



# 8. KONGRES NASTAVNIKA MATEMATIKE

## 3D bilježnice

**Svjetlana Jakšić,**  
**profesor mentor,**  
Zagreb, 3.-5. srpnja 2018.

# Uuuuh, ta matematika...

- U nastavi matematike često se susrećemo s nezainteresiranošću učenika
- Proizlazi najčešće iz nerazumijevanja nastavnih sadržaja
- Postoje sadržaji koje je zaista teško predočiti na zanimljiv način, npr. algebarski razlomci.



# Kreativnost, zornost



# motivacija, interes



# SAVRŠENSTVO GEOMETRIJE



Frank  
Gehry



Dvije cjeline nastavnog programa 8. razreda osnovne škole te 2. razreda gimnazija, kao i nekih strukovnih programa, odnose se na proučavanje odnosa i preslikavanja geometrijskih objekata u prostoru te izračunavanje oplošja i obujma geometrijskih tijela.



- Geometrijski sadržaji su oni za koje učenike i nije teško zainteresirati.
- ...dok ne dođemo do geometrije prostora i kose projekcije ravnina, geometrijskih tijela, presjeka...
- Potrebno je vidjeti, uočiti, potrebna je prostorna percepcija...
- Zornost je narušena...
- Posljedica - gubljenje interesa i za te sadržaje...





# Modeli tijela?

- Školski modeli (žičani, plastični...)
  - po jedno tijelo pojedine vrste i ne može biti svakom učeniku dostupno
- Modeli koje učenici izrađuju lijepljenjem mreža
  - nježni i krhki...
- Geogebra
  - odlična, iako se ne može dodirnuti, osjetiti
  - potrebno je znanje da bi aktivno mogli koristiti...

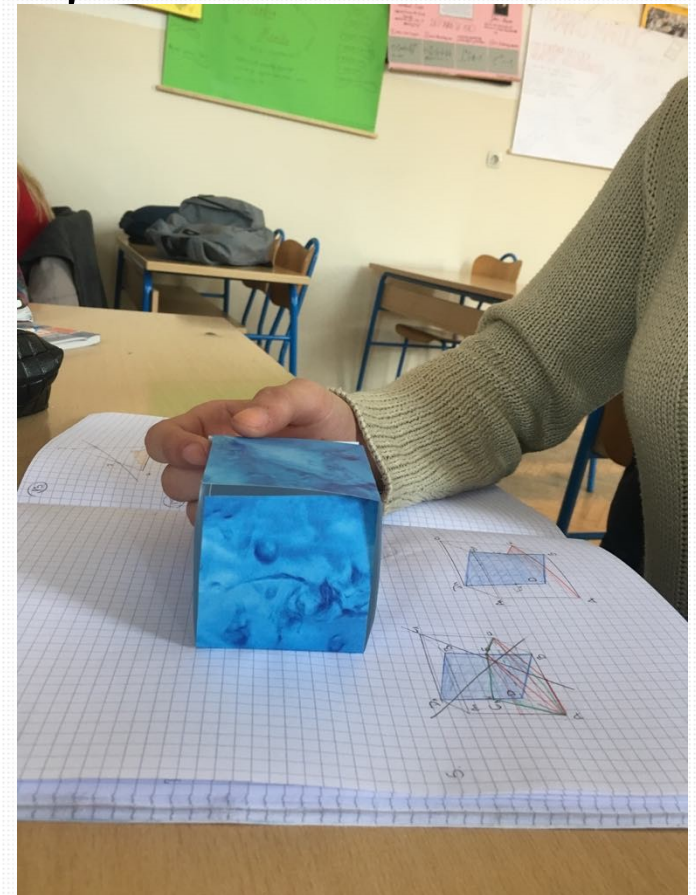
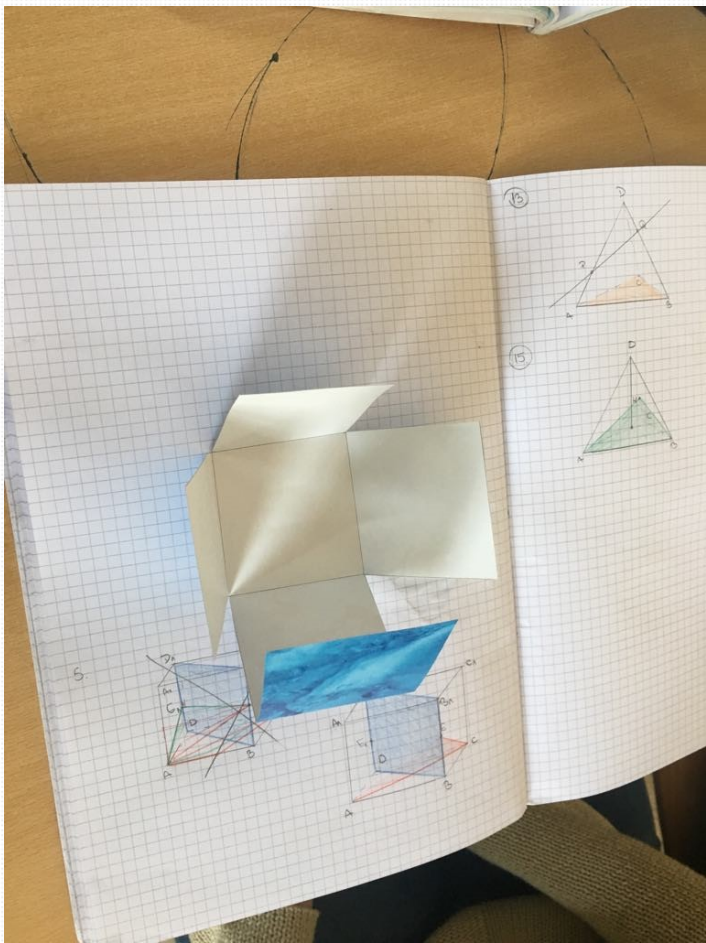




