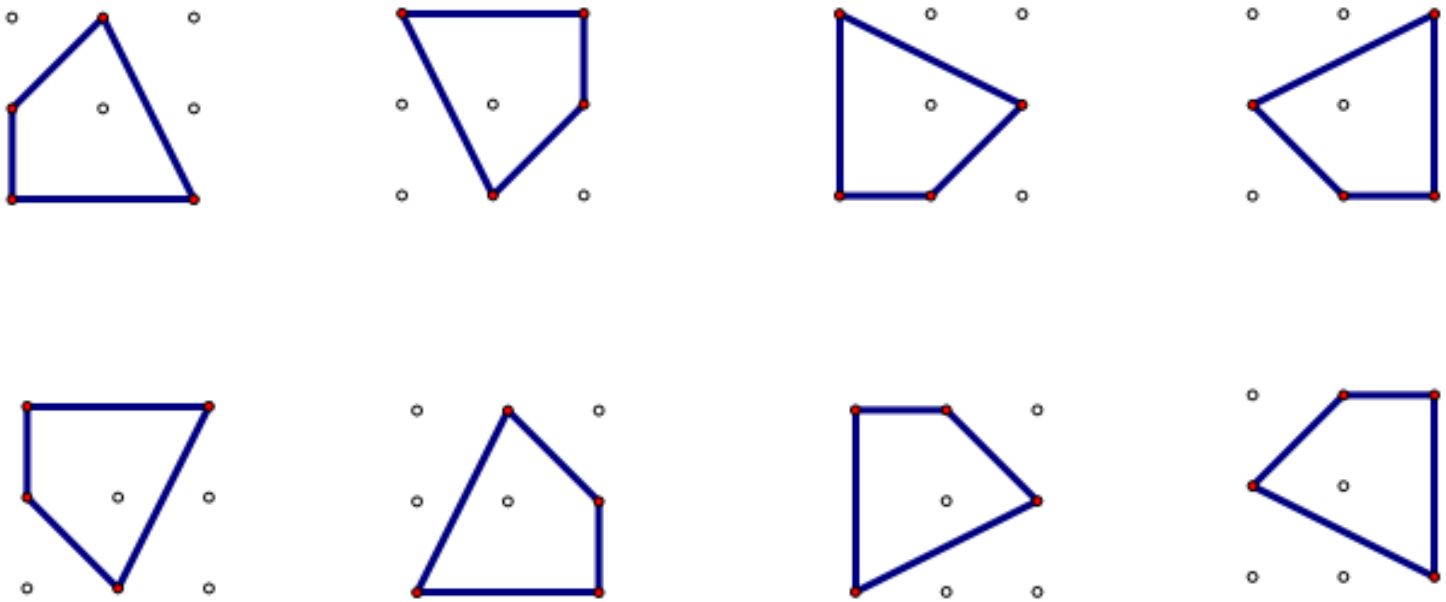
$$p = \frac{3}{2}\ \text{cm}^2$$



$o = 2(1+\sqrt{5})$ cm

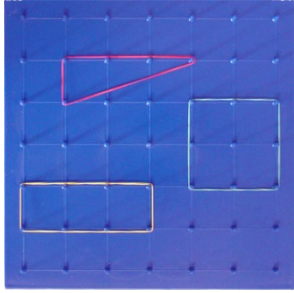
$p = 2$ cm^2

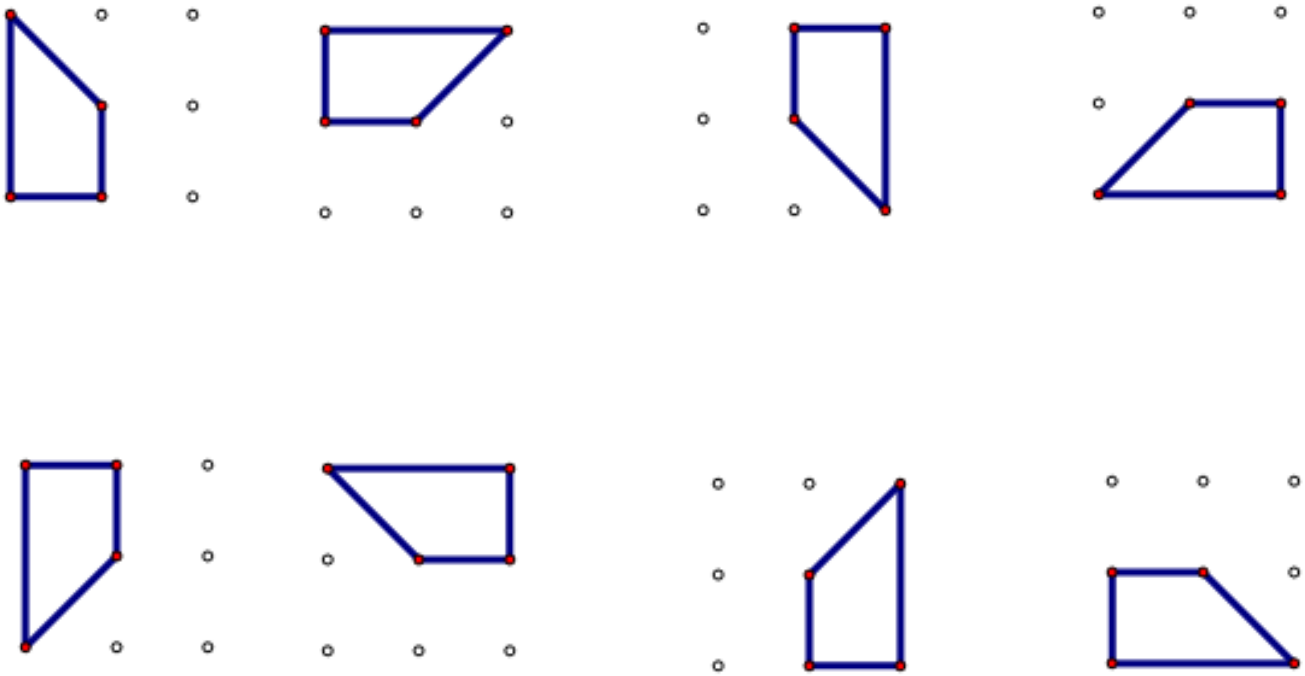




$o = 3 + \sqrt{5} + \sqrt{2}$ cm

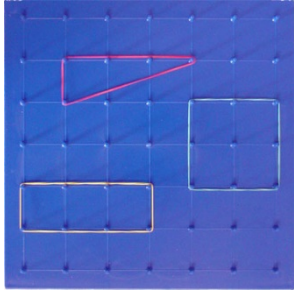
$p = 2\ \text{cm}^2$

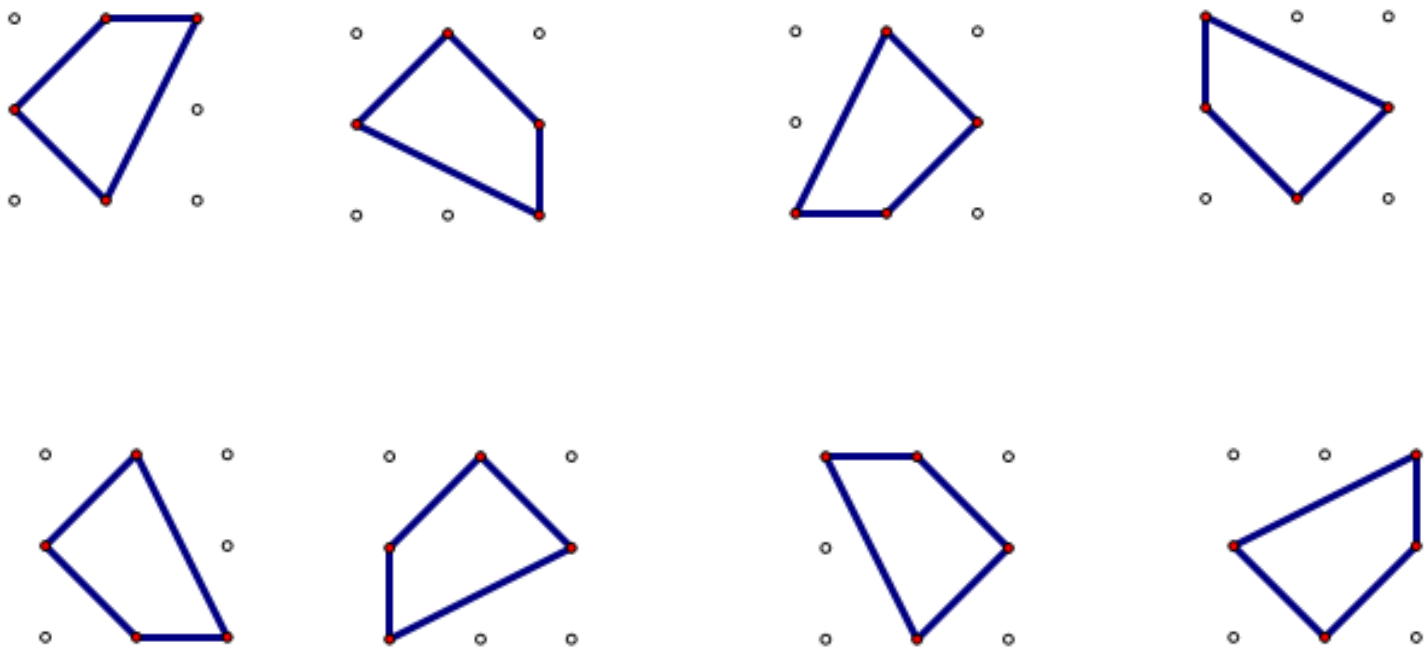




$o = 4 + \sqrt{2}$ cm

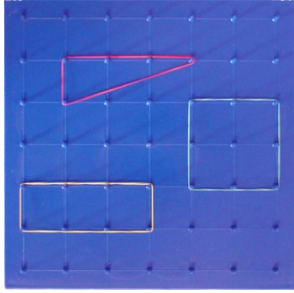
$p = \frac{3}{2}$ cm^2

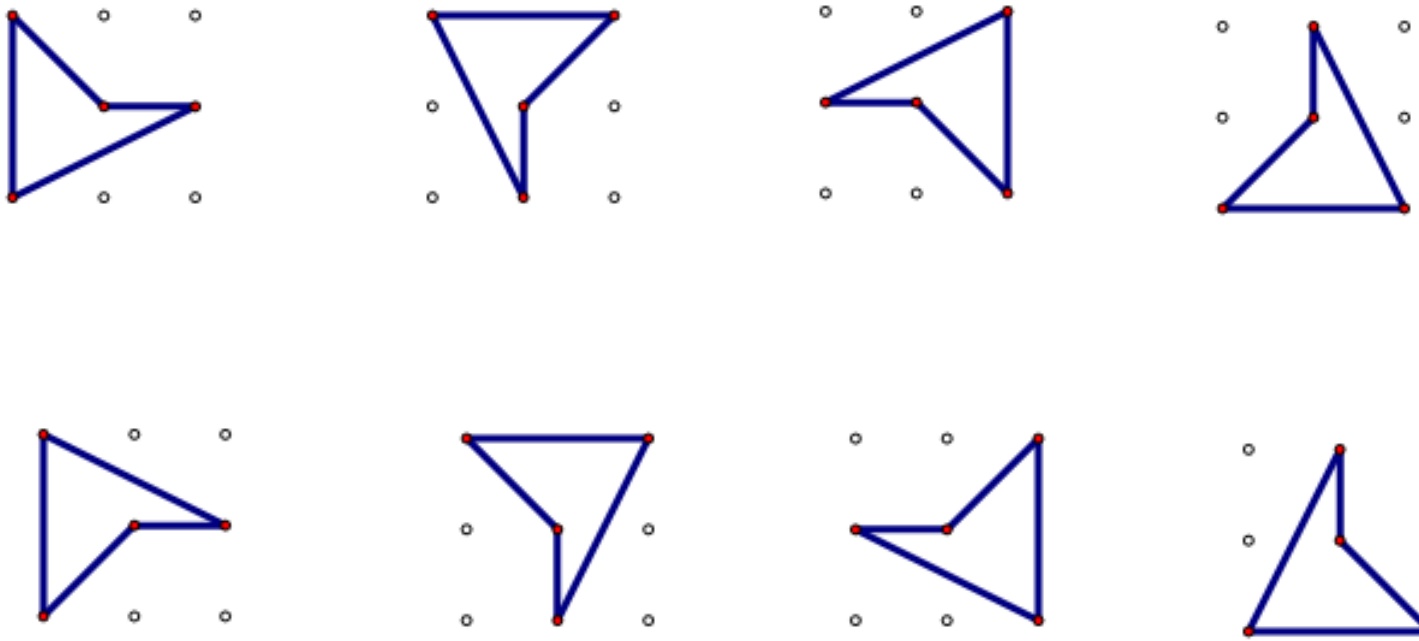




$o = 1 + \sqrt{5} + 2\sqrt{2}$ cm

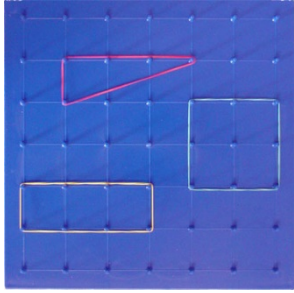
$p = 2\ \text{cm}^2$

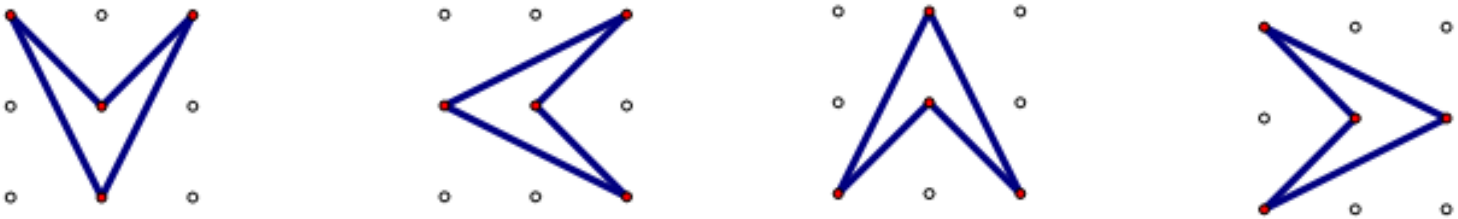




$$o = 3 + \sqrt{5} + \sqrt{2}\ \text{cm}$$

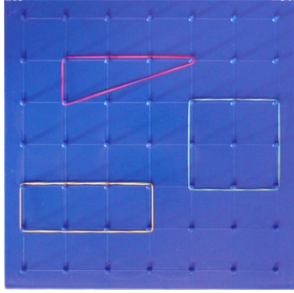
$$p = \frac{3}{2}\ \text{cm}^2$$





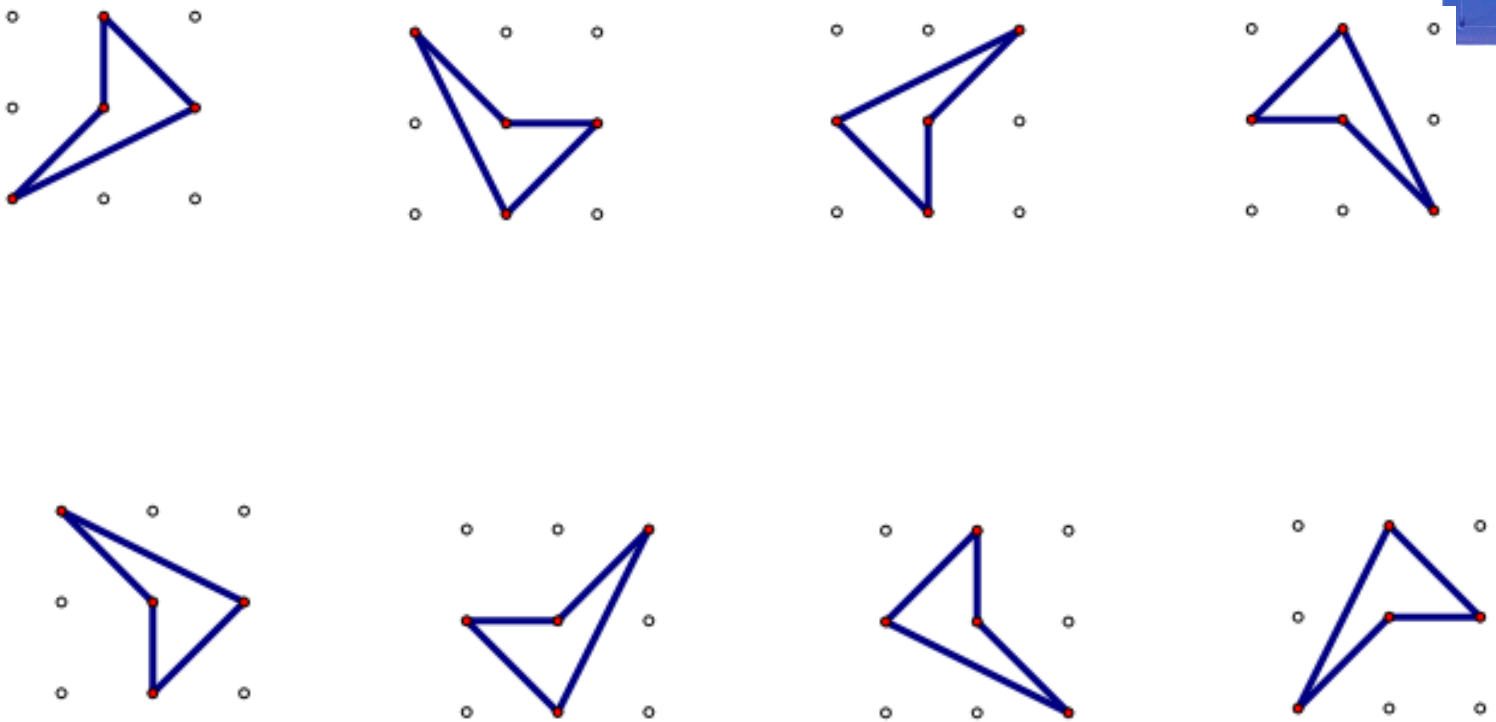
$o = 2(\sqrt{2} + \sqrt{5})\,\text{cm}$

$p = 1\,\text{cm}^2$



$o = 2(1+\sqrt{5})\,\text{cm}$

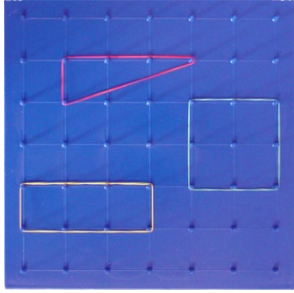
$p = 1\,\text{cm}^2$



$o = 1 + \sqrt{5} + 2\sqrt{2}$ cm

$p = 1\,\text{cm}^2$

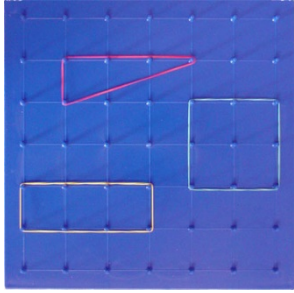
Paralelogrami [1]



Ciljevi:

- razvoj prostornog zora i podizanje razine geometrijskog mišljenja s razine vizualizacije na razinu neformalne dedukcije [5]
- prepoznavanje svojstava paralelograma kao karakteristike klase likova te uočavanje i formuliranje logičkih odnosa među svojstvima
- razvoj osjećaja za opseg paralelograma
- primjena koncepta sukladnosti i sličnosti na paralelograme
- eksperimentiranje s geometrijskim uzorcima

