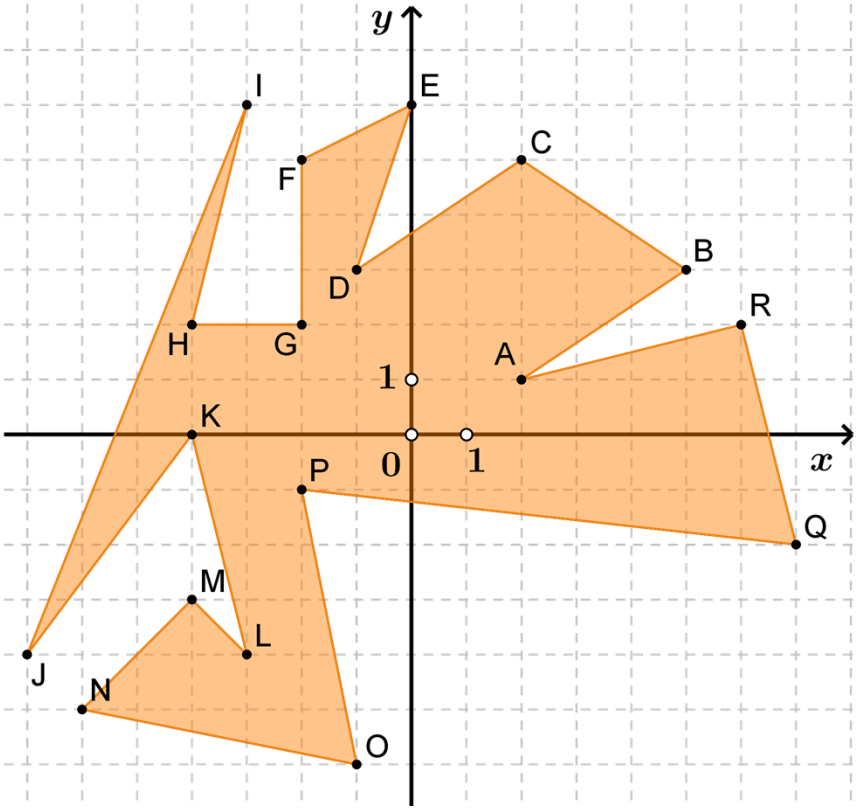
METODA VEZICA

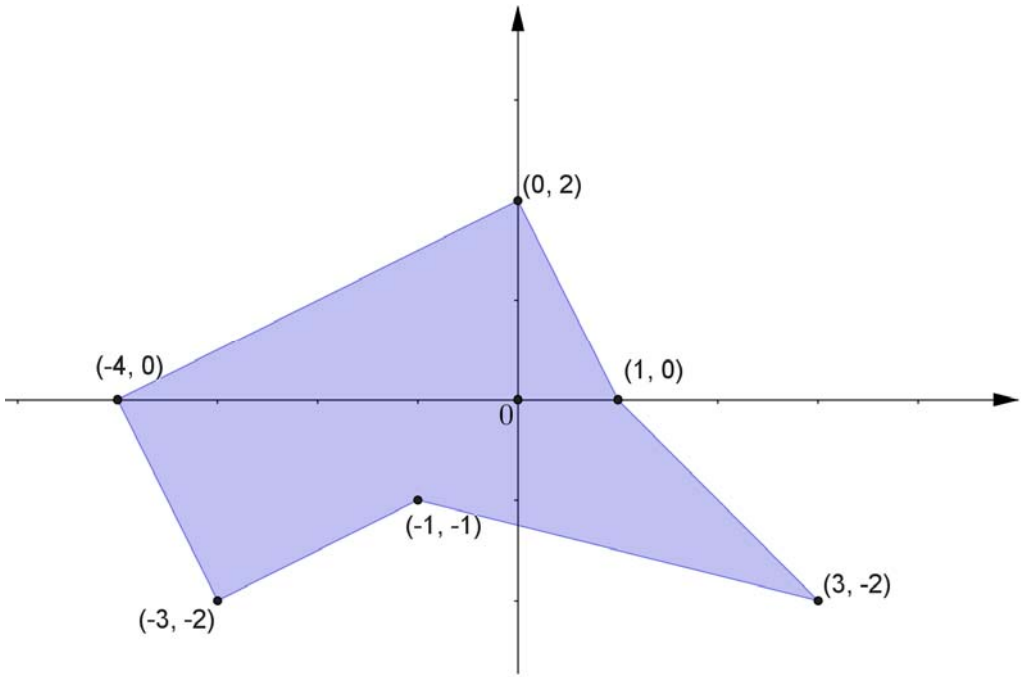