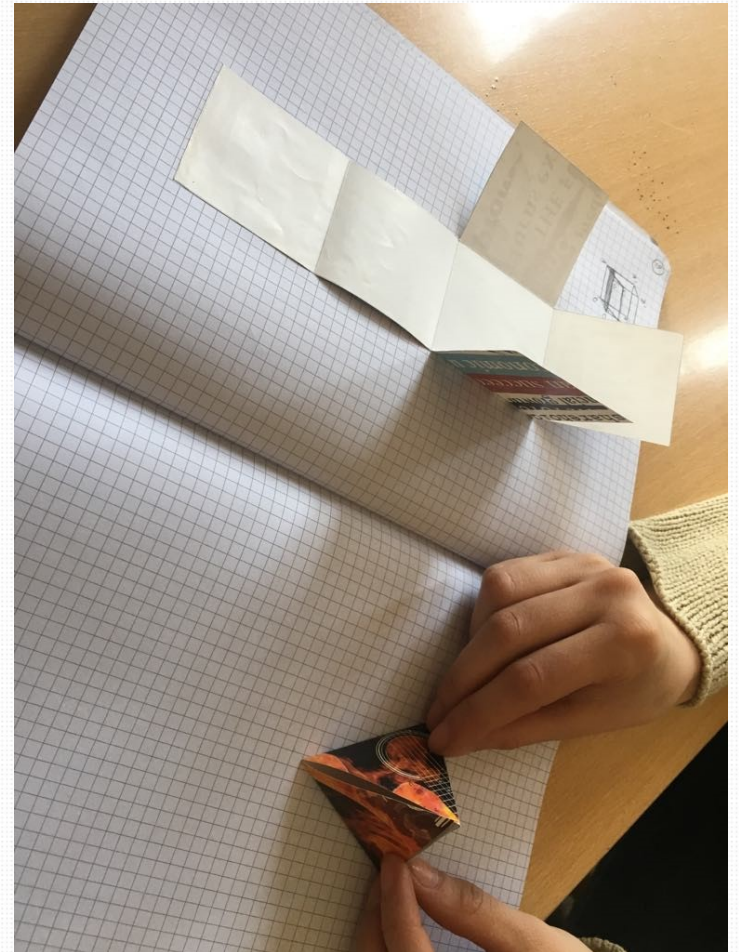
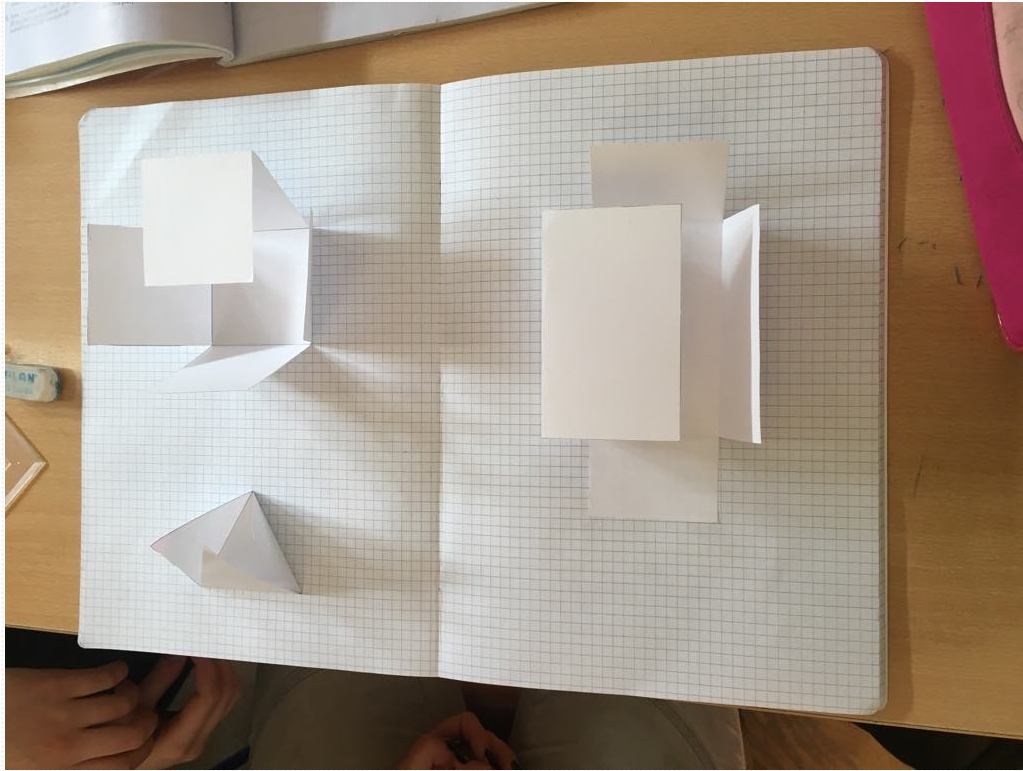
# Izrada 3D matematičke bilježnice