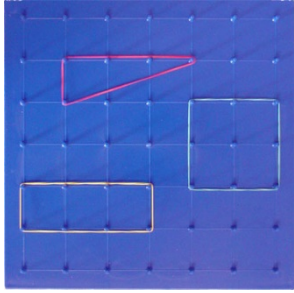
Paralelogrami [2]



Potrebni materijali:

- za svaki par učenika geoploča 5 × 5 i dovoljan broj elastičnih vrpca
- za svakog učenika nastavni listić za zadacima
- za svakog učenika točkasti papir 5 × 5 točaka i točkasti papir 11 × 11 točaka
- za učitelja interaktivna geoploča za analizu rješenja

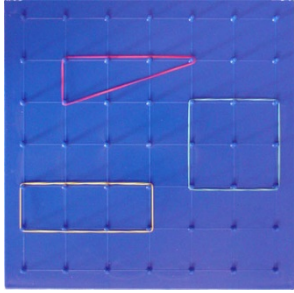
Paralelogrami [3]



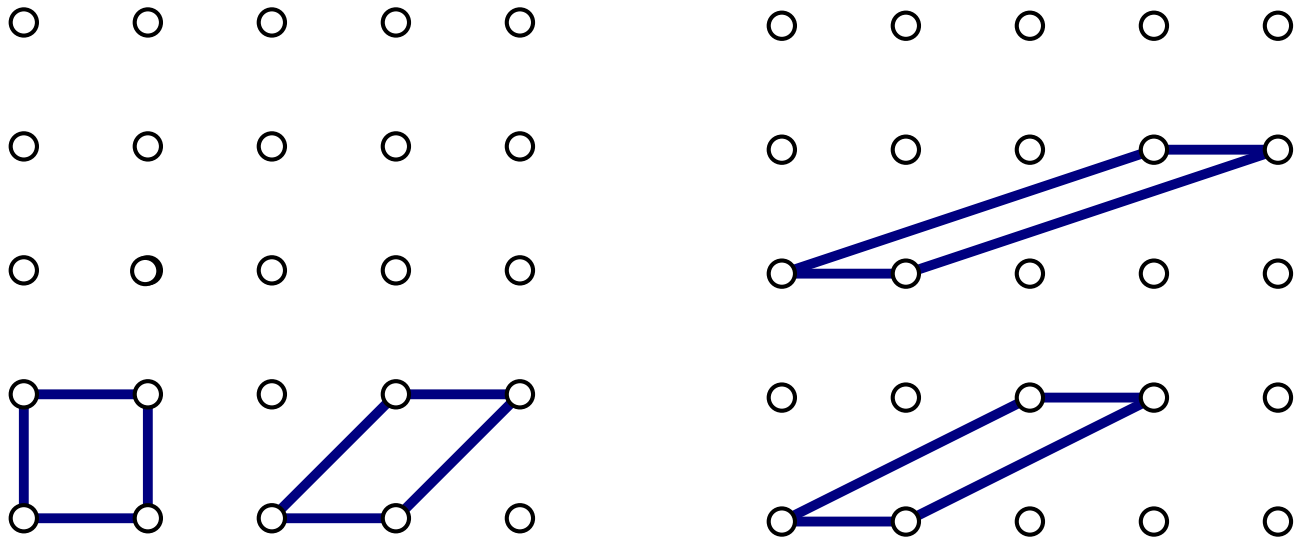
Uvodni zadaci:

1. Na geoploči dimenzija 5 × 5 napravite paralelogram s osnovicom duljine 1.

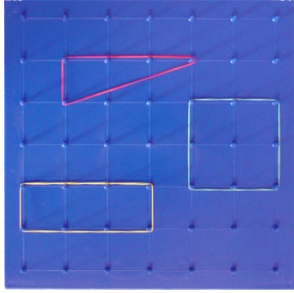
Paralelogrami [4]



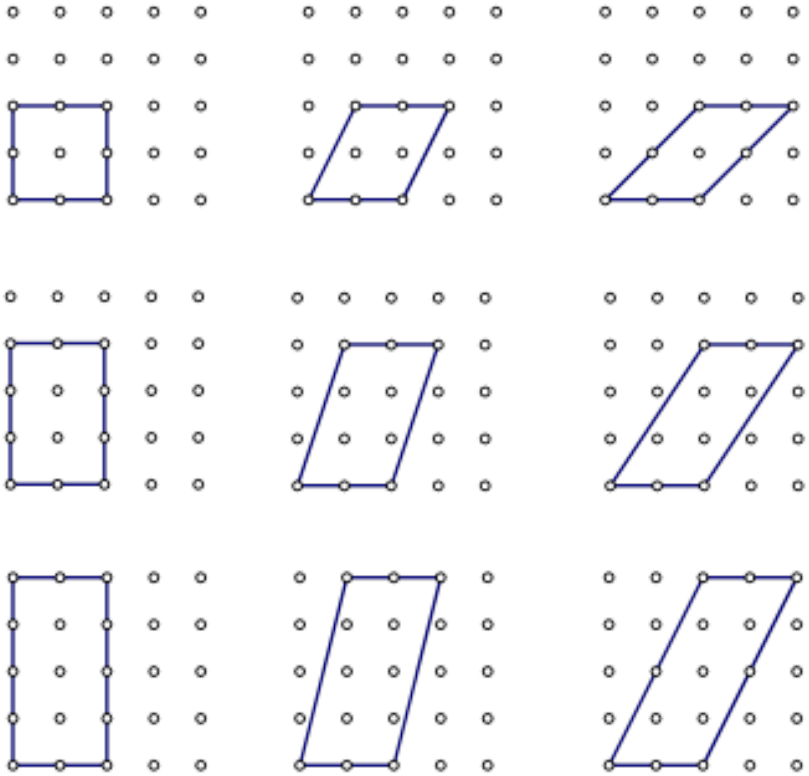
2. Na geoploči dimenzija 5 × 5 napravite još tri paralelograma s osnovicom duljine 1 koji imaju jednaku površinu kao paralelogram iz zadatka 1.



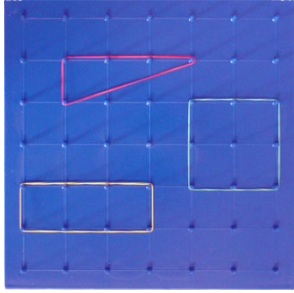
Paralelogrami [5]



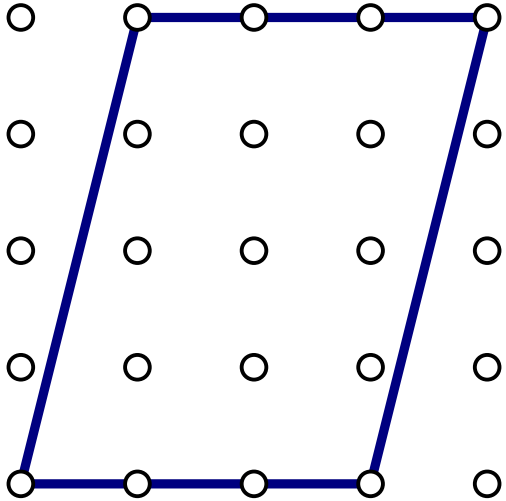
3. Na geoploči dimenzija 5 × 5 napravite sve međusobno nesukladne paralelograme s osnovicom duljine 2. Koliko ih ima?



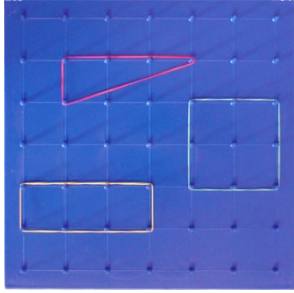
Paralelogrami [6]



4. Na geoploči dimenzija 5 × 5 napravite nepravokutni paralelogram najveće moguće površine. Kolika je ta površina? Koliki mu je opseg?



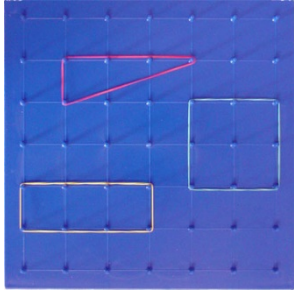
Paralelogrami [7]

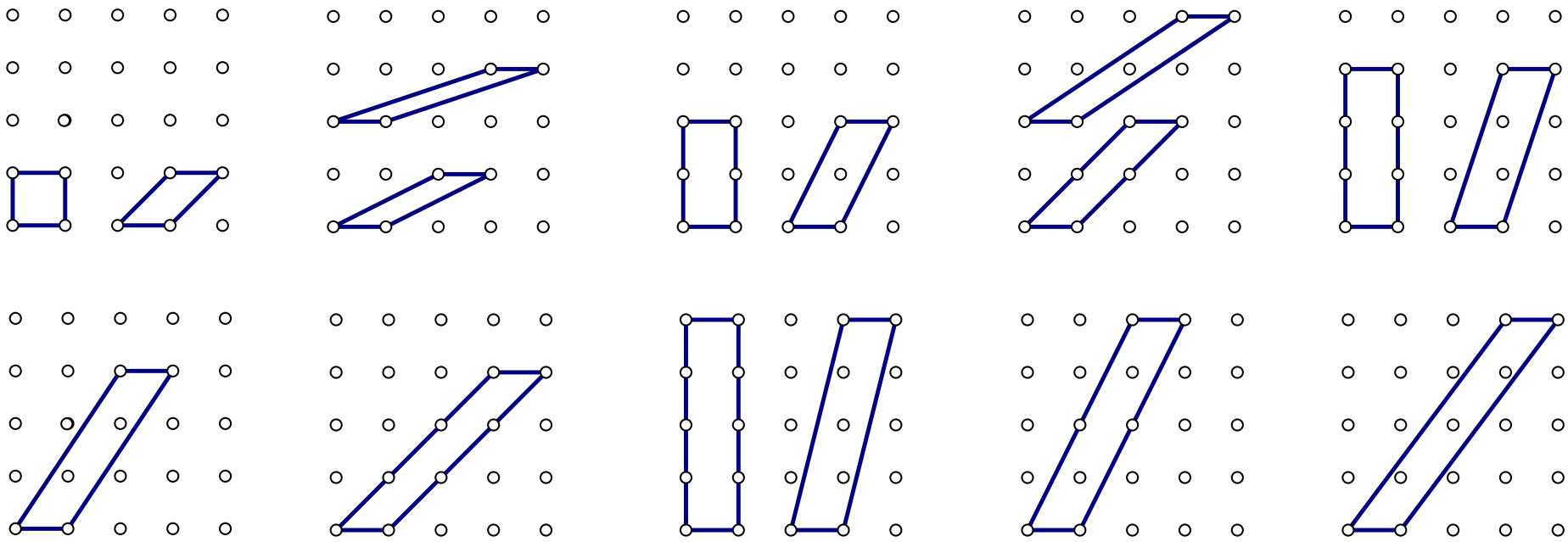


Glavni zadatak:

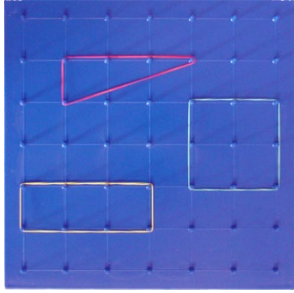
Na geoploči dimenzija 5 × 5 napravite sve međusobno nesukladne paralelograme. Nacrtajte odgovarajuće slike na točkastom papiru.