Aneta Copić

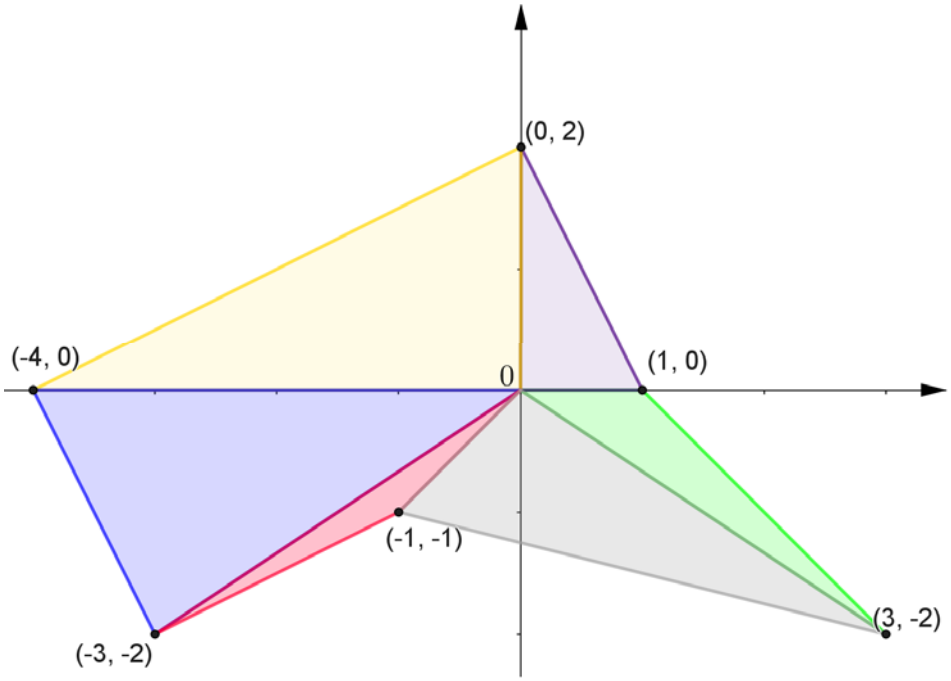
XV. Gimnazija, Zagreb

Površina je?