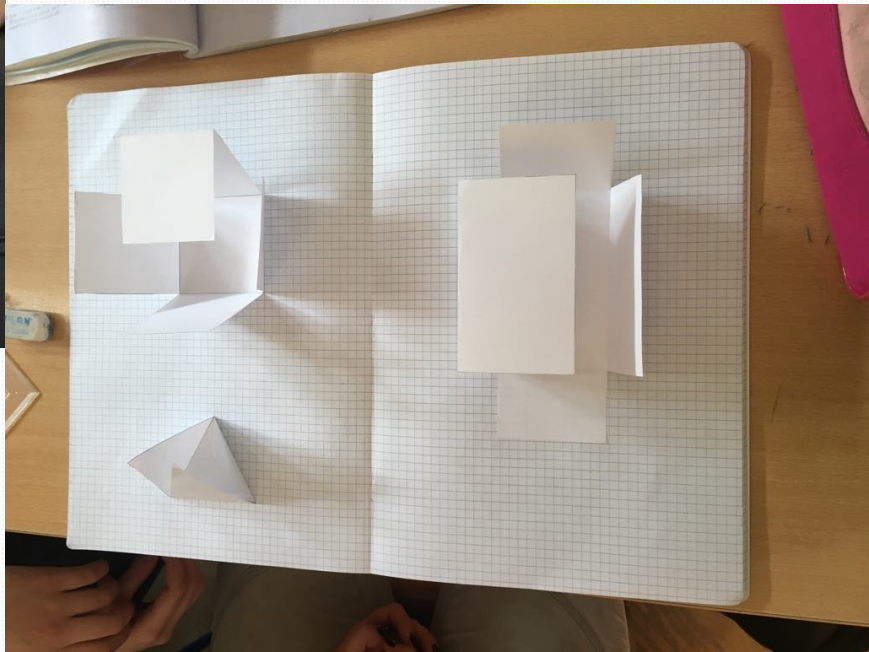
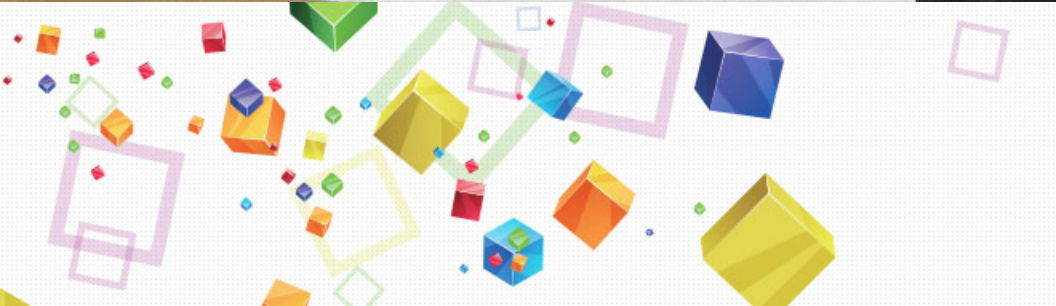
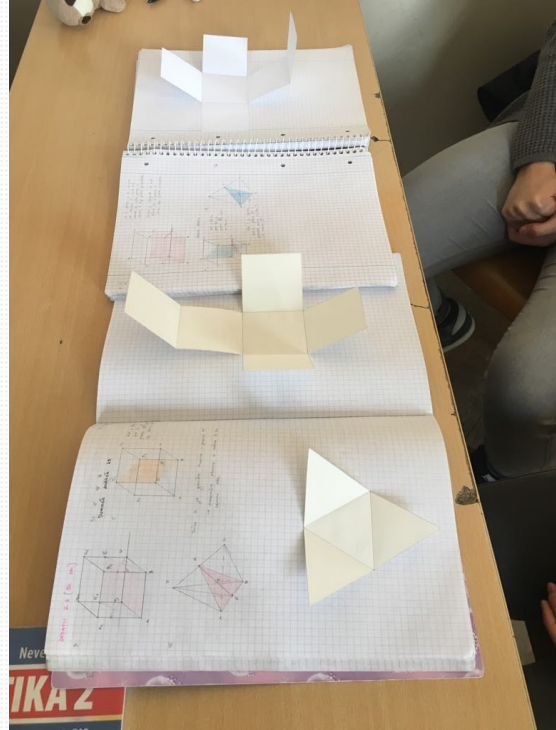
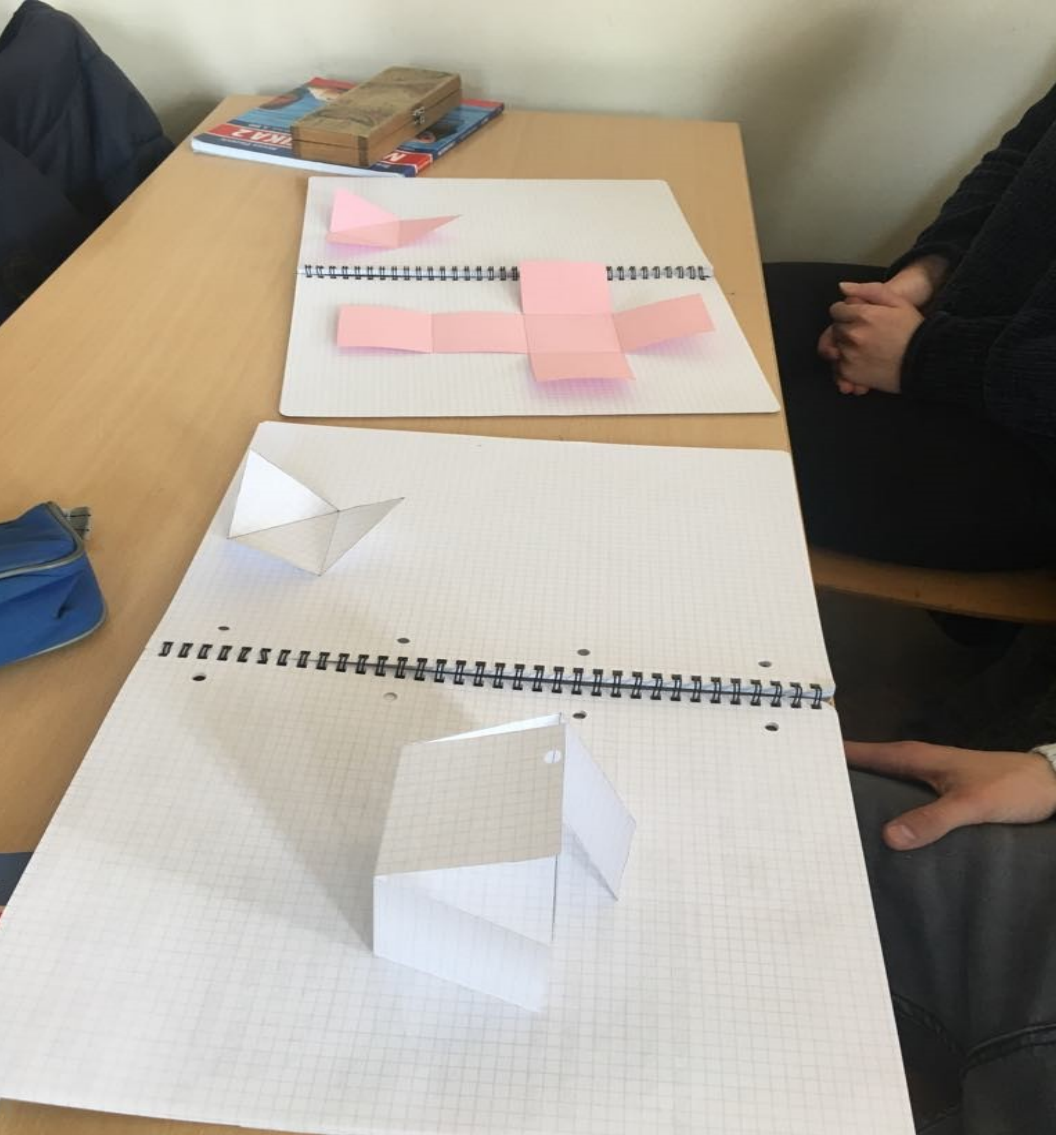
- Učenici samostalno (najčešće je to domaća zadaća) izrezuju mreže tijela i lijepo ih u bilježnice, ali samo jednu plohu, ostavljajući otvorenu mrežu -primjeri učeničkih bilježnica
- Krećemo s kvadrom i kockom na kojima prezentiramo prostor i odnose objekata u prostoru (cjelina „Geometrija prostora“ – 2. razred gimnazije i neki strukovni programi)