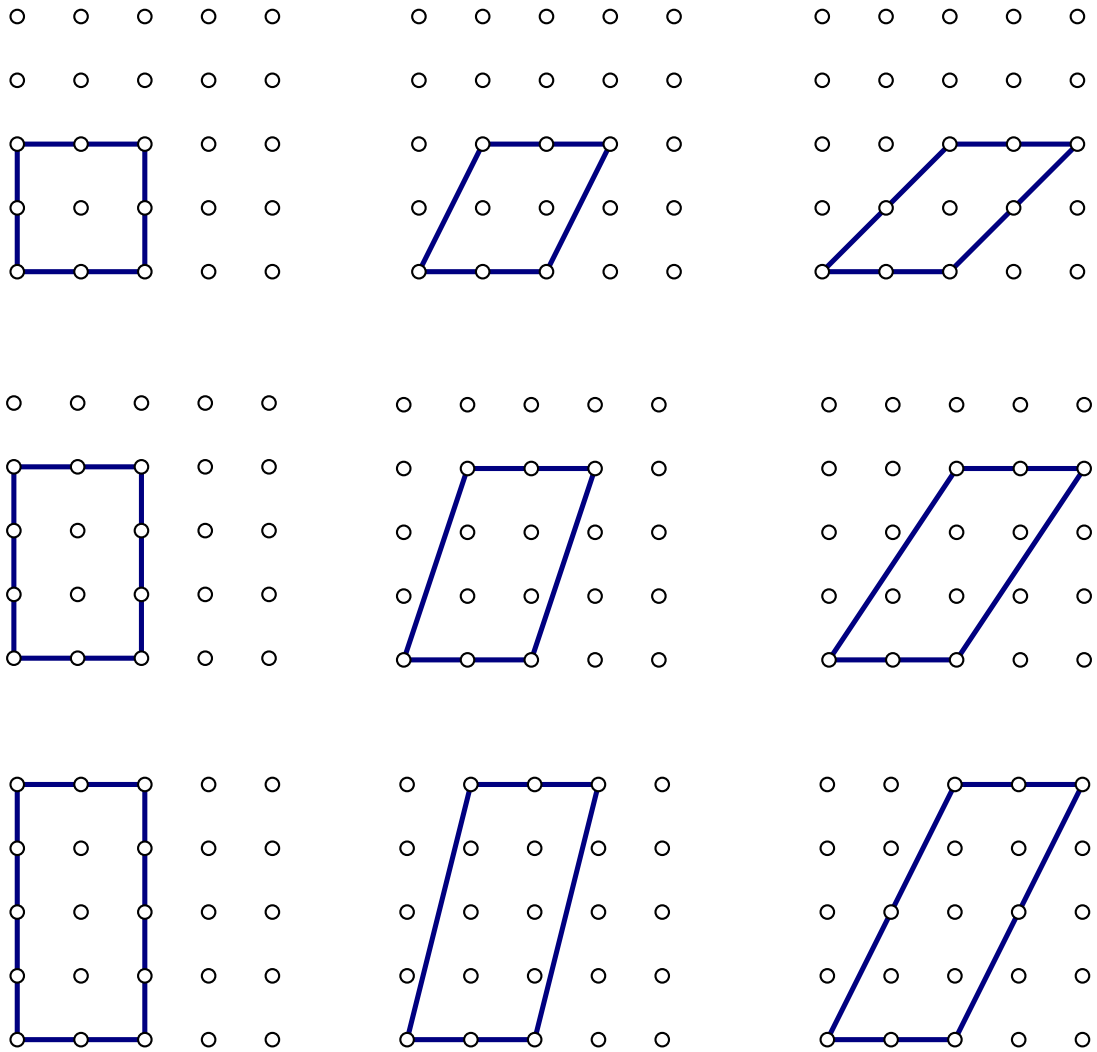
Paralelogrami [8]



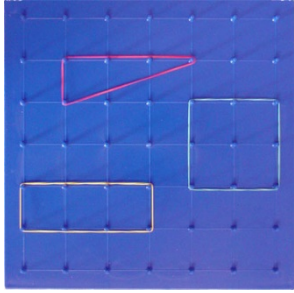


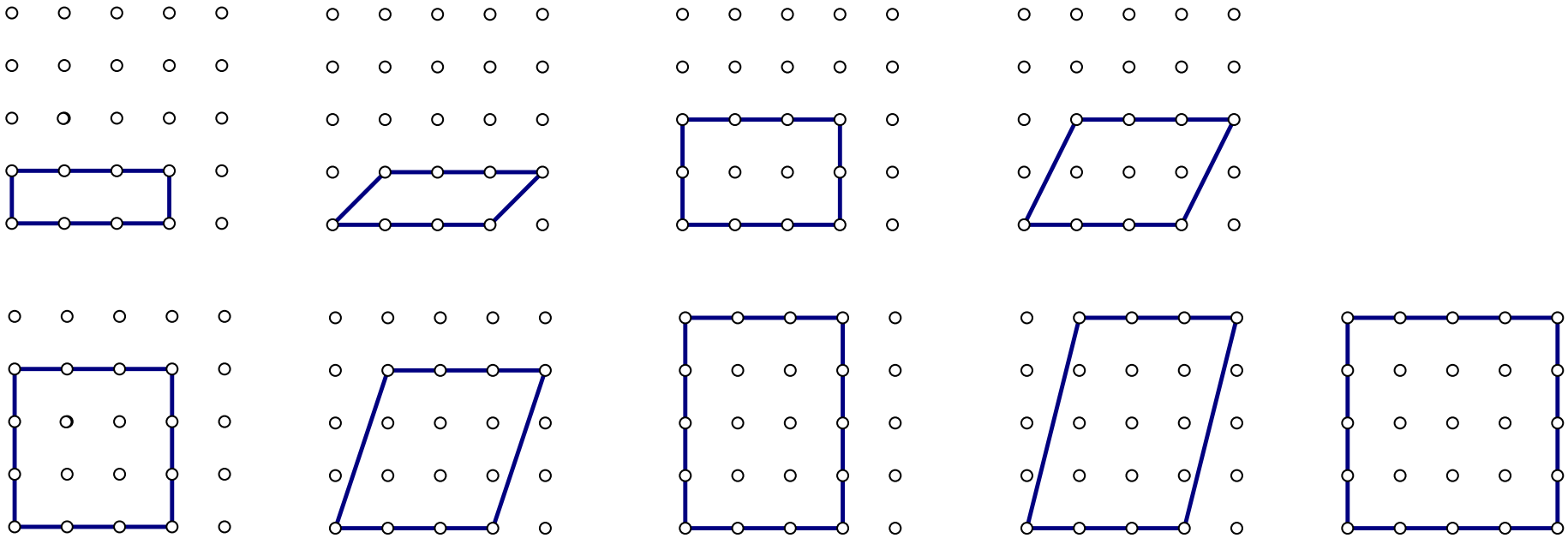
Paralelogrami [9]



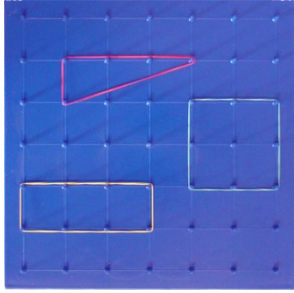


Paralelogrami [10]



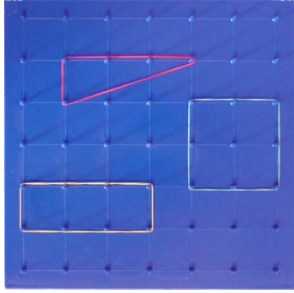


Paralelogrami [11]

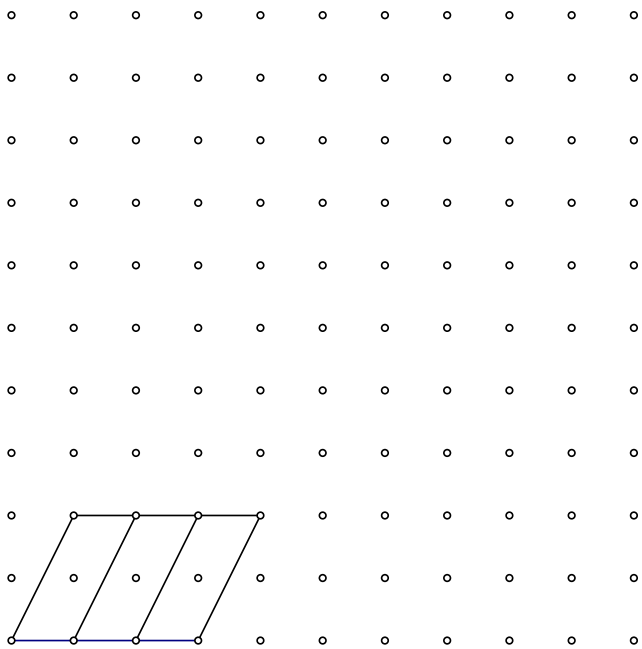


Izračunajte opseg svakoga od paralelograma iz prethodnog zadatka ako je udaljenost dviju susjednih točaka u retku/stupcu jednaka 1 cm.

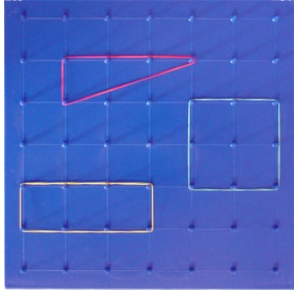
Paralelogrami [12]



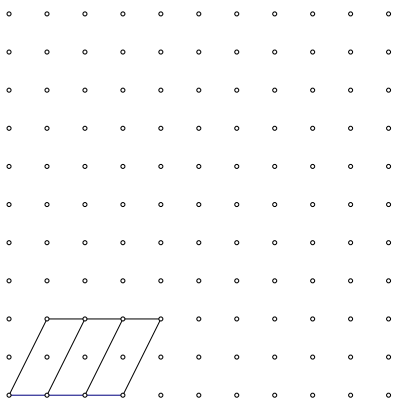
Nastavite niz paralelograma na geoploči dimenzija 11 × 11.



Paralelogrami [13]



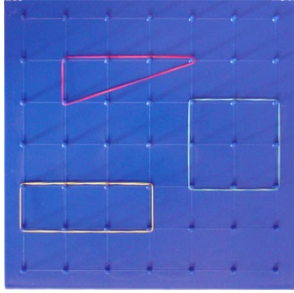
Nastavite niz paralelograma na geoploči dimenzija 11 × 11.



Nacrtajte odgovarajuću sliku na točkastom papiru i ispunite tablicu:

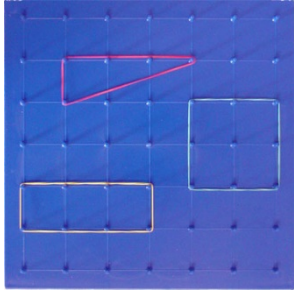
Duljina stranice	1	2	3	4	5	6	7	8	9	10
Opseg	$2(1+\sqrt{5})$	$2(2+\sqrt{5})$	$2(3+\sqrt{5})$							
Broj čavlića (točaka) na rubu	4	6	8							
Broj čavlića (točaka) u unutrašnjosti	1	2	3							

Preslikavanja ravnine



Prikazane učeničke aktivnosti učenicima će pomoći uvježbati navedene vještine te istražiti neka od svojstava preslikavanja ravnine.

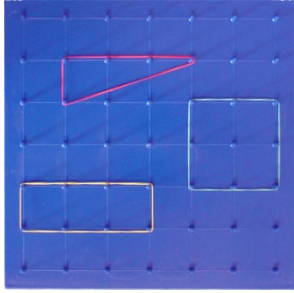
Osna simetrija [1]



Izvodeći ovu aktivnost, učenici će:

- preslikavati lik osnom simetrijom s obzirom na zadanu os simetrije,
- odrediti os simetrije zadanog lika.

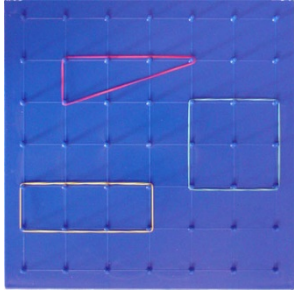
Osna simetrija [2]



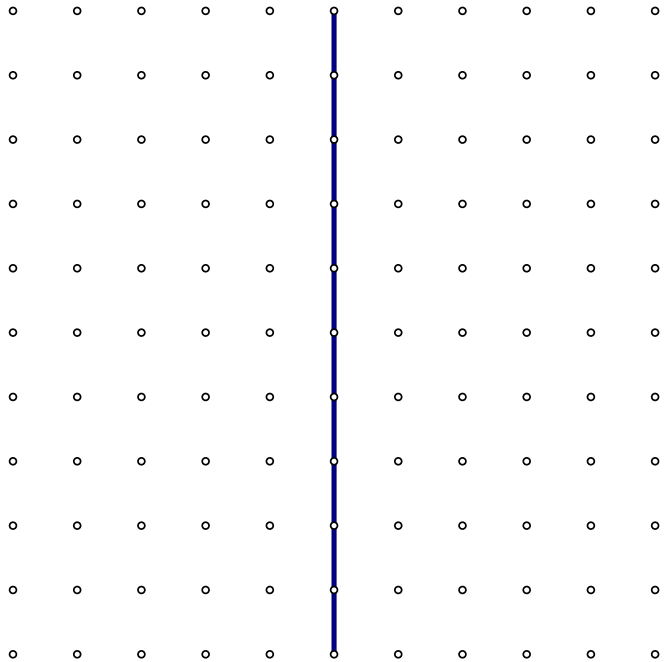
Potreban materijal:

- geoploča 11 × 11 na kojoj je gumicom istaknuta os simetrije
- više gumica različitih boja i duljina
- nastavni listić za svakog učenika

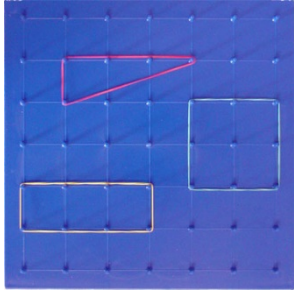
Osna simetrija [3]



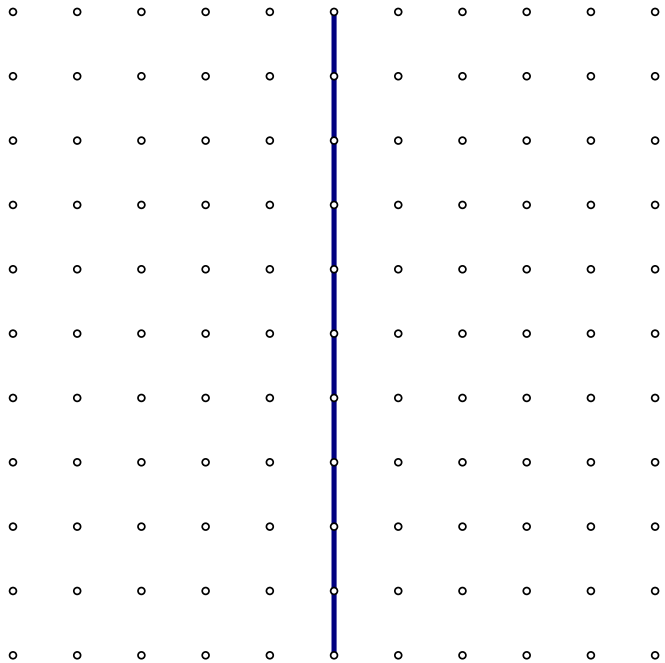
1. Istaknuti pravac je os simetrije. Na geoploči prikažite trokut tako da se cijeli trokut nalazi s desne strane istaknutog pravca. Prikažite njegovu osnosimetričnu sliku.



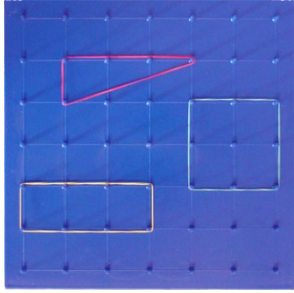
Osna simetrija [4]



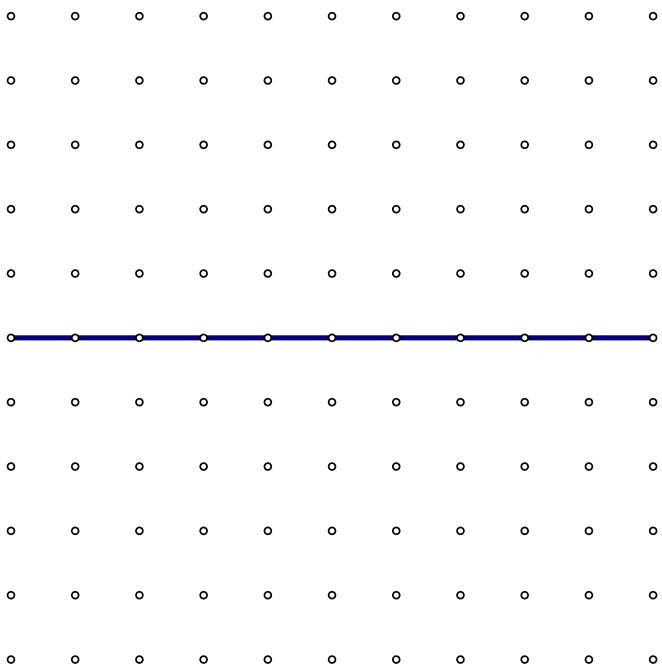
2. Istaknuti pravac je os simetrije. Na geoploči prikažite trokut tako da se jedan njegov vrh nalazi na zadanom pravcu, a dva vrha s lijeve strane zadane pravce. Prikažite njegovu osnosimetričnu sliku.