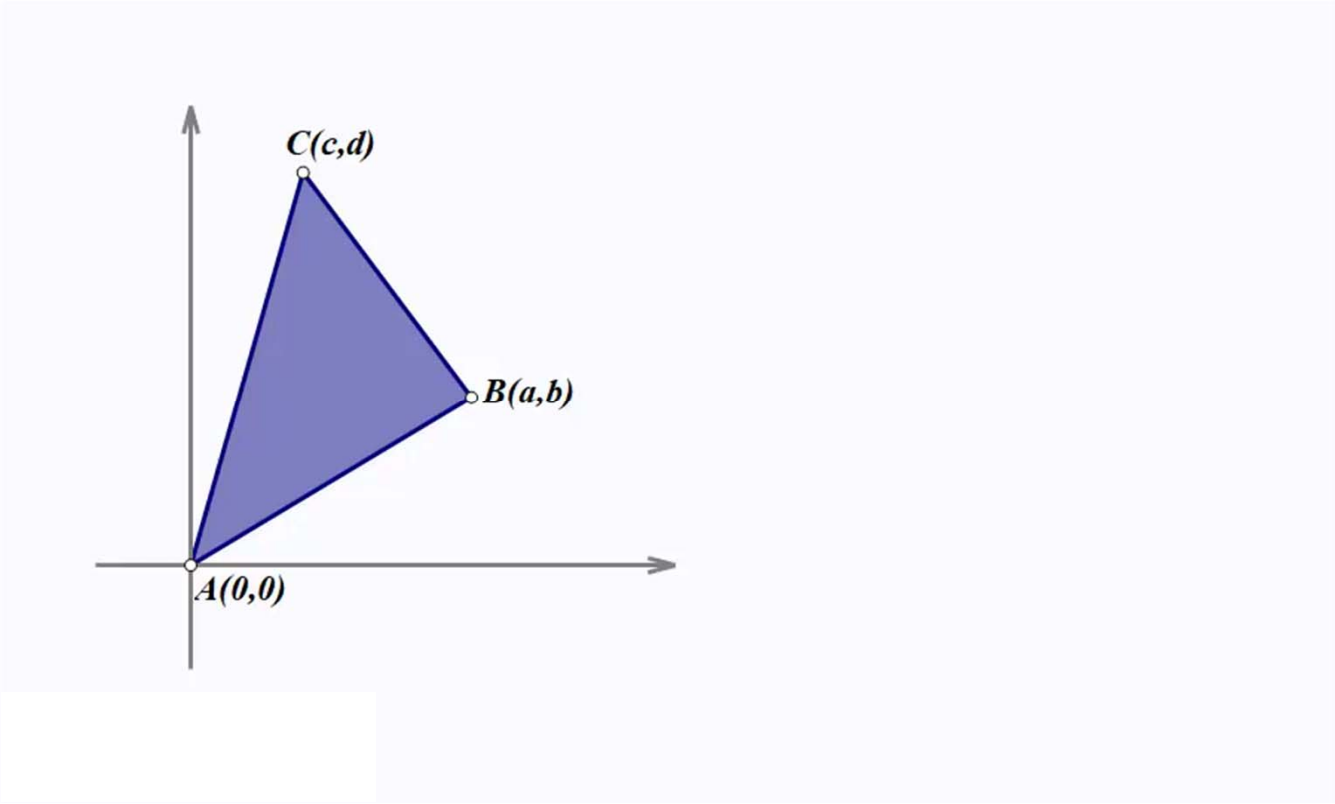


Površina poligona

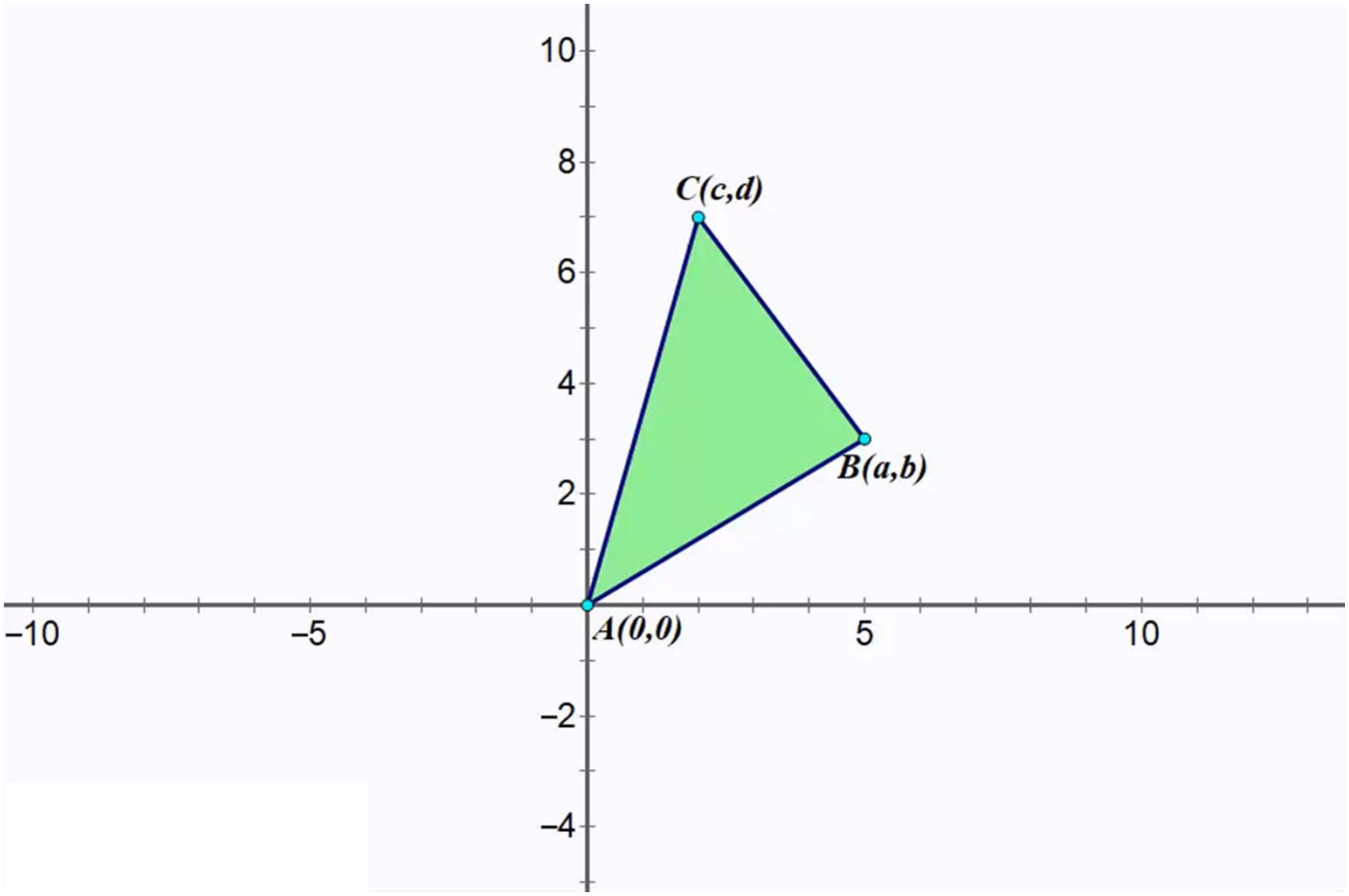




Analitički



Geometrijski