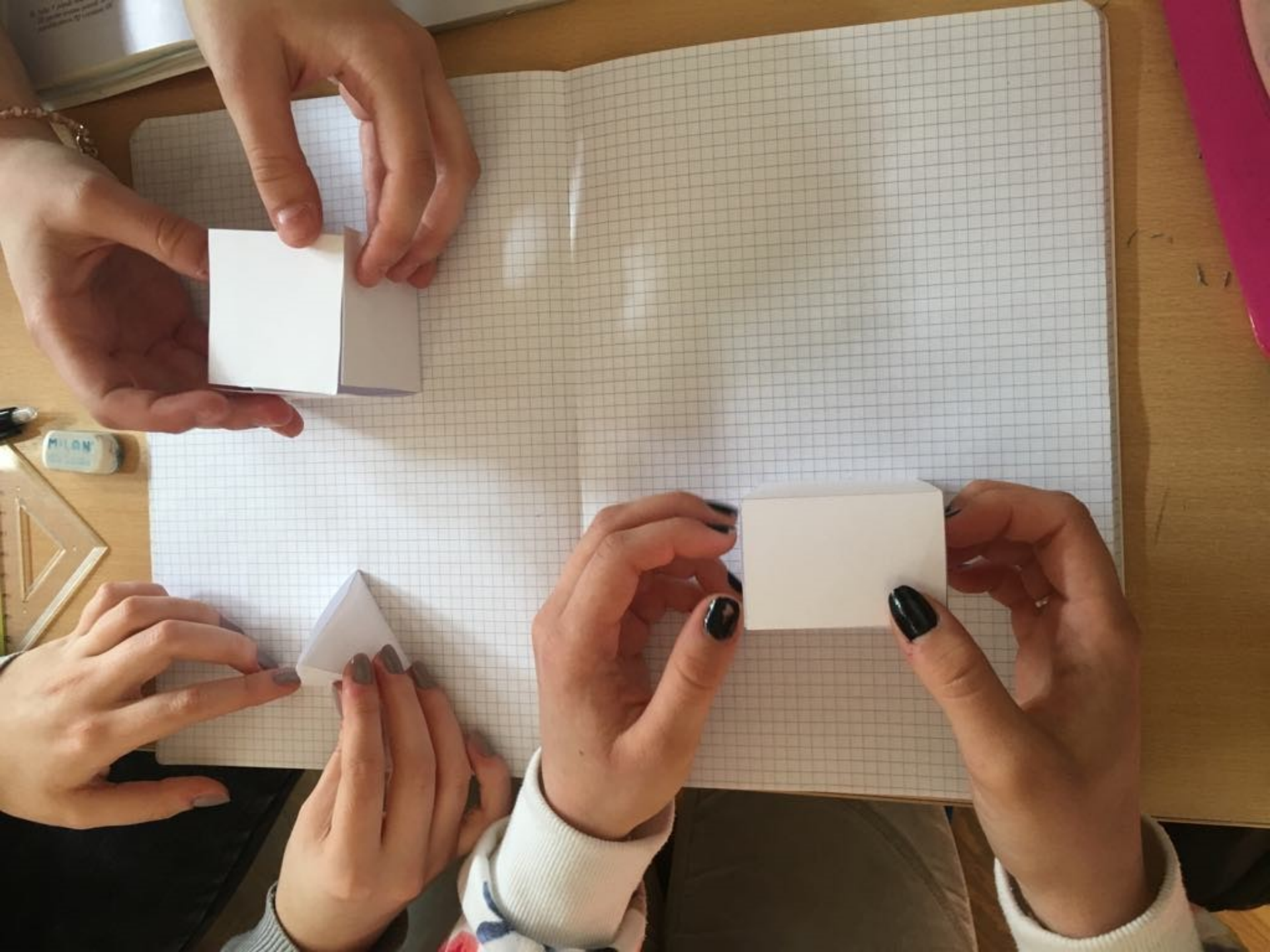
Mreža je u bilježnici sklopljena, a po potrebi se može podignuti i sklopiti te tako uočiti tražene odnose te odgovoriti na pitanja i riješiti postavljene zadatke.



# Slijedi tetraedar...

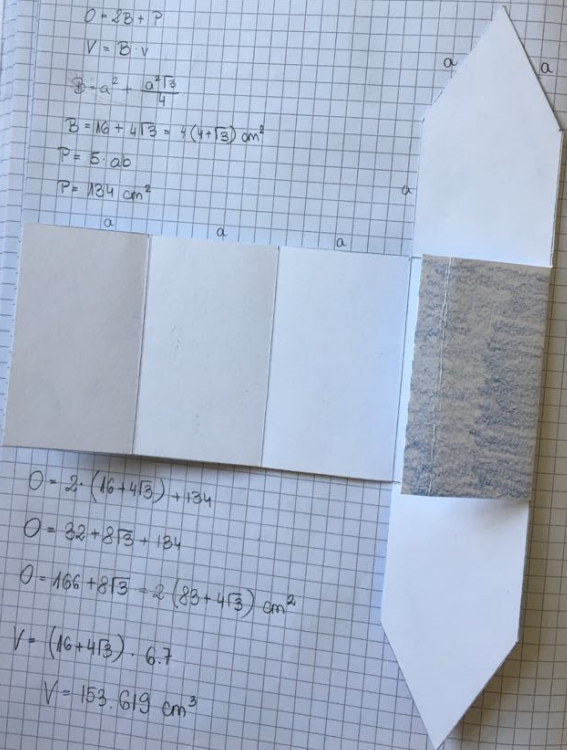




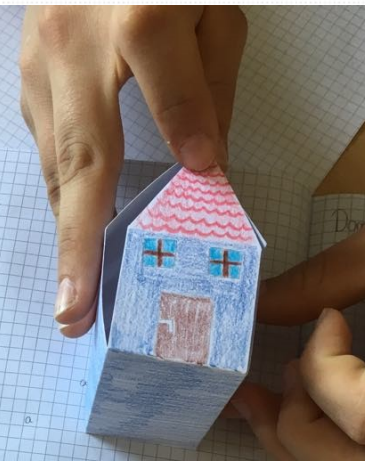


# Prizme

$a = 4 \text{ cm}$   
 $b = 6.7 \text{ cm}$   
 $y = b$   
 $v = 6.7 \text{ cm}$   
 $O = ?$   
 $O = 2B + P$   
 $V = B \cdot v$   
 $S = a^2 + \frac{a^2 \sqrt{3}}{4}$   
 $B = 16 + 4\sqrt{3} = 4(4 + \sqrt{3}) \text{ cm}^2$   
 $P = 5 \cdot ab$   
 $P = 184 \text{ cm}^2$



$O = 2 \cdot (16 + 4\sqrt{3}) + 184$   
 $O = 32 + 8\sqrt{3} + 184$   
 $O = 166 + 8\sqrt{3} = 2(83 + 4\sqrt{3}) \text{ cm}^2$   
 $V = (16 + 4\sqrt{3}) \cdot 6.7$   
 $V = 153.619 \text{ cm}^3$