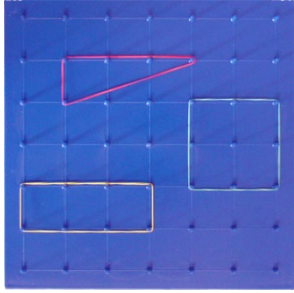
Osna simetrija [5]



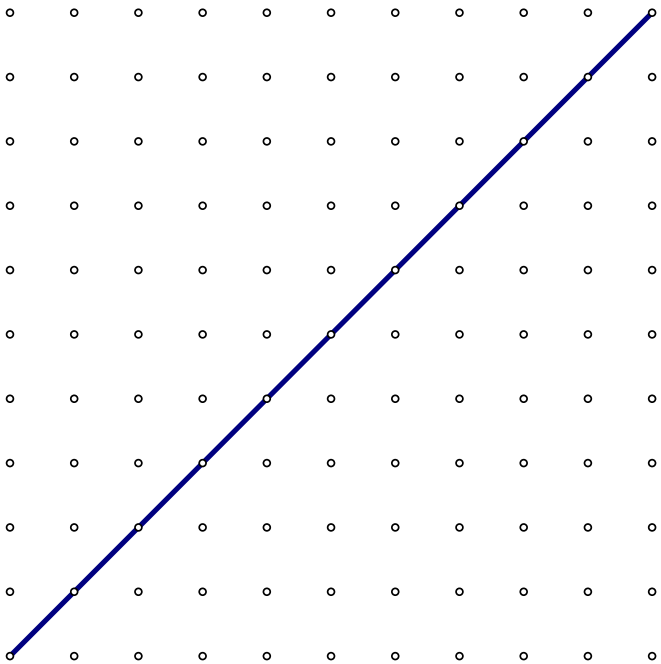
3. Istaknuti pravac je os simetrije. Na geoploči prikažite trokut tako da se dva vrha nalaze iznad, a jedna ispod zadanog pravca. Prikažite njegovu osnosimetričnu sliku.



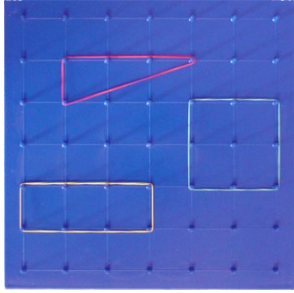
Osna simetrija [6]



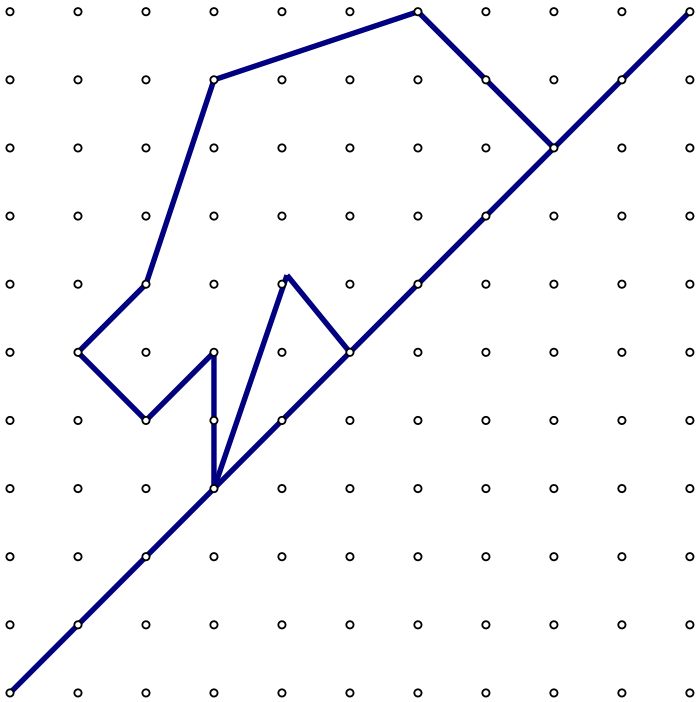
4. Istaknuti pravac je os simetrije. Na geoploči prikažite neki trokut te njegovu osnosimetričnu sliku.



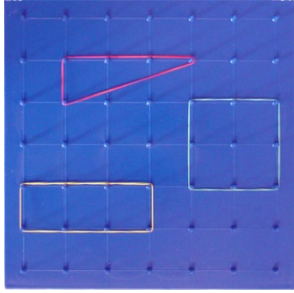
Osna simetrija [7]



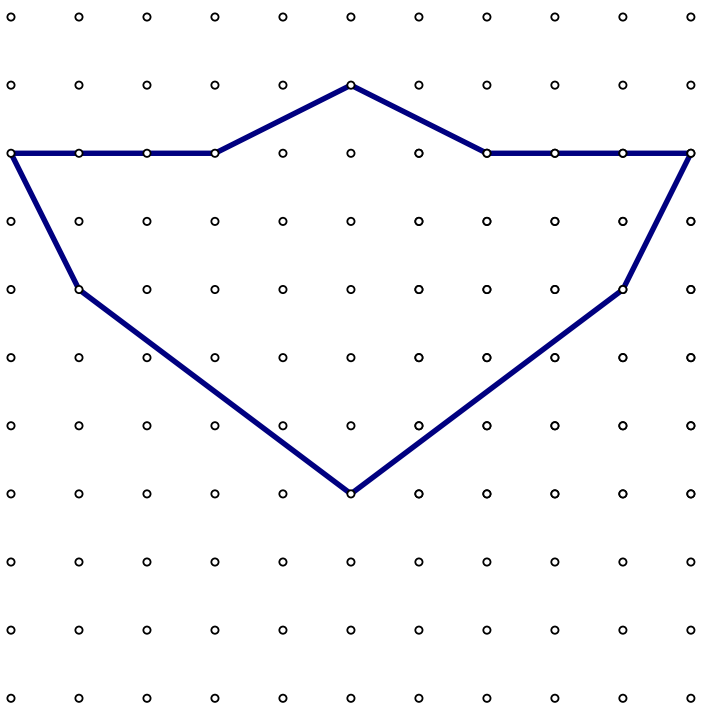
5. Istaknuti pravac je os simetrije. Koristeći geoploču dovršite sliku osnosimetričnog lika.

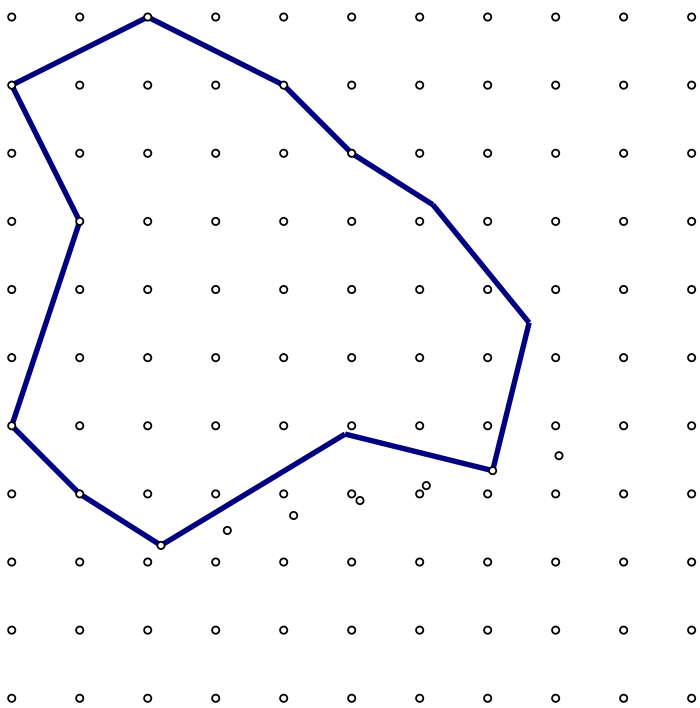


Osna simetrija [8]

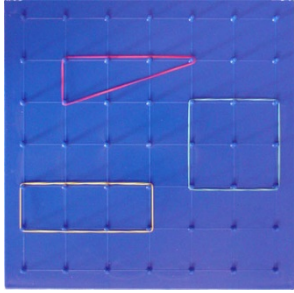


6. Prikazani likovi su osnosimetrični. Prikažite svakoga od njih na geoploči te istakni njihovu os simetrije .





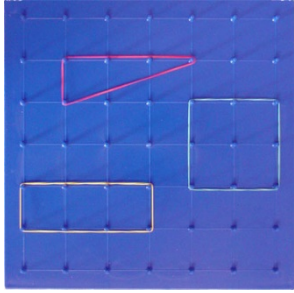
Osna simetrija [9]



Izvodeći ovu aktivnost, učenici će:

- istraživati svojstva osne simetrije.

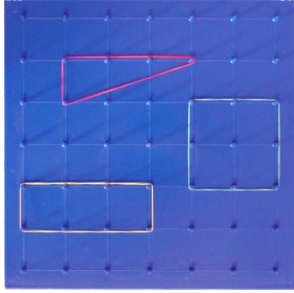
Osna simetrija [10]



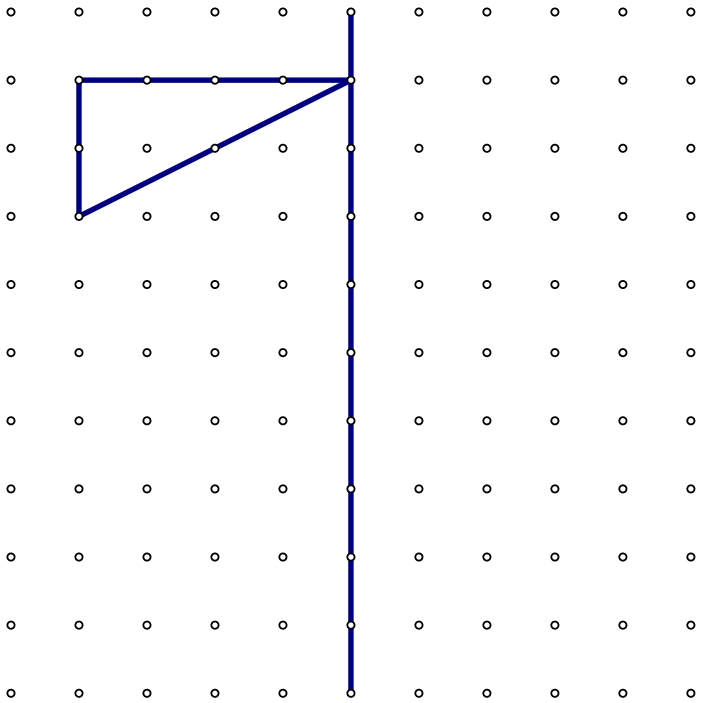
Potreban materijal:

- geoploča 11 × 11 na kojoj je gumicom istaknuta os simetrije
- više gumica različitih boja i duljina
- nastavni listić za svakog učenika

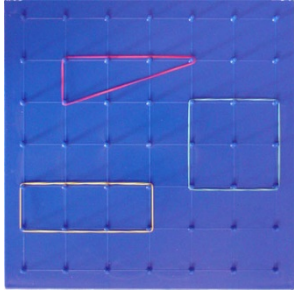
Osna simetrija [11]



1. Prikažite osnosimetričnu sliku zadanog trokuta s obzirom na zadanu os simetrije.



Osna simetrija [12]



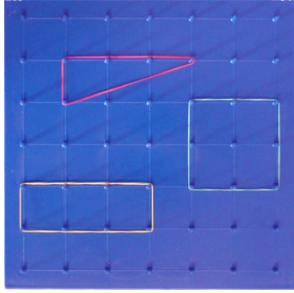
a) Izmjerite duljine stranica zadanog trokuta te duljine stranica njegove osnosimetrične slike. Što primjećujete?

b) Koje je vrste zadani trokut s obzirom na veličinu kutova?

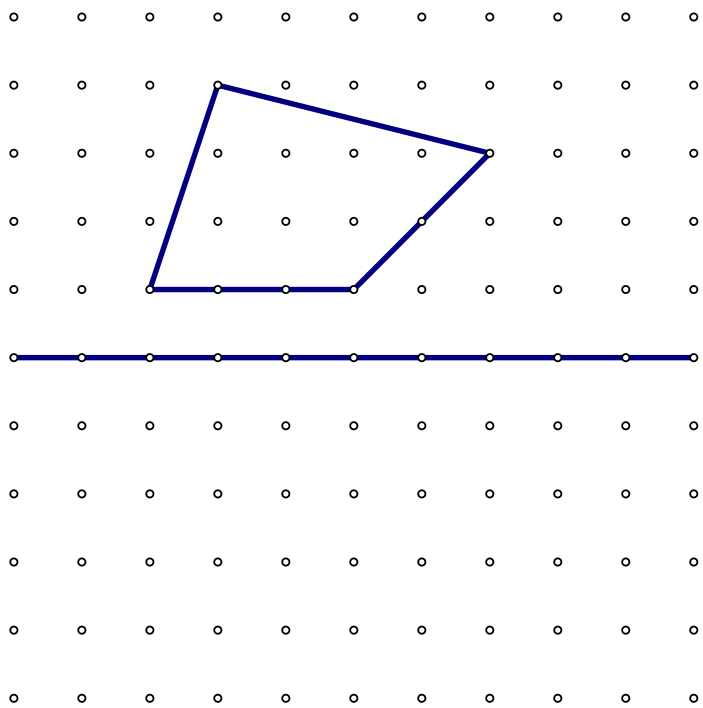
c) Koje je vrste novonastali trokut s obzirom na veličinu kutova?

d) Izmjerite kutove zadanog trokuta te kutove njemu osnosimetričnog trokuta. Što primjećujete?

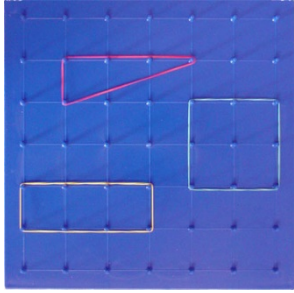
Osna simetrija [13]



2. Prikažite osnosimetričnu sliku zadanog četverokuta s obzirom na zadanu os simetrije.



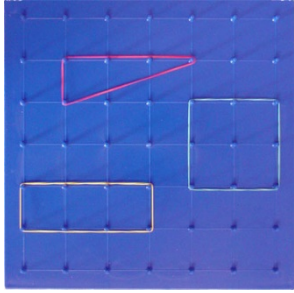
Osna simetrija [14]



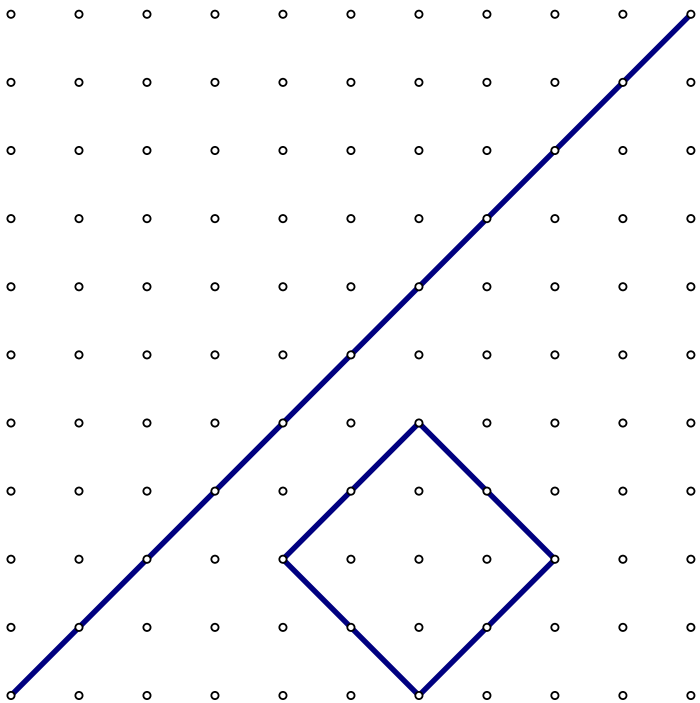
a) Izmjerite duljine stranica zadanog četverokuta te duljine stranica njegove osnosimetrične slike. Što primjećujete?

b) Izmjerite veličinu kutova zadanog četverokuta te veličine kutova njegove osnosimetrične slike. Što primjećujete?