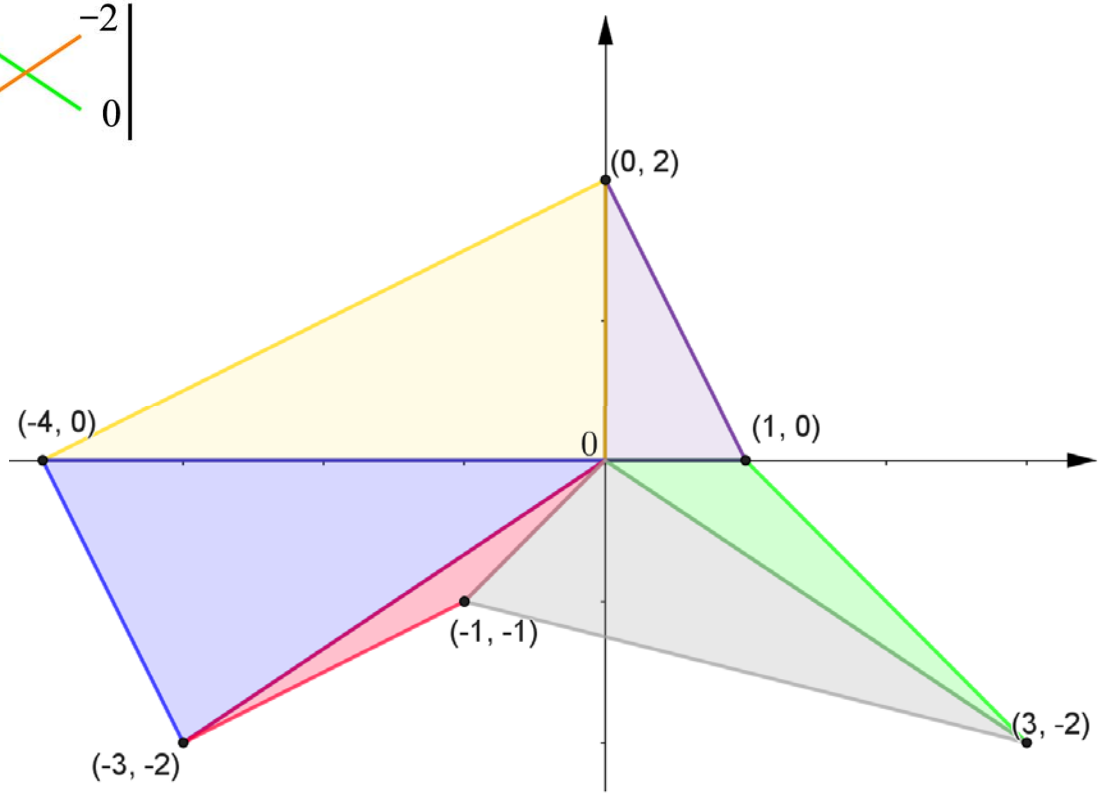
Solomon Wolf Golomb, 1932. – 2016.,USA

Površina trokuta s vrhovima $A(0,0)$, $B(a,b)$, $C(c,d)$:

$$p(\Delta ABC) = \frac{1}{2}\,|ad-bc| = \frac{1}{2}\cdot \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

$$P = \frac{1}{2}\begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix} + \frac{1}{2}\begin{vmatrix} 0 & 2 \\ -4 & 0 \end{vmatrix} + \frac{1}{2}\begin{vmatrix} -4 & 0 \\ -3 & -2 \end{vmatrix} \cdots + \frac{1}{2}\begin{vmatrix} 3 & -2 \\ 1 & 0 \end{vmatrix}$$

$$= \frac{1}{2} \cdot \begin{vmatrix} 1 & 0 \\ 0 & 2 \\ -4 & 0 \\ -3 & -2 \\ -1 & -1 \\ 3 & -2 \\ 1 & 0 \end{vmatrix} = \frac{1}{2} \begin{vmatrix} - & \\ 1 \cdot 2 & 0 \cdot 0 \\ + & + \\ 0 \cdot 0 & 2 \cdot -4 \\ + & + \\ -4 \cdot -2 & 0 \cdot -3 \\ + & + \\ -3 \cdot -1 & -2 \cdot -1 \\ + & + \\ -1 \cdot -2 & -1 \cdot 3 \\ + & + \\ 3 \cdot 0 & -2 \cdot 1 \end{vmatrix}$$

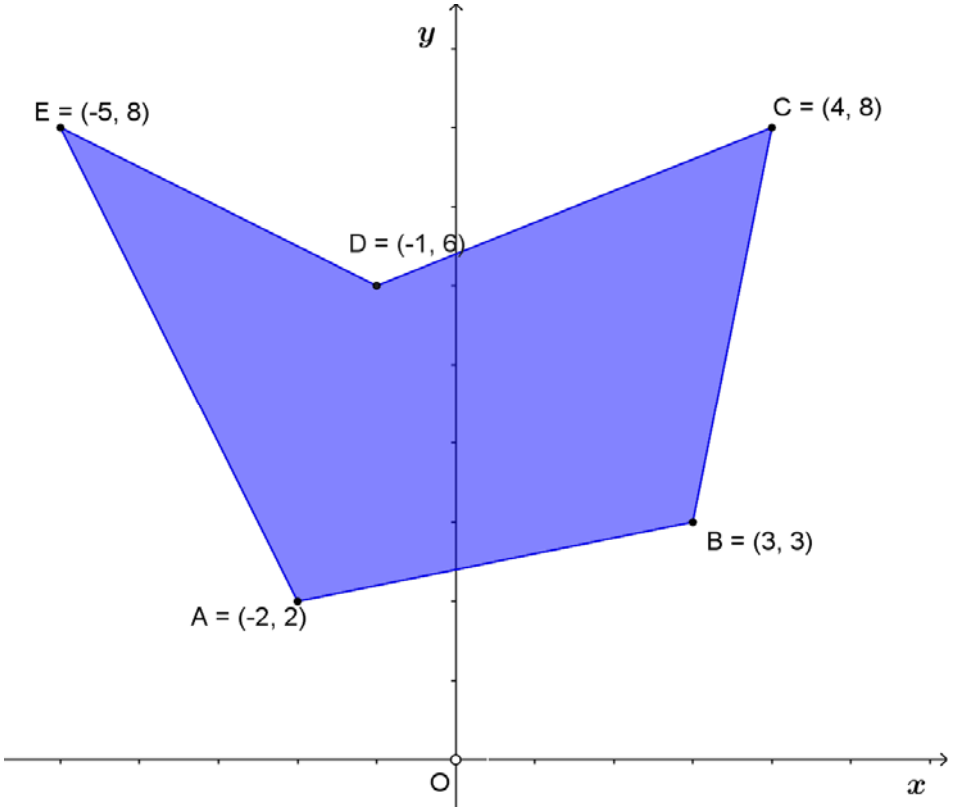