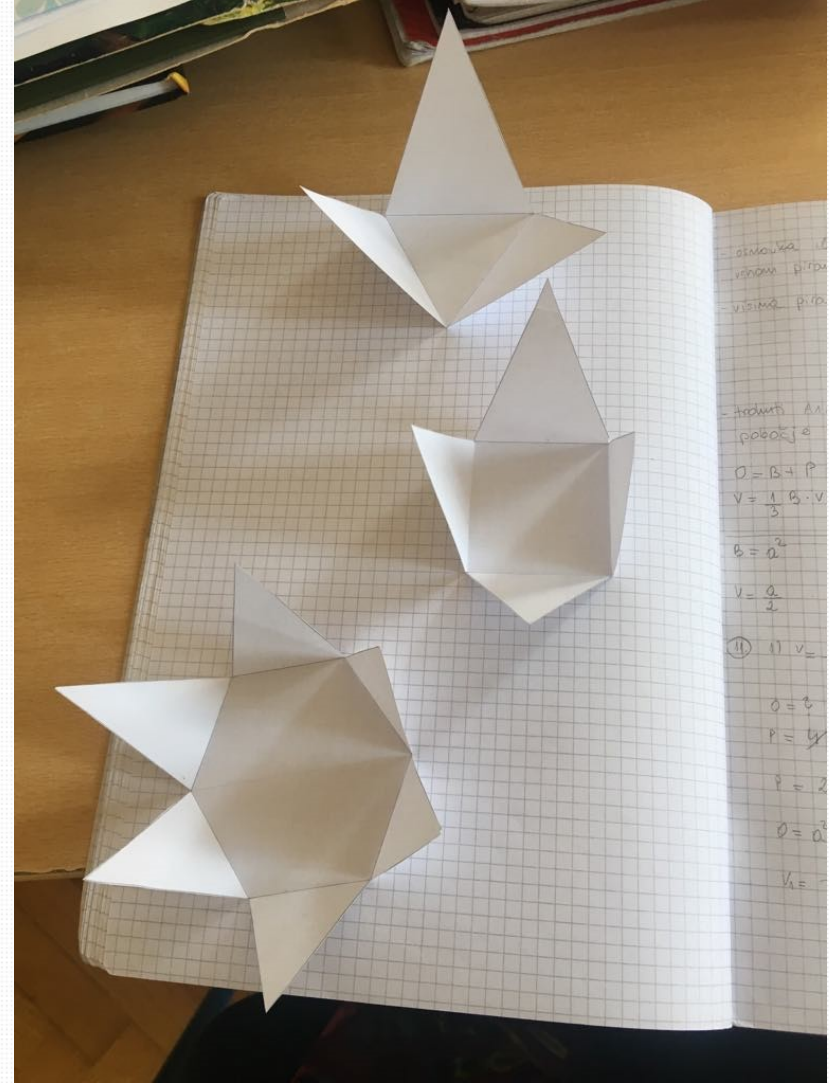
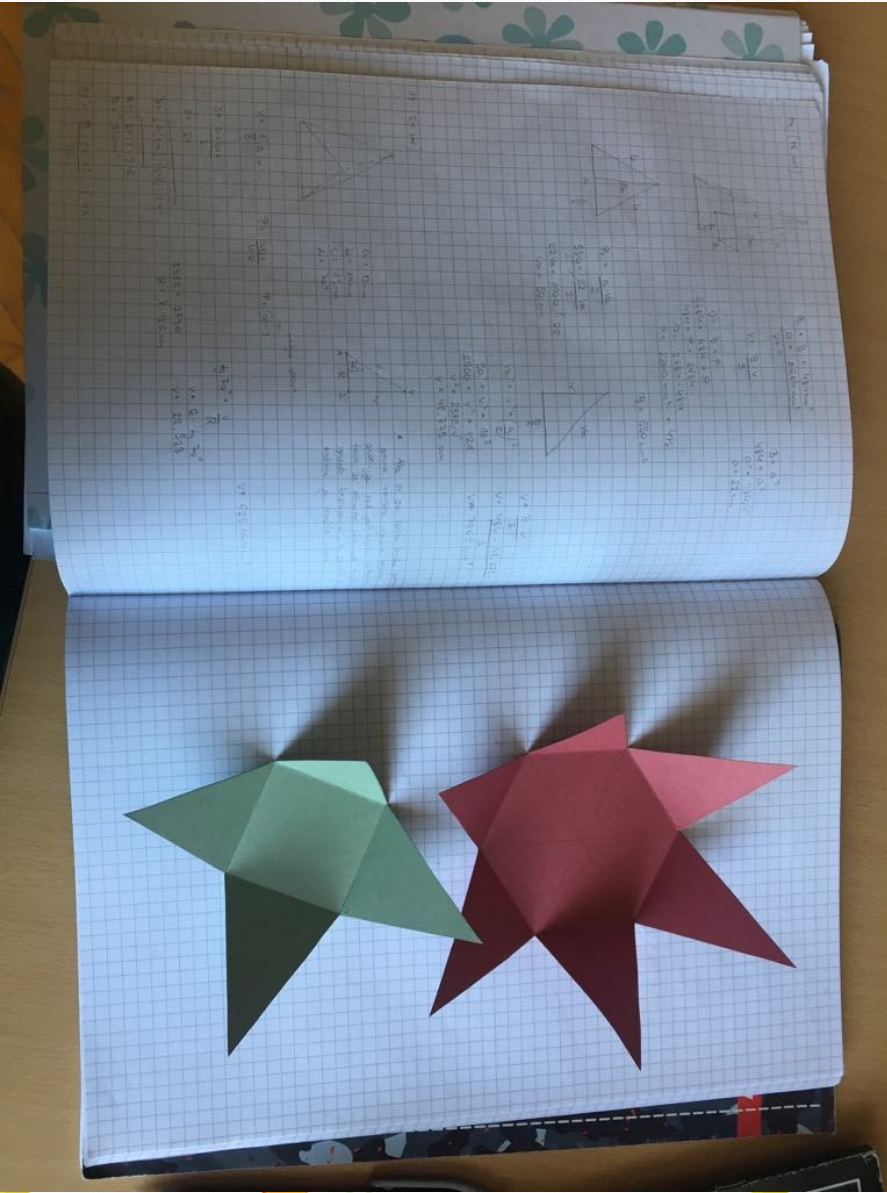