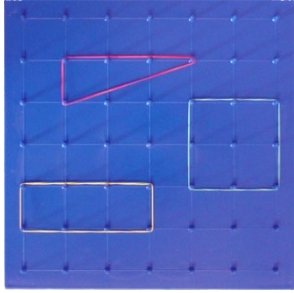
Osna simetrija [15]



3. Prikažite osnosimetričnu sliku zadanog četverokuta s obzirom na zadanu os simetrije.

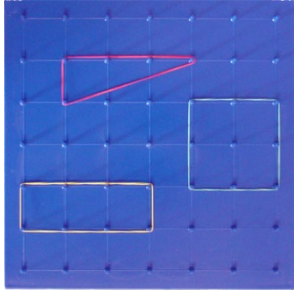


Osna simetrija [16]



Na zadanom kvadratu istakni međusobno usporedne stranice. Jesu li ti parovi stranica ostali usporedni nakon što je kvadrat preslikan osnom simetrijom?

Osna simetrija [17]

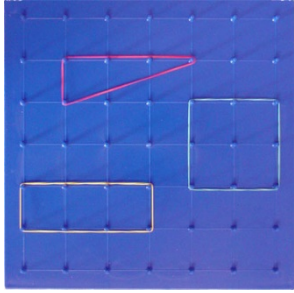


Pri sljedećoj aktivnosti geoploču ćemo poistovjetiti s koordinatnim sustavom. Aktivnost je namijenjena učenicima osmoga razreda.

Izvodeći ovu aktivnost, učenici će:

- preslikavati trokut s obzirom na os simetrije.

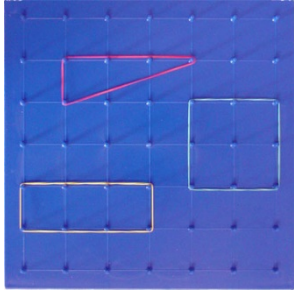
Osna simetrija [18]



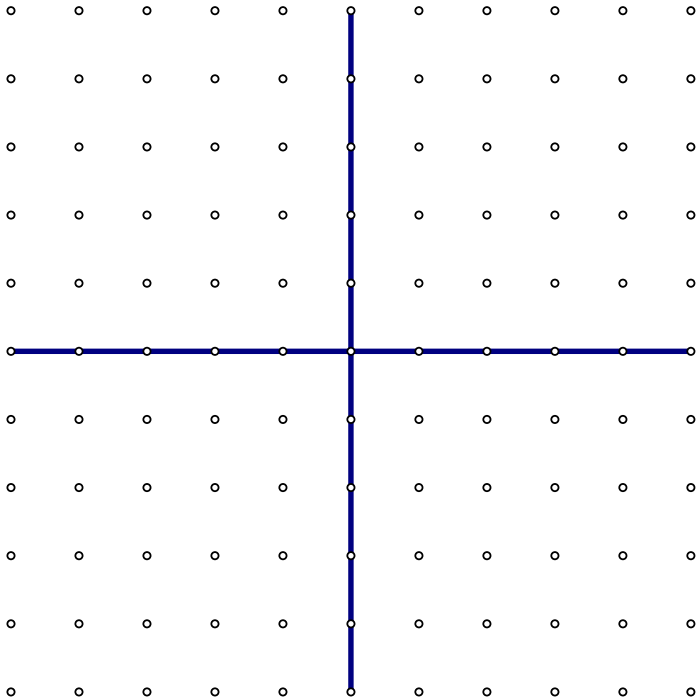
Potreban materijal:

- geoploča 11 × 11 na kojoj je gumicama definirana koordinatna ravnina
- više gumica različitih boja i duljina
- nastavni listić za svakog učenika

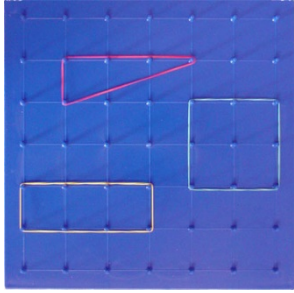
Osna simetrija [19]



1. Na geoploči prikažite trokut tako da se cijeli trokut nalazi u drugom kvadrantu. Preslikajte trokut s obzirom na *x*-os i zapišite koordinate novonastalih vrhova.

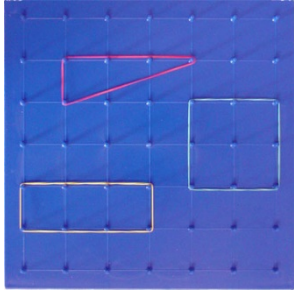


Osna simetrija [20]

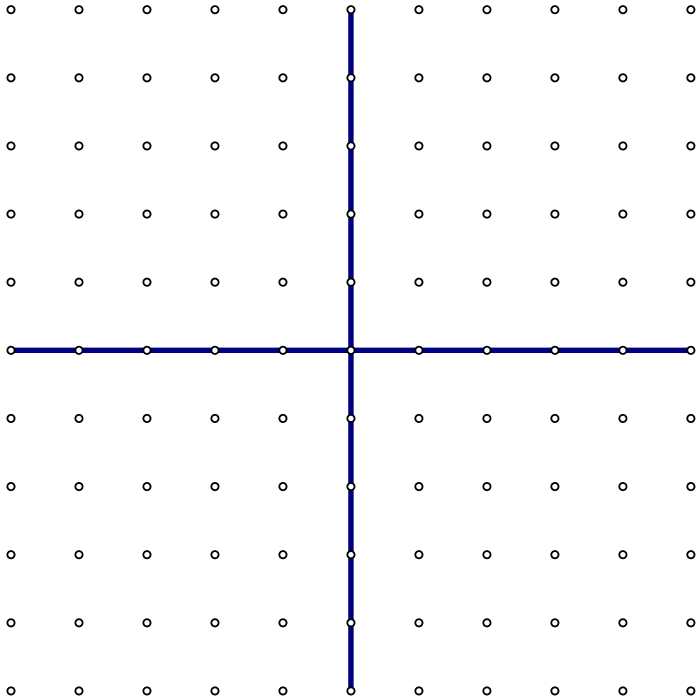


- Ako preslikamo mnogokut s vrhom u (x, y) s obzirom na x - os, kako će se to odraziti na njegove koordinate?

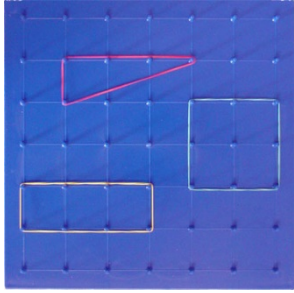
Osna simetrija [21]



2. Na geoploči prikažite trokut tako da se cijeli trokut nalazi u trećem kvadrantu. Preslikajte trokut s obzirom na pravac s jednadžbom $x = 2$ i zapišite koordinate novonastalih vrhova.

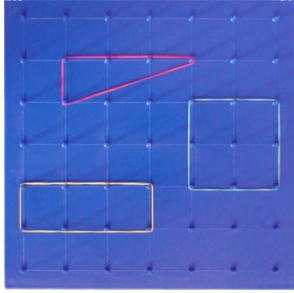


Osna simetrija [22]

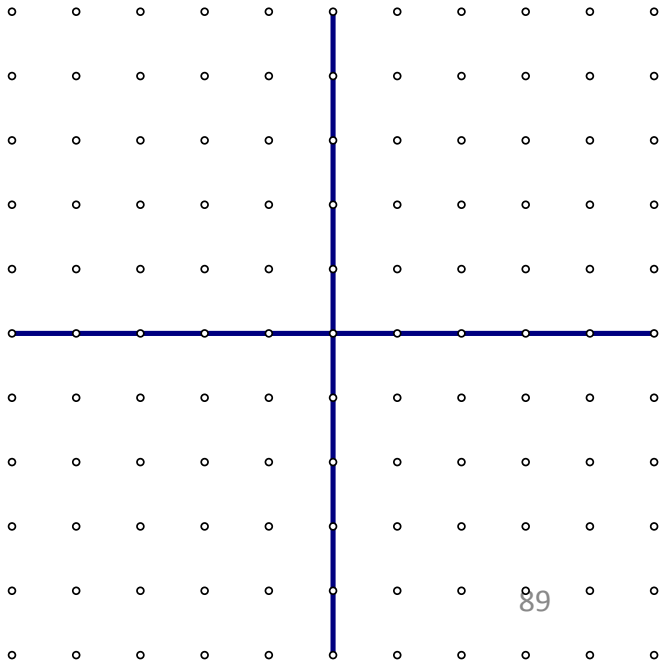


- Ako mnogokut s vrhom u (x, y) preslikamo s obzirom na pravac s jednadžbom $x = 2$, kako će se to odraziti na njegove koordinate?

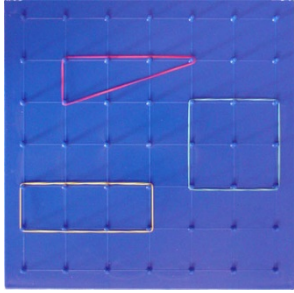
Osna simetrija [23]



3. Na geoploči gumicama prikažite trokut tako da se cijeli trokut nalazi u prvom kvadrantu. Preslikajte trokut s obzirom na pravac s jednadžbom $x = -1$ i s obzirom na pravac s jednadžbom $y = 0$ te zapišite koordinate novonastalih vrhova.

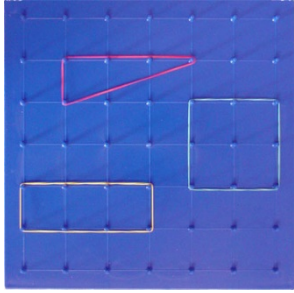


Osna simetrija [24]

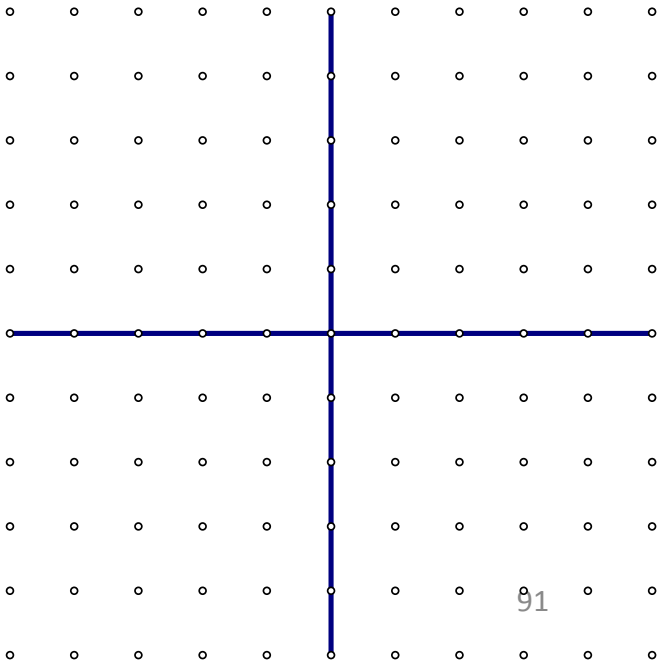


- Ako preslikamo mnogokut s vrhom u (x, y) s obzirom na pravac s jednadžbom $x = -1$, zatim novonastalu sliku s obzirom na $y = 0$, kako će se to odraziti na njegove koordinate?

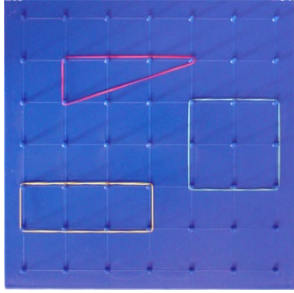
Osna simetrija [25]



4. Na geoploči prikažite trokut tako da se cijeli trokut nalazi u četvrtom kvadrantu. Preslikajte trokut s obzirom na pravac s jednadžbom $y = -x$ i s obzirom na pravac s jednadžbom $y = x$ te zabilježite koordinate novonastalih vrhova.

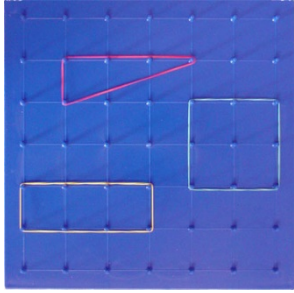


Osna simetrija [26]



- Ako preslikamo mnogokut s vrhom u (x, y) s obzirom na pravac s jednadžbom $y = -x$, zatim novonastalu sliku s obzirom na pravac s jednadžbom $y = x$, kako će se to odraziti na njegove koordinate?
- Hoćemo li dobiti jednaku sliku ako preslikamo mnogokut s vrhom u (x, y) s obzirom na pravac s jednadžbom $y = x$ i onda novonastalu sliku preslikamo s obzirom na pravac s jednadžbom $y = -x$? Objasnite svoj odgovor

Translacija [1]

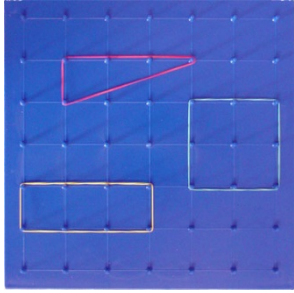


Izvodeći ovu aktivnost, učenici će:

- translatirati trokut u smjeru x-osi, a zatim u smjeru y-osi.

Aktivnost je namijenjena učenicima 8. razred osnovne škole.

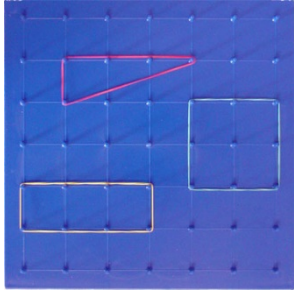
Translacija [2]



Potreban materijal:

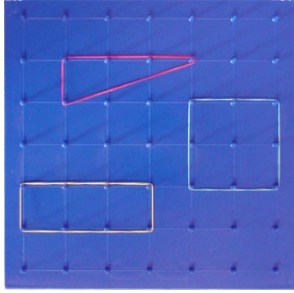
- geoploča 11 × 11 na kojoj je gumicama definirana koordinatna ravnina
- više gumica različitih boja i duljina
- nastavni listić

Translacija [3]



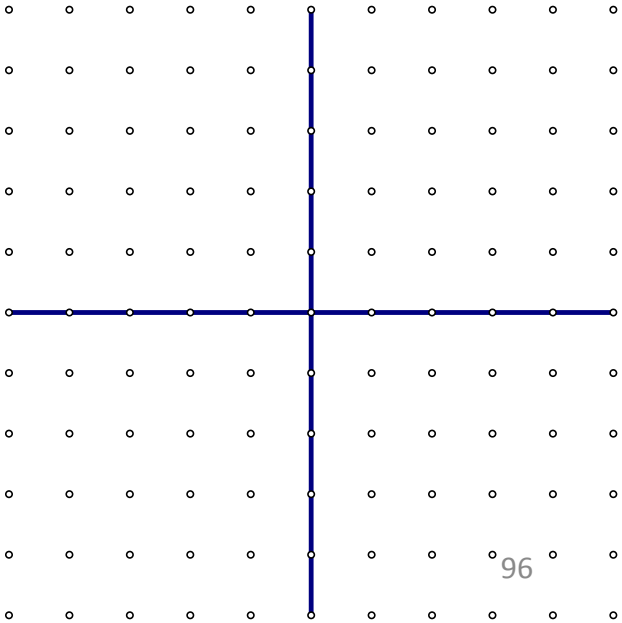
- Učenike podijelimo u četiri grupe.
- Nakon što su učenici markerom označili vrijednosti na odgovarajućim koordinatnim osima, gumicama kreiraju zadane trokute.
- Svaka grupa kreira trokut u svom kvadrantu i translatira ga s obzirom na dane vektore.
- Kada grupe izvrše svoj zadatak, redom riješe zadatke ostalih grupa.
- Na kraju učenici međusobno usporede dobivena rješenja.