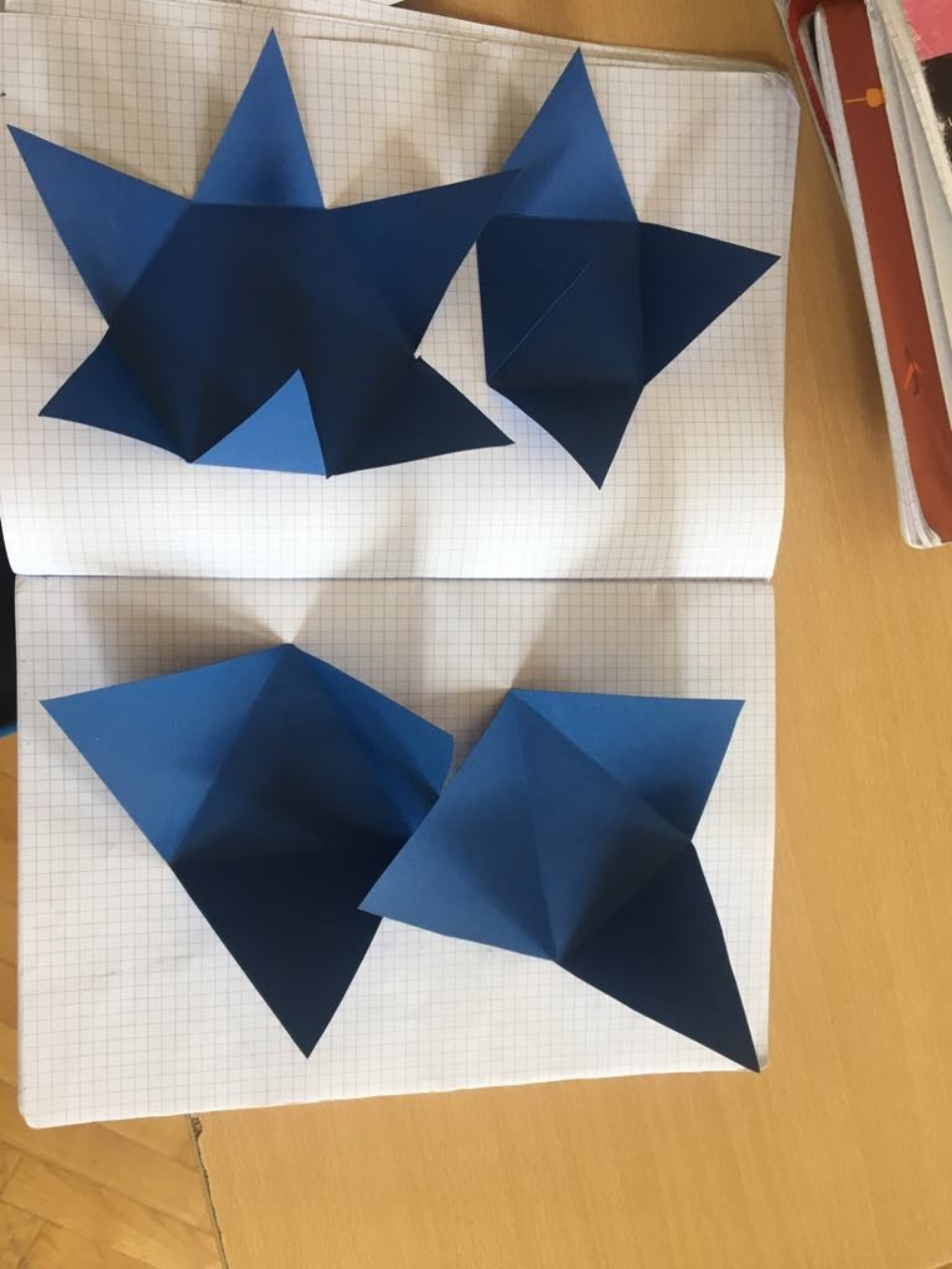
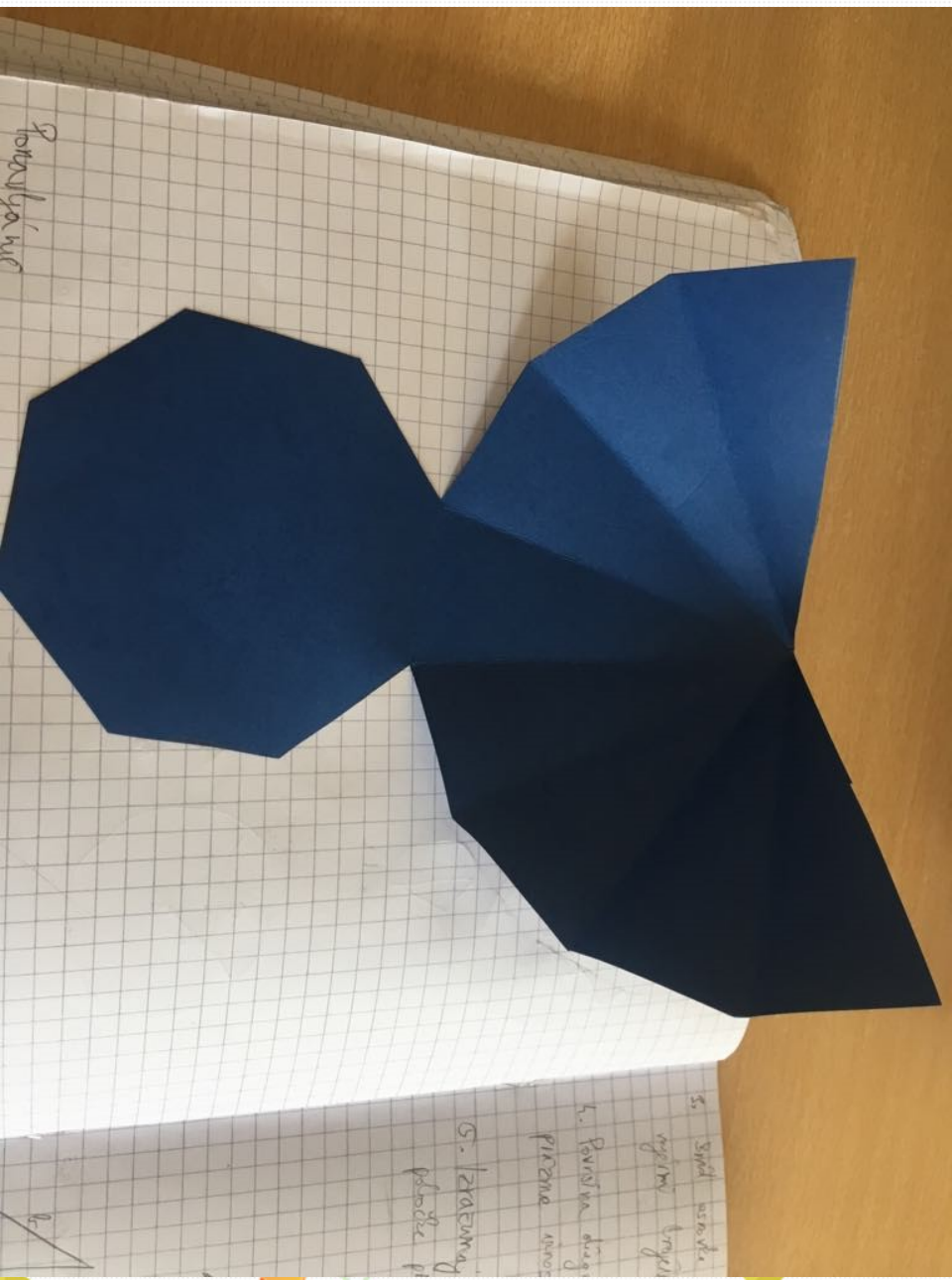
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# Piramide





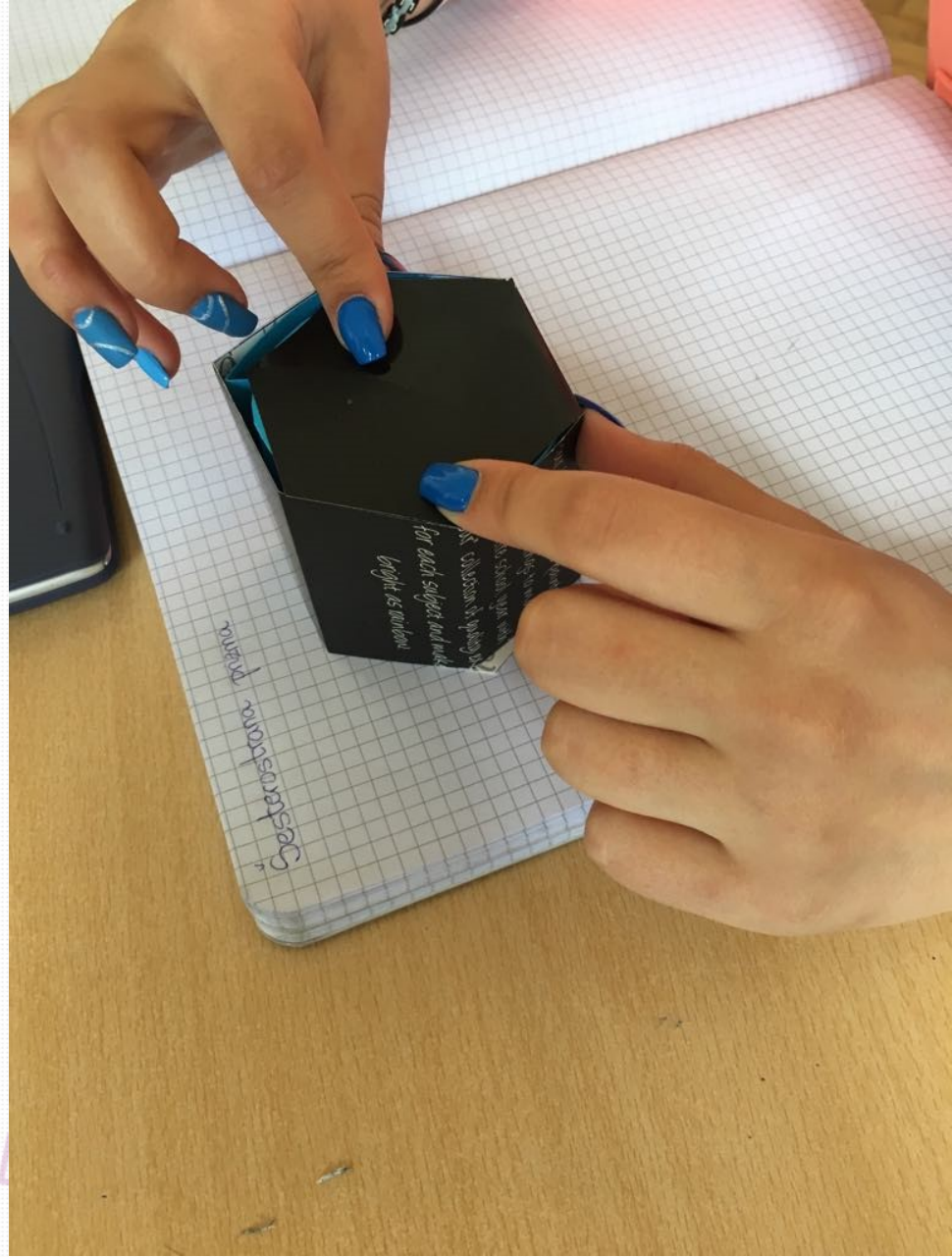
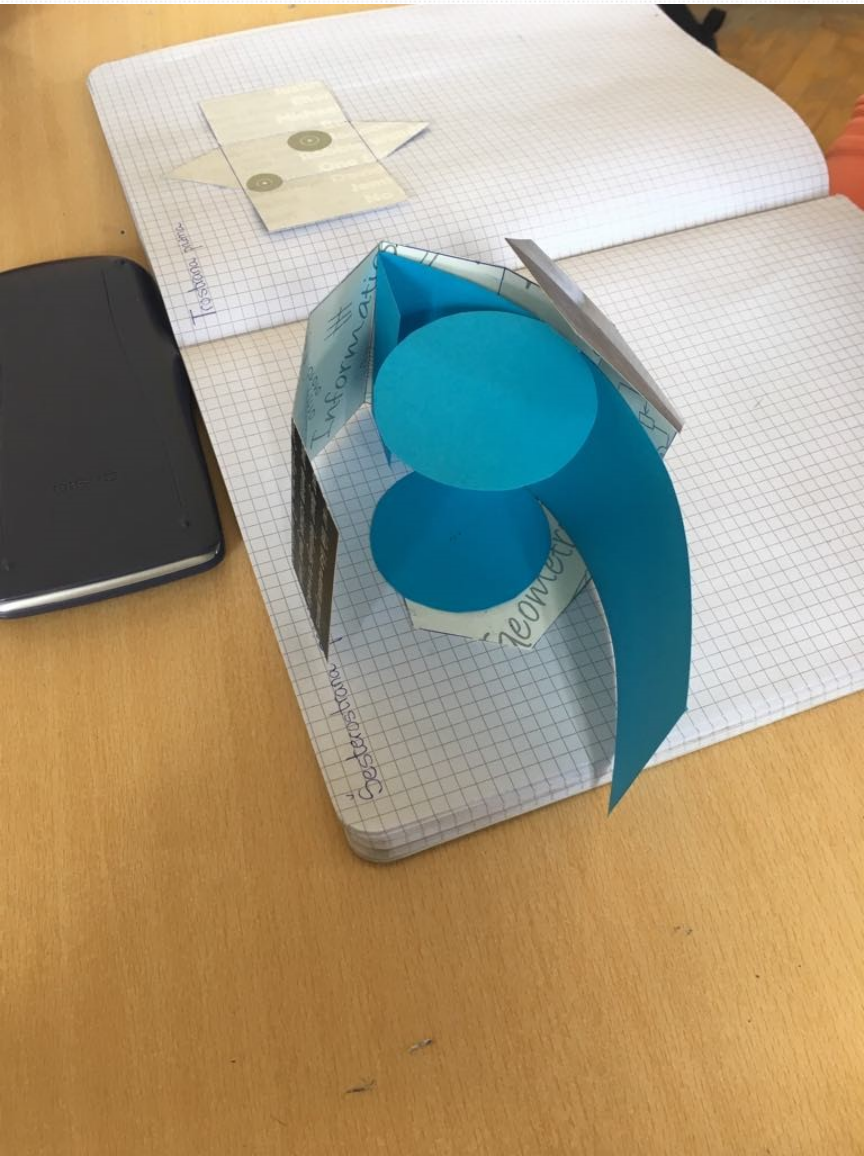




# Zadaci otvorenog tipa

## Primjer:

- valjak upisan u pravilnu šesterostranu prizmu proizvoljnih dimenzija koju su ranije izradili i zalijepili u bilježnice
- Tijekom analize domaće zadaće provjerili smo koliko su uspješno riješili taj zadatak otvorenog tipa, tj. može li se taj valjak „posložiti“, upisati u prizmu.



# Rotacijska tijela

„zastavice“



Izračunali smo  
oplošja i obujme  
tijela nastalih  
rotacijom  
različitih  
proizvoljnih  
ravninskih likova.



Handwritten mathematical work on grid paper showing the calculation of surface area and volume for a cone and a cylinder, with a 3D model of a green car wheel on a stick.

Diagrams show a right-angled triangle with legs 8 cm and 4 cm, and a hypotenuse of  $4\sqrt{2}$  cm. A rotation arrow indicates the cone formed. Another diagram shows a cylinder with height 8 cm and radius  $4\sqrt{2}$  cm.

$r = 4\sqrt{2}$  cm  
 $V = 8$  cm

$Q_v = 2 \cdot (r^2 \pi) + 2r\pi \cdot V$   
 $Q_v = 2 \cdot ((4\sqrt{2})^2 \pi) + 2 \cdot 4\sqrt{2}$   
 $Q_v = 2 \cdot 32\pi + 8\sqrt{2} \cdot \pi \cdot 8$   
 $Q_v = 64\pi + 64\sqrt{2}\pi$   
 $Q_v = 485.406 \text{ cm}^2$

$Q_{\text{tijelo}} = Q_v \cdot 2 = Q_s$   
 $Q_t = 485.406 \cdot 2 = 970.812$   
 $Q_t = 71.086 \text{ cm}^2$

$V = \frac{1}{2} \cdot 8 \cdot 4 = 16$   
 $V_t = V \cdot 2 = 32$   
 $V_t = 32 \text{ cm}^3$

$V_v = \pi r^2 V$   
 $V_v = (4\sqrt{2})^2 \cdot \pi \cdot 2$   
 $V_v = 32\pi \cdot 2$   
 $V_v = 64\pi \text{ cm}^3$   
 $V_v = 201.062 \text{ cm}^3$