Translacija [4]

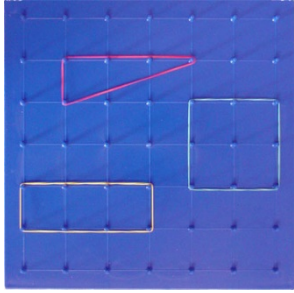


SKUPINA A

- Na geoploči gumicama prikažite trokut s vrhovima (1, 2), (1, 1), (3, 1). Trokut translatirajte za -3 jedinice u smjeru *x* - osi i -4 jedinice u smjeru *y* - osi. Nacrtajte odgovarajuću sliku te odredite koordinate vrhova translatiranog trokuta.

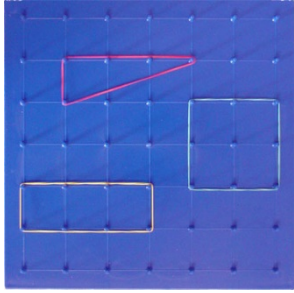


Translacija [5]



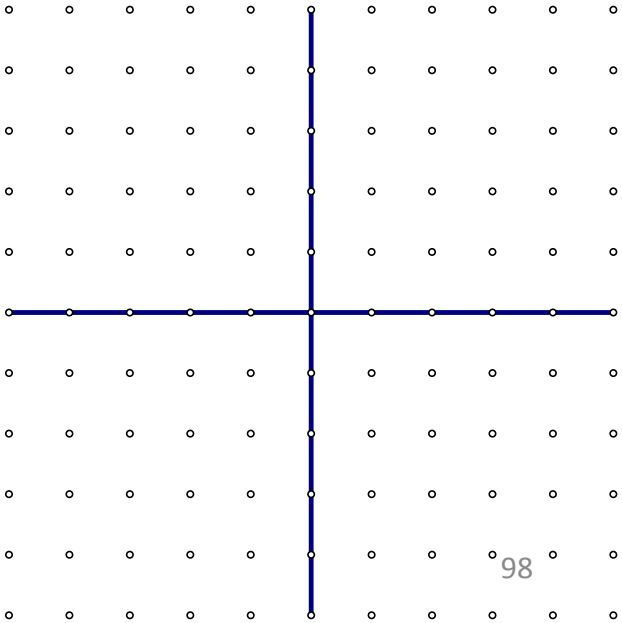
- Ako translatirate mnogokut čiji je jedan vrh u (x, y) za -3 jedinice u smjeru x-osi i -4 jedinice u smjeru y-osi, koje će koordinate poprimiti taj vrh?

Translacija [6]

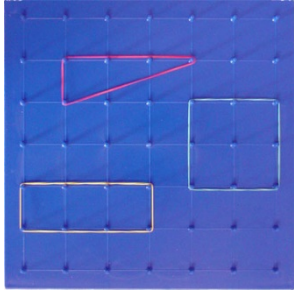


SKUPINA B

- Na geoploči gumicama prikažite trokut s vrhovima (-2, 2), (-3, 1), (-1, 0). Trokut treba translatirati za 4 jedinice u smjeru *x* - osi i -3 jedinice u smjeru *y* - osi. Nacrtajte odgovarajuću sliku te odredite koordinate vrhova translatiranog trokuta.

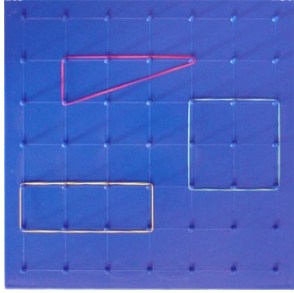


Translacija [7]



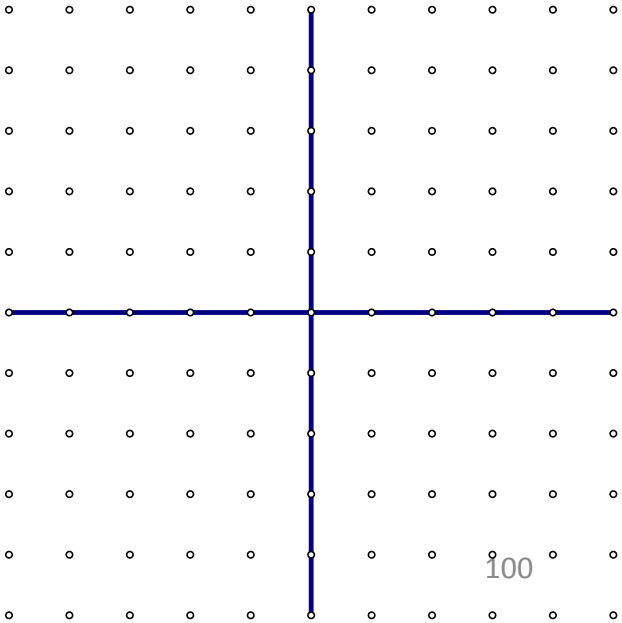
- Ako translatirate mnogokut čiji je jedan vrh u (x, y) za 4 jedinice u smjeru x - osi i -3 jedinice u smjeru y - osi, koje će koordinate poprimiti taj vrh?

Translacija [8]

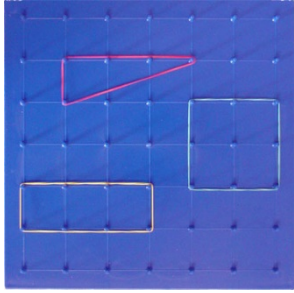


SKUPINA C

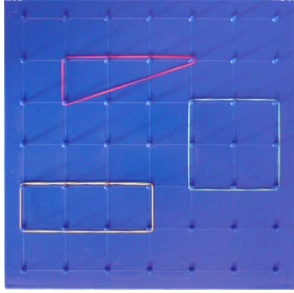
- Na geoploči gumicama prikažite trokut s vrhovima (-2, 0), (-3, -1), (0, -1). Trokut treba translatirati za 3 jedinice u smjeru *x* – osi i 2 jedinice u smjeru *y* - osi. Nacrtajte odgovarajuću sliku te odredite koordinate vrhova translatiranog trokuta.



Translacija [9]



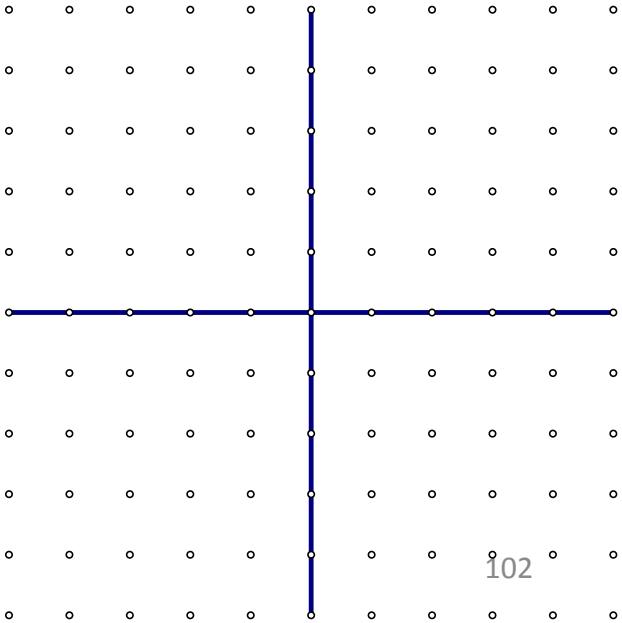
- Ako translatirate mnogokut čiji je jedan vrh u (x, y) za 3 jedinice u smjeru x - osi i 2 jedinice u smjeru y - osi, koje će koordinate poprimiti taj vrh?



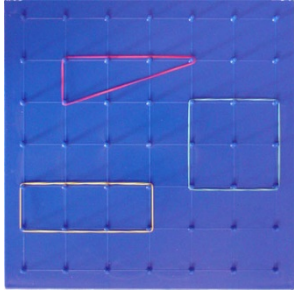
Translacija [10]

SKUPINA D

- Na geoploči gumicama prikažite trokut s vrhovima (4, -3), (3, -1), (1, -2). Trokut treba translatirati za -5 jedinice u smjeru *x* – osi i 3 jedinice u smjeru *y* - osi. Nacrtajte odgovarajuću sliku te odredite koordinate vrhova translatiranog trokuta.

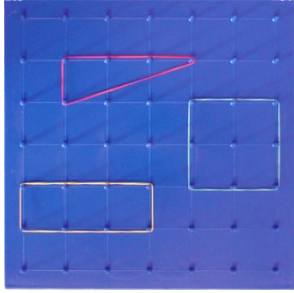


Translacija [11]



- Ako translatirate mnogokut čiji je jedan vrh u (x, y) za -5 jedinica u smjeru x – osi i 3 jedinice u smjeru y – osi, koje će koordinate poprimiti taj vrh?

Translacija [12]

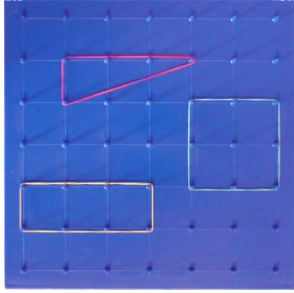


Izvodeći ovu aktivnost, učenici će:

- translatirati mnogokute za zadani vektor,
- odrediti vektor translacije.

Aktivnost je namijenjena učenicima 8. razred osnovne škole.

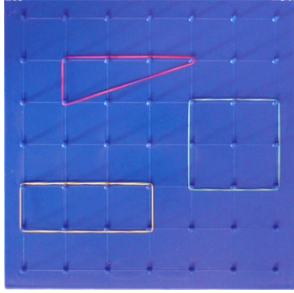
Translacija [13]



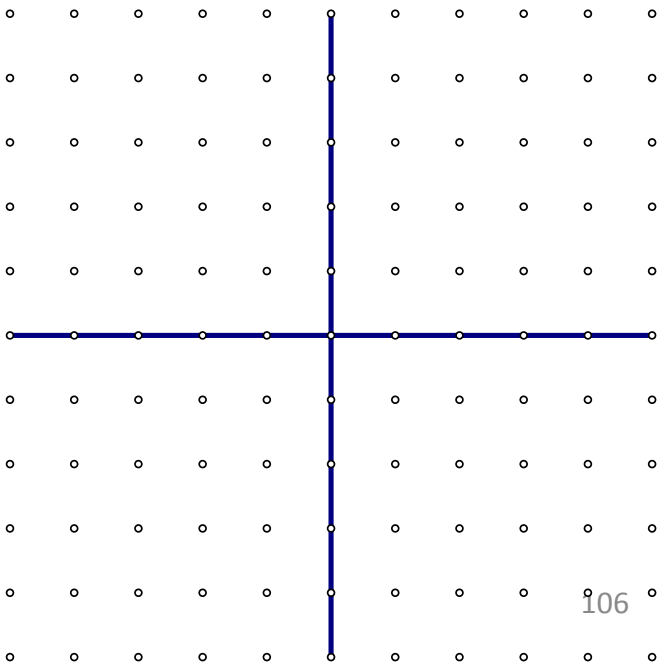
Potreban materijal:

- geoploča 11 × 11 na kojoj je gumicama definirana koordinatna ravnina
- više gumica različitih boja i duljina
- nastavni listić

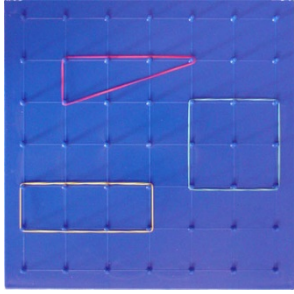
Translacija [14]



1. Na geoploči gumicama prikažite vektor koji počinje u točki *A*(-4, 4), a završava u toči *B*(-2, 5) te trokut s vrhovima (0, 2), (3, -1), (2, 3). Nacrtajte odgovarajuću sliku te odredite koordinate vrhova translatiranog trokuta.



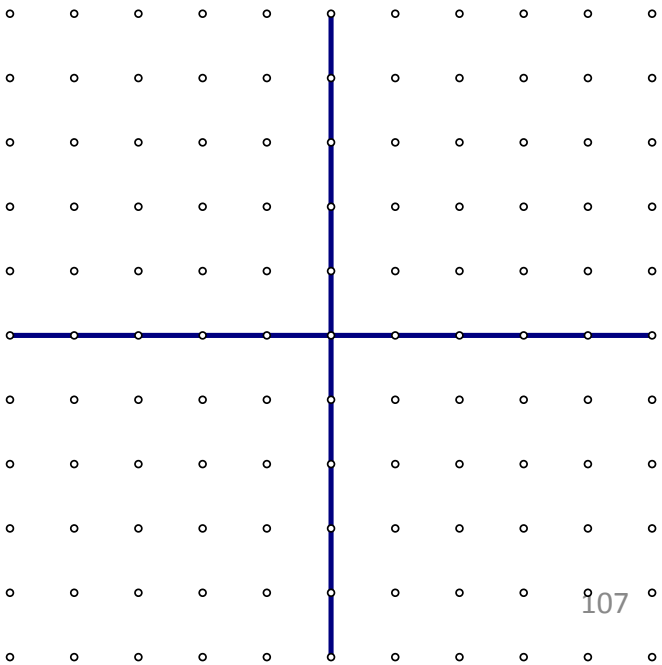
Translacija [15]



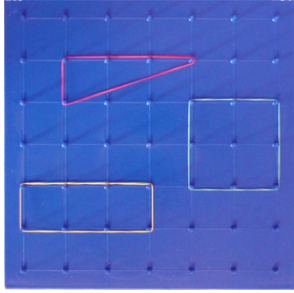
2. Na geoploči gumicama prikažite vektor duljine $\sqrt{5}$ čija je krajnja točka zadana koordinatama (-1, 3) te trokut s vrhovima (-2, -3), (1, -2), (3, 3). Nacrtajte odgovarajuću sliku te odredite koordinate vrhova translatiranog trokuta.

Je li rješenje jednoznačno?

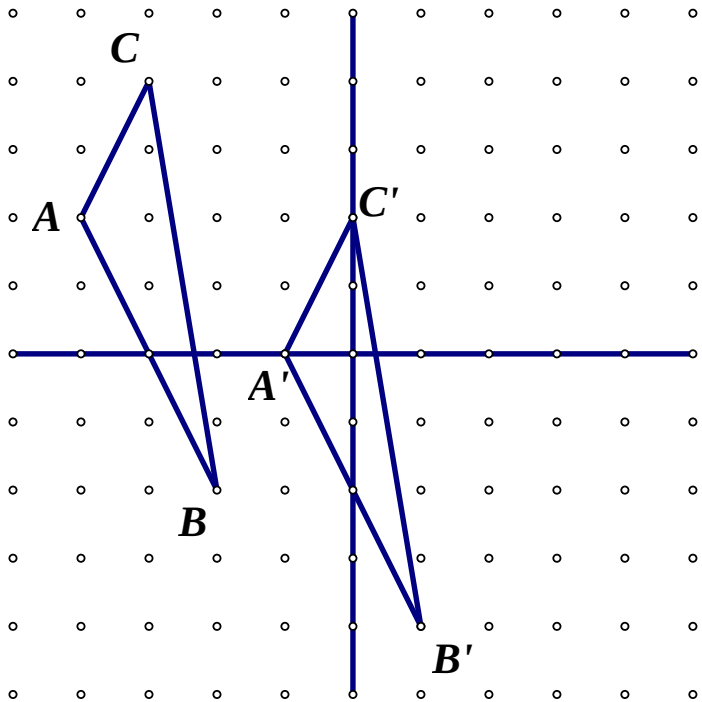
Objasnite.