$V_s = \frac{\pi r^2 V}{3}$   
 $V_s = \frac{(4\sqrt{2})^2 \cdot \pi \cdot 2}{3}$   
 $V_s = \frac{32\pi \cdot 2}{3}$   
 $V_s = \frac{64\pi}{3}$   
 $V_s = 21.333 \pi \text{ cm}^3$   
 $V_s = 67.021 \text{ cm}^3$

$V_t = V_v - 2 \cdot V_s$   
 $V_t = 201.062 - 2 \cdot 67.021$   
 $V_t = 201.062 - 134.041$   
 $V_t = 67.021 \text{ cm}^3$

Andrea Jakovljević, 3.b

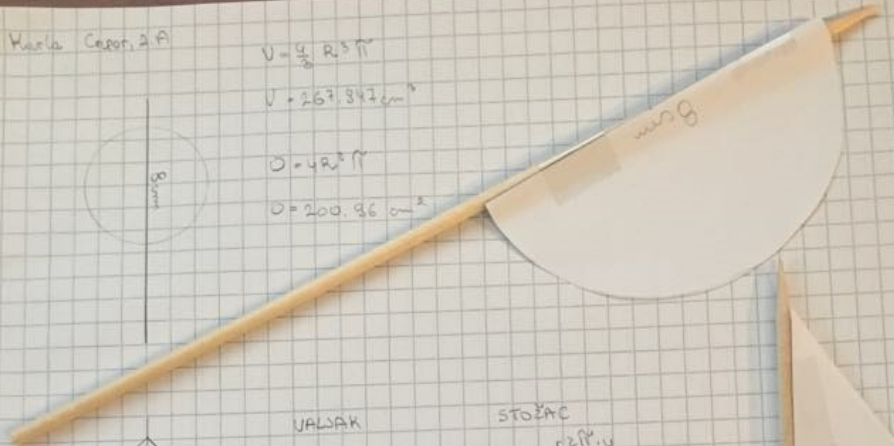
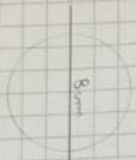
Wielka Ciężka, 2. A

$$V = \frac{4}{3} R^3 \pi$$

$$V = 267.842 \text{ cm}^3$$

$$O = 4R^2 \pi$$

$$O = 200.96 \text{ cm}^2$$



WALIAK

$$V = r^2 \pi \cdot u$$

$$V = 187.32 \text{ cm}^3$$

STOŻEC

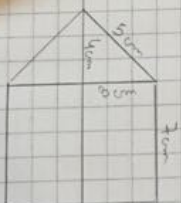
$$V = \frac{r^2 \pi \cdot u}{3}$$

$$V = 62.768 \text{ cm}^3$$

$$V_{\text{wsumy}} = 250.5 \text{ cm}^3$$

$$O = r^2 \pi s + r^2 \pi + r^2 \pi u$$

$$O = 255.5 \text{ cm}^2$$



$$d = 6.8$$

$$r = 3.45$$

$$s = 4 \text{ cm}$$

$$V_1 = 4.088 \text{ cm}^3$$

$$V_2 = \frac{r^2 \pi u}{3} \cdot 2$$

$$V_2 = 102.106 \text{ cm}^3$$

$$V_{\text{wielka}} = 4 \text{ cm}^3$$

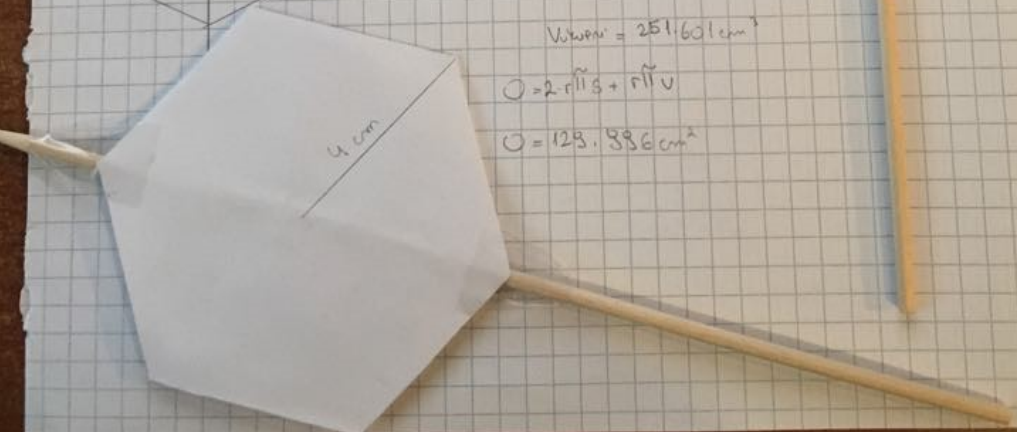
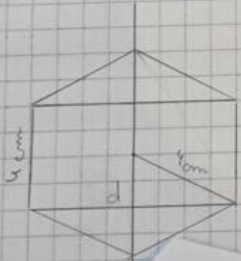
$$V_3 = r^2 \pi \cdot u$$

$$V_3 = 148.435 \text{ cm}^3$$

$$V_{\text{wsumy}} = 251.601 \text{ cm}^3$$

$$O = 2 \cdot r^2 \pi s + r^2 \pi u$$

$$O = 128.886 \text{ cm}^2$$



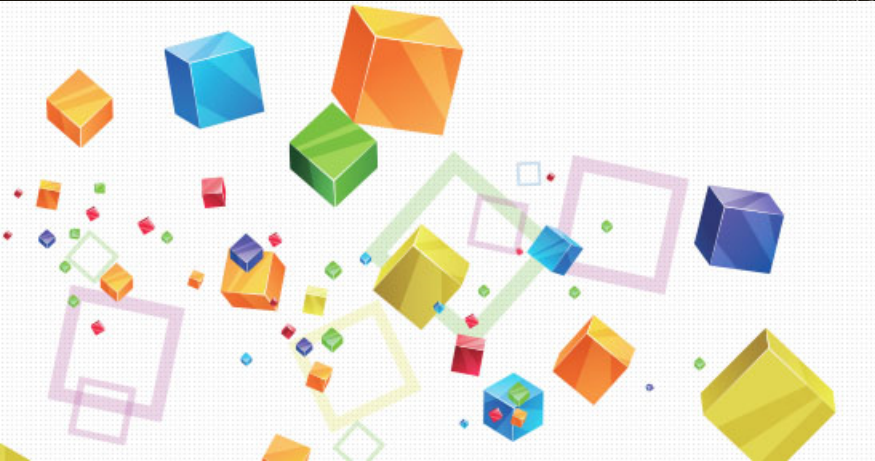
Nekima se  
svidjelo pa  
su napravili  
i više 😊

# A Platonova tijela iz buketa s početka priče?



# Kraj?

To je tek  
početak  
jedne lijepo  
priče 😊



**Pomoću matematike mi je lakše rješavati neke životne probleme jer me matematika kroz zadatke tjera da razmišljam, uključim male sive stanice i logiku da bih došla do rješenja.**

**Lucija**