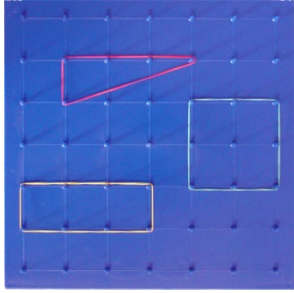
Translacija [16]



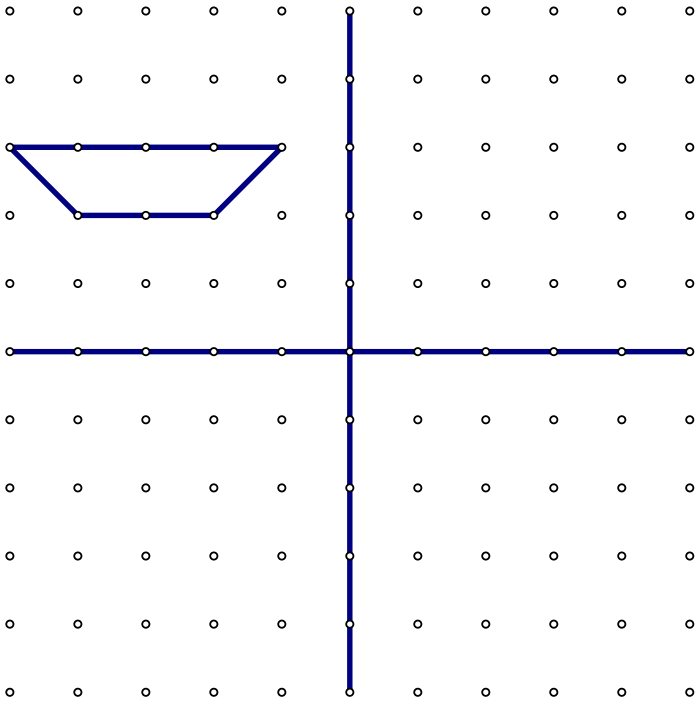
3. Odredite vektor kojim je trokut *ABC* preslikan u trokut *A'B'C'*. Je li rješenje jednoznačno? Objasnite.



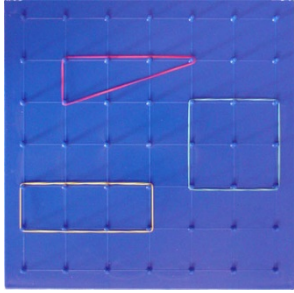
Translacija [17]



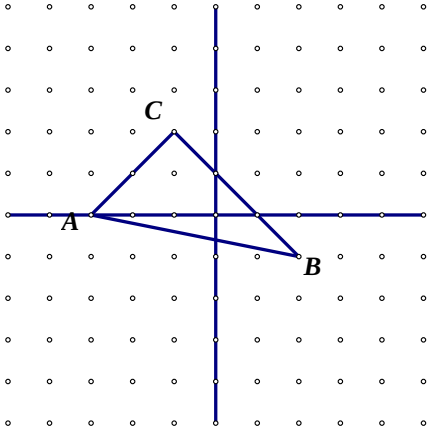
4. Odredite vektor koji će trapez preslikati iz drugog u četvrti kvadrant. Je li rješenje jednoznačno? Objasnite.

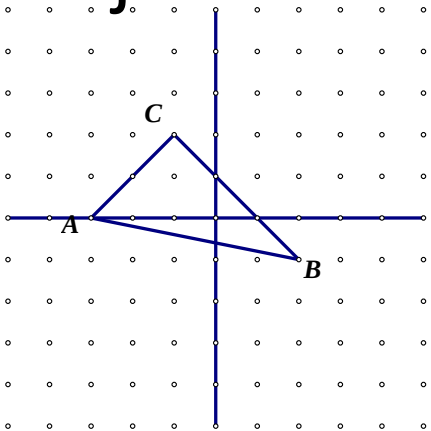


Translacija [18]

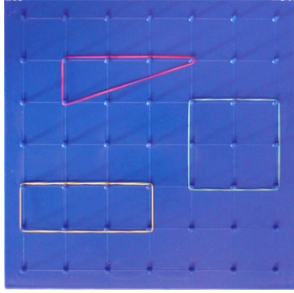


5. Trokut *ABC* preslikajte za vektor koji počinje u točki (2, -5), a završava u točki (3, -4), a zatim za vektor koji počinje u točki (5, -3), a završava u točki (3, -2). Zatim isti trokut preslikajte za vektor koji počinje u točki (5, -3), a završava u točki (3, -2) te za vektor koji počinje u točki (2, -5), a završava u točki (3, -4). Hoće li rezultat translacija biti jednak? Objasnite.





Centralna simetrija [1]

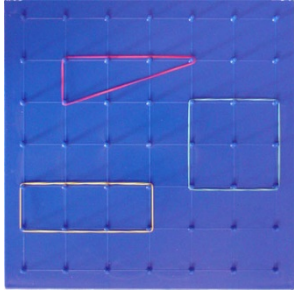


Pri sljedećoj aktivnosti geoploču ćemo poistovjetiti s koordinatnim sustavom. Aktivnost je namijenjena učenicima osmoga razreda.

Izvodeći ovu aktivnost, učenici će:

- preslikavati trokut s obzirom na zadanu točku (centar simetrije).

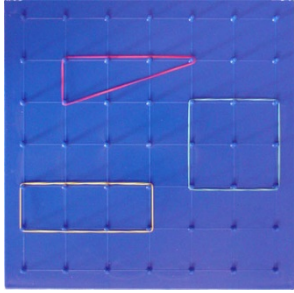
Centralna simetrija [2]



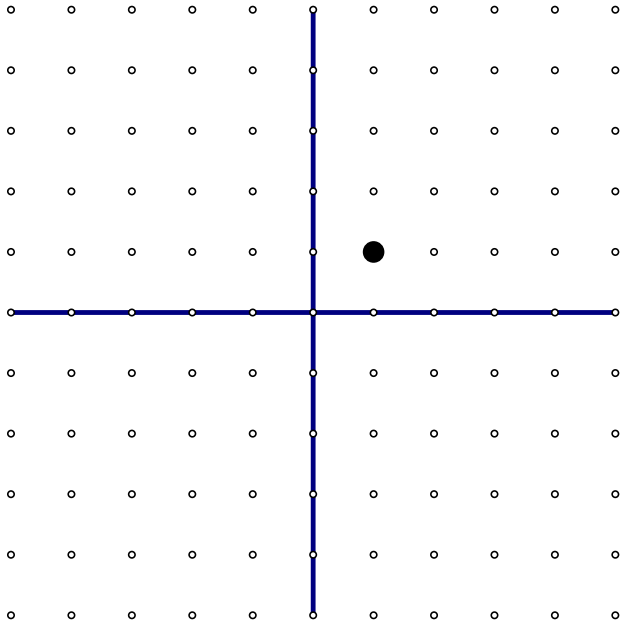
Potreban materijal:

- geoploča 11 × 11 na kojoj je gumicama definirana koordinatna ravnina
- više gumica različitih boja i duljina
- nastavni listić za svakog učenika
- mali kružni vijenac izrezan iz papira koji se može staviti na čavlić tako da predstavlja centar simetrije

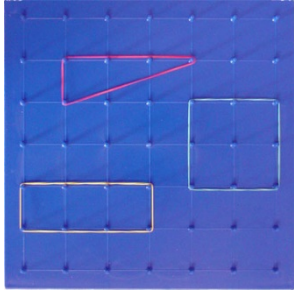
Centralna simetrija [3]



1. Neka je točka *A* s koordinatama (1, 1) središte centralne simetrije. Na geoploči prikažite trokut tako da se cijeli trokut nalazi u prvom kvadrantu. Preslikajte trokut s obzirom na točku *A* i zapišite koordinate novonastalih vrhova.

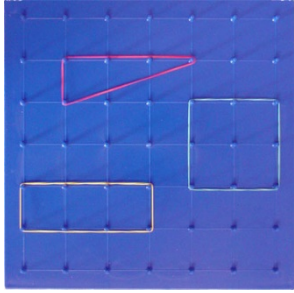


Centralna simetrija [4]

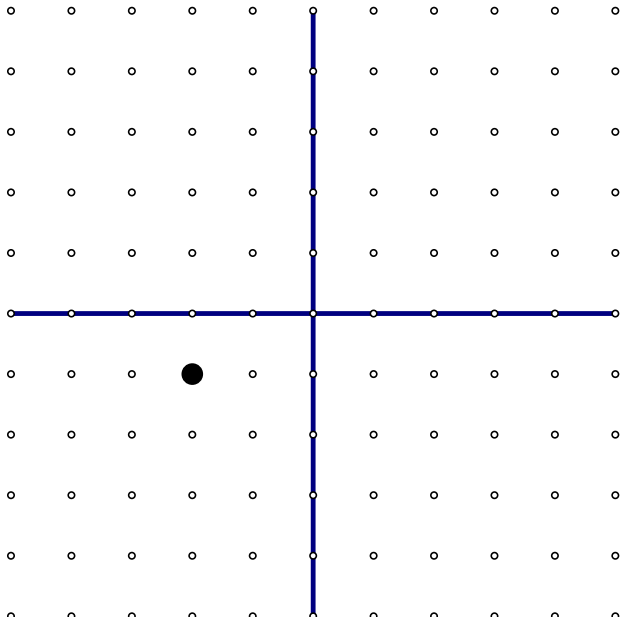


- Izmjerite duljine stranica originalnog trokuta te njegove centralnosimetrične slike. Što primjećujete?

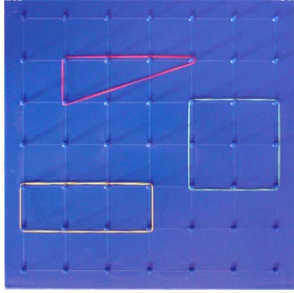
Centralna simetrija [5]



2. Neka je točka *B* s koordinatama (-2, -1) središte centralne simetrije. Na geoploči prikažite pravokutni trokut tako da se cijeli trokut nalazi u drugom kvadrantu. Preslikajte trokut s obzirom na točku *B* i zapišite koordinate novonastalih vrhova.

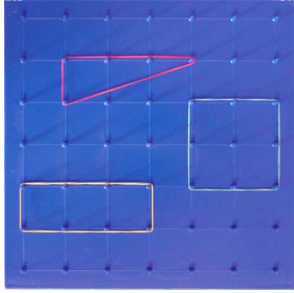


Centralna simetrija [6]

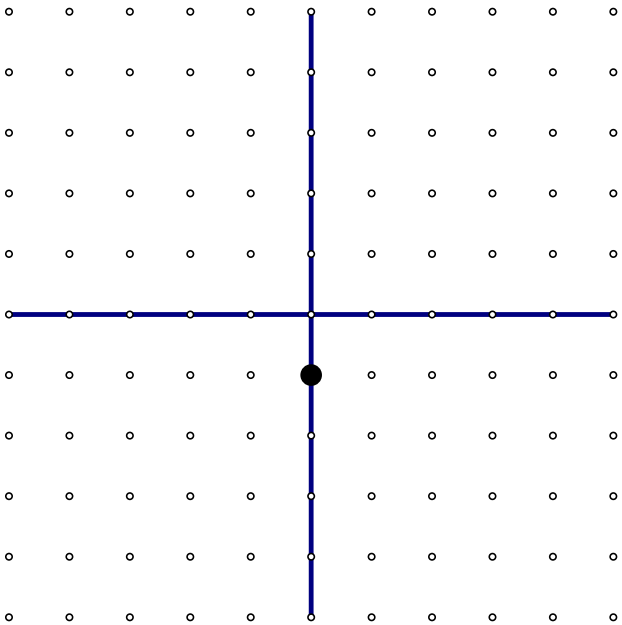


- Koje je vrste trokut dobiven centralnom simetrijom s obzirom na veličinu kutova?
- Izmjerite veličine kutova originalnog trokuta i njegove centralnosimetrične slike. Što primjećujete?

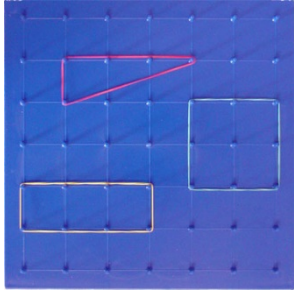
Centralna simetrija [7]



3. Neka je točka *C* s koordinatama (0, -1) središte centralne simetrije. Na geoploči prikažite dvije paralelne dužine. Preslikajte obje dužine s obzirom na točku *C*.

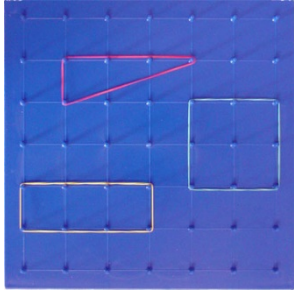


Centralna simetrija [8]

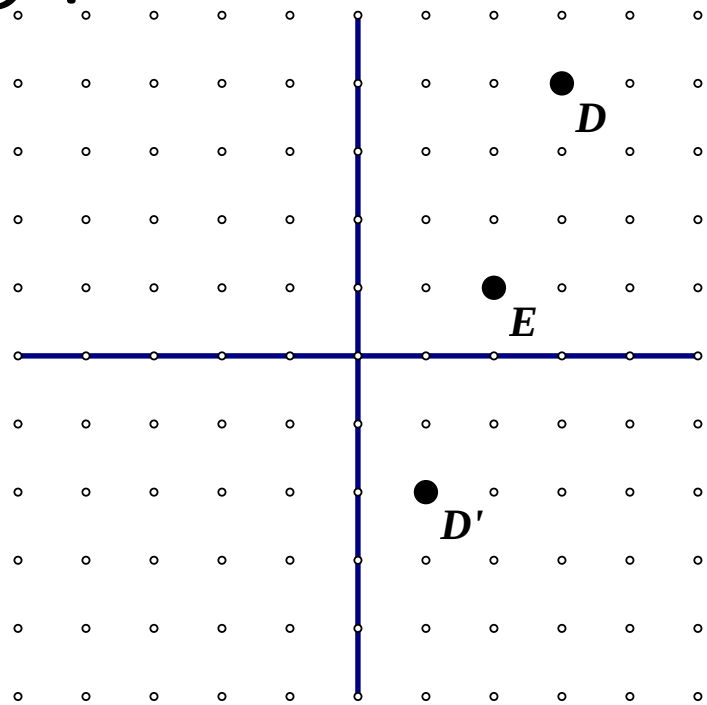


- Jesu li dužine dobivene centralnom simetrijom i dalje usporedne?
- Jesu li dužine dobivene centralnom simetrijom usporedne s originalnim dužinama?

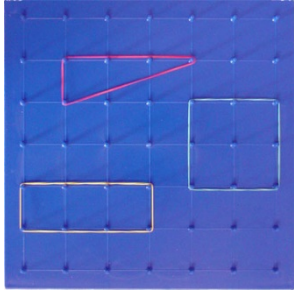
Centralna simetrija [9]



4. Koja svojstva čuva centralna simetrija?
5. Promotrimo točku *D* i njenu centralnosimetričnu sliku s obzirom na točku *E*, centar simetrije. Što je točka *E* dužini $\overline{DD'}$?



Centralna simetrija [10]

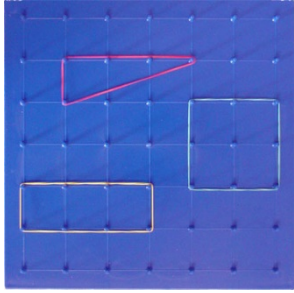


Aktivnost je namijenjena učenicima osmoga razreda.

Izvodeći ovu aktivnost, učenici će:

- primjenjivati svojstva centralnosimetričnog lika

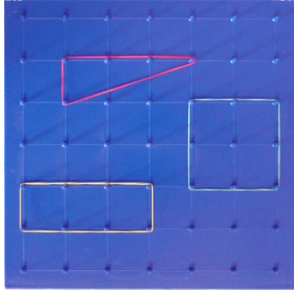
Centralna simetrija [11]



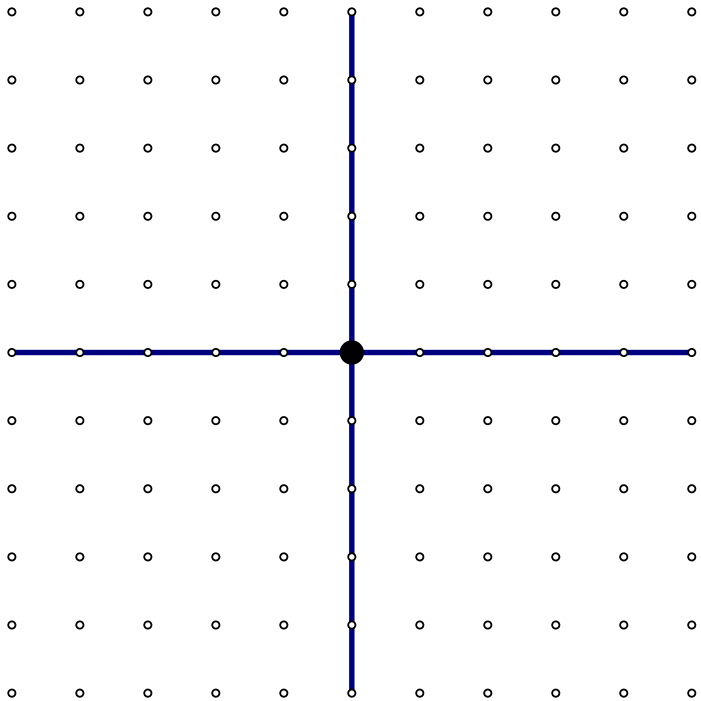
Potreban materijal:

- geoploča
- više gumica različitih boja i duljina
- nastavni listić za svakog učenika

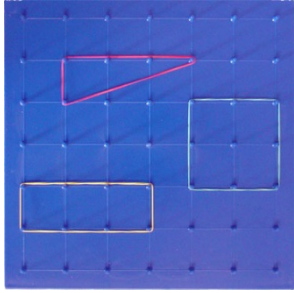
Centralna simetrija [12]



1. Na geoploči prikažite najmanje tri centralnosimetrična lika s ishodištem kao centrom simetrije.

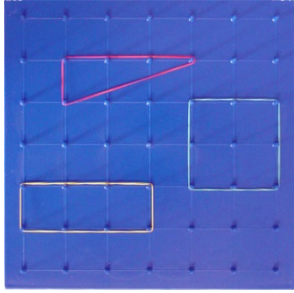


Centralna simetrija [13]

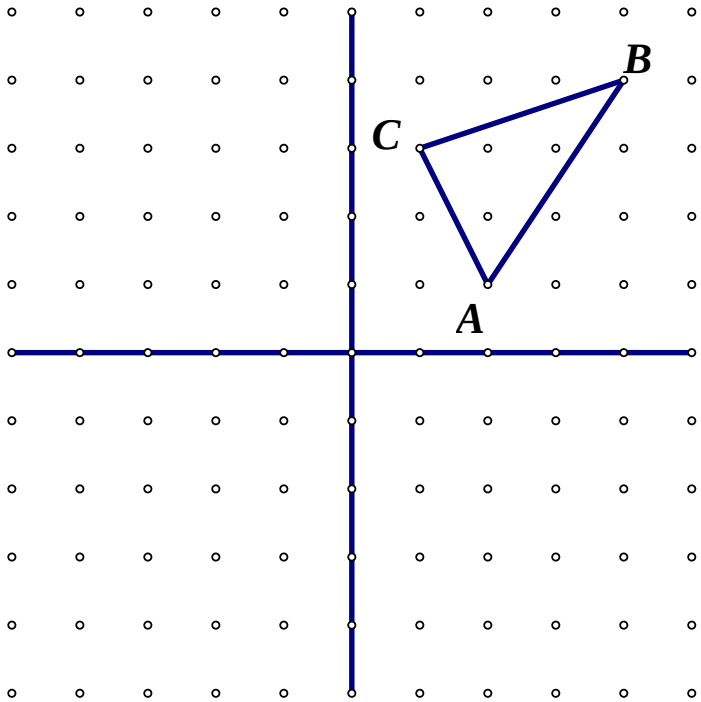


2. Može li centralnosimetričan lik imati neparan broj vrhova? Objasnite svoj odgovor.

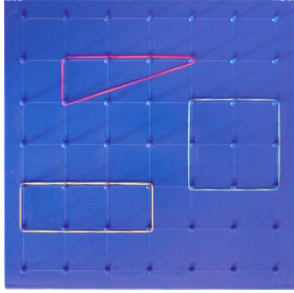
Centralna simetrija [14]



3. Neka je ishodište koordinatnog sustava centar simetrije. Centralnom simetrijom preslikajte trokut *ABC*.

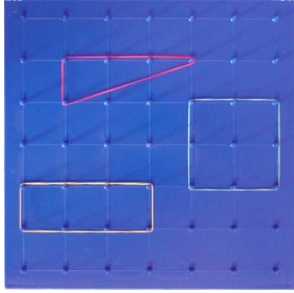


Centralna simetrija [15]

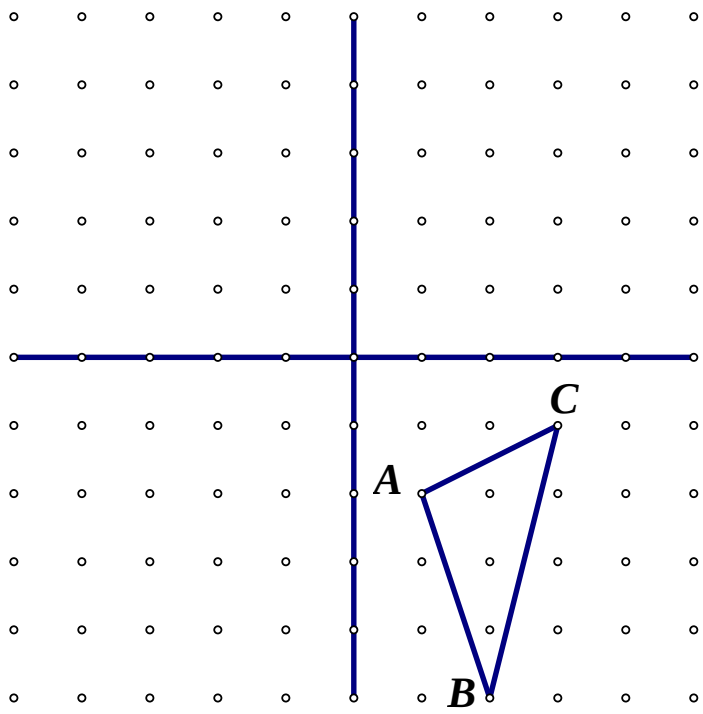


- Da se jedan od vrhova zadanog trokuta nalazio u ishodištu koordinatnog sustava, koju bi koordinatu taj vrh poprimio nakon preslikavanja centralnom simetrijom?

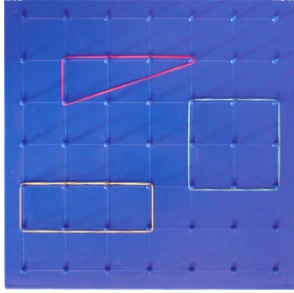
Centralna simetrija [16]



4. Neka je ishodište koordinatnog sustava centar simetrije. Centralnom simetrijom preslikajte trokut *ABC*.

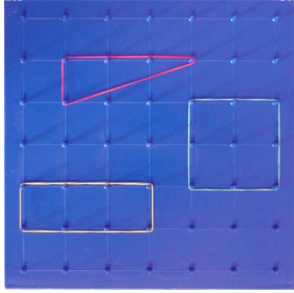


Centralna simetrija [17]



- Ako centralnom simetrijom preslikate mnogokut čiji je jedan vrh u s obzirom na ishodište, koje će koordinate poprimiti taj vrh?

Rotacija [1]

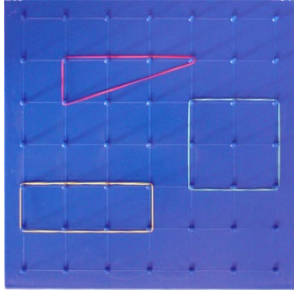


Izvodeći ovu aktivnost, učenici će:

- rotirati trokut oko ishodišta za kut od 180°, 90°, -90°.

Aktivnost je namijenjena učenicima 8. razred osnovne škole.

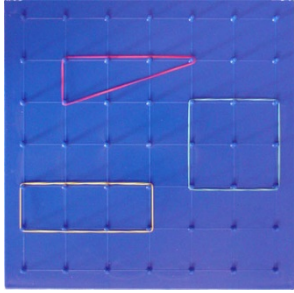
Rotacija [2]



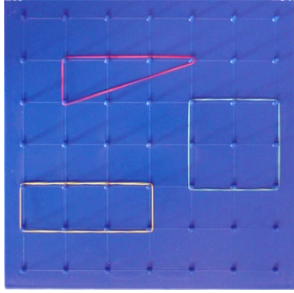
Potreban materijal:

- geoploča 11 × 11 na kojoj je gumicama definirana koordinatna ravnina
- više gumica različitih boja i duljina
- nastavni listić

Rotacija [3]



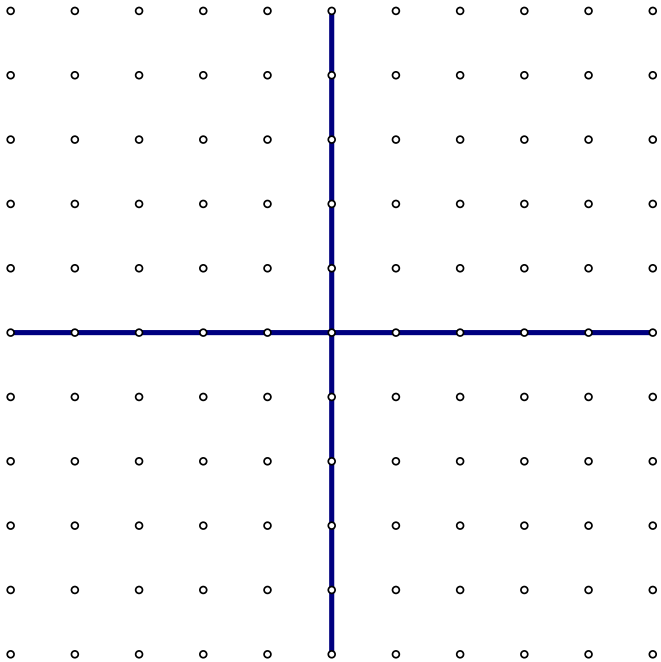
- Učenike podijelimo u tri skupine.
- Svaka grupa treba rotirati trokut oko ishodišta, jedna grupa za 180°, druga grupa za 90° i treća za -90°.
- Učenici trebaju svoja rješenja zabilježiti na dobiveni nastavni listić i odgovoriti na postavljena pitanja.
- Kada grupe izvrše svoj zadatak, redom riješe zadatke ostalih grupa.
- Na kraju učenici međusobno usporede dobivena rješenja.



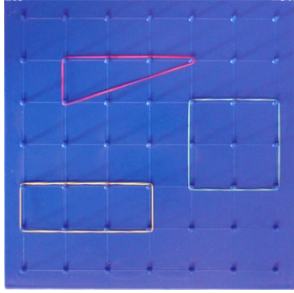
Rotacija [4]

SKUPINA A

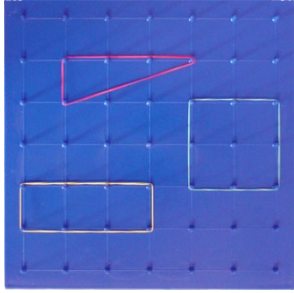
Na geoploči gumicama prikažite trokut s vrhovima (1, 3), (2, 1), (3, 1). Rotirajte trokut za 180° oko ishodišta. Nacrtajte odgovarajuću sliku i odredite koordinate dobivenih vrhova.



Rotacija [5]



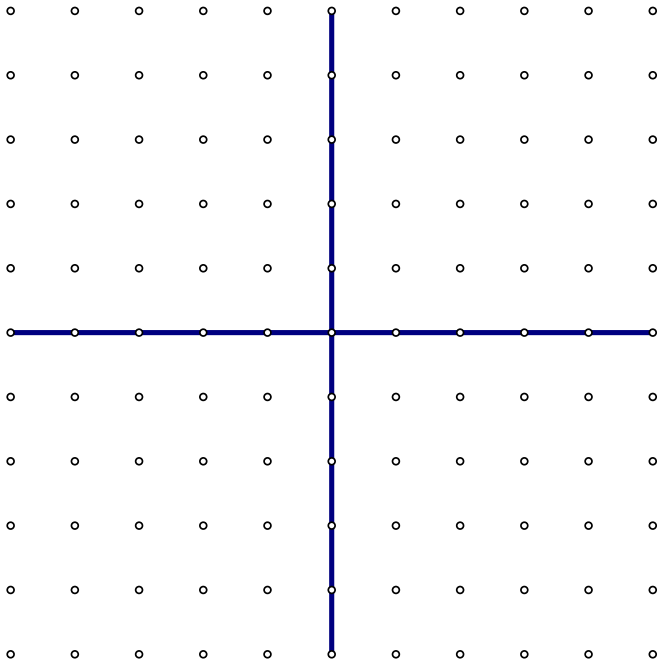
- Ako rotirate mnogokut s vrhom u (x, y) za 180° oko ishodišta, koje će koordinate poprimiti taj vrh?
- Možete li rotaciju mnogokuta oko ishodišta za kut od 180° poistovjetiti s nekom drugom metodom preslikavanja? Ako možete, koja bi to metoda bila?



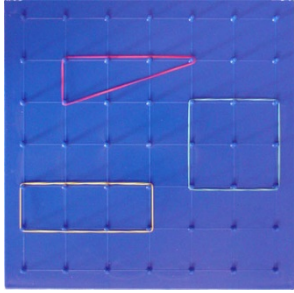
Rotacija [6]

SKUPINA B

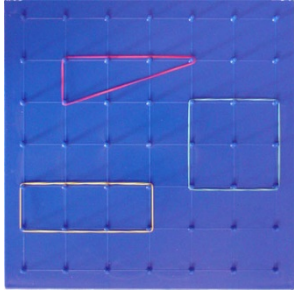
Na geoploči gumicama prikažite trokut s vrhovima (1, 3), (2, 1), (3, 1). Rotirajte trokut za +90° oko ishodišta. Nacrtajte odgovarajuću sliku i odredite koordinate dobivenih vrhova.



Rotacija [7]



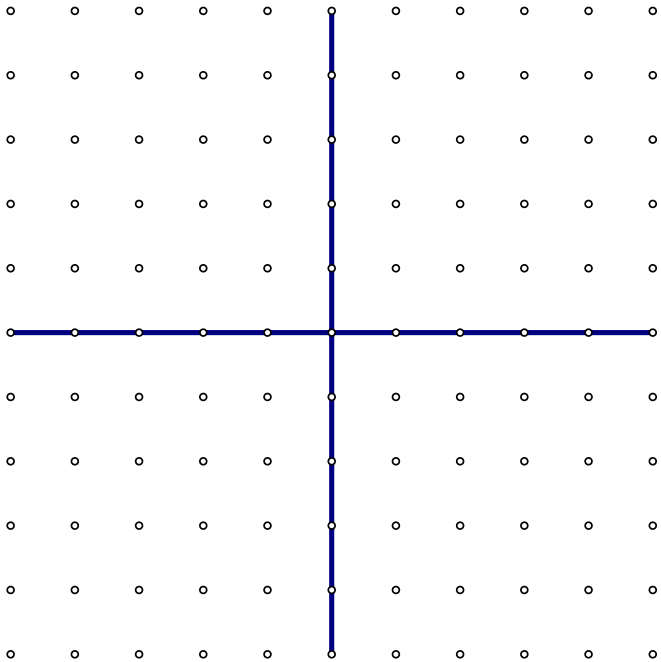
- Ako rotirate mnogokut s vrhom u (x, y) za +90° oko ishodišta, koje će koordinate poprimiti taj vrh?



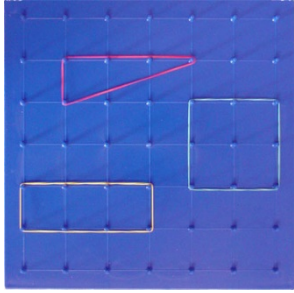
Rotacija [8]

SKUPINA C

Na geoploči gumicama prikažite trokut s vrhovima (1, 3), (2, 1), (3, 1). Rotirajte trokut za -90° oko ishodišta. Nacrtajte odgovarajuću sliku i odredite koordinate dobivenih vrhova.



Rotacija [9]



- Ako rotirate mnogokut s vrhom u (*x*, *y*) za -90° oko ishodišta, koje će koordinate poprimiti taj vrh?

Final Egad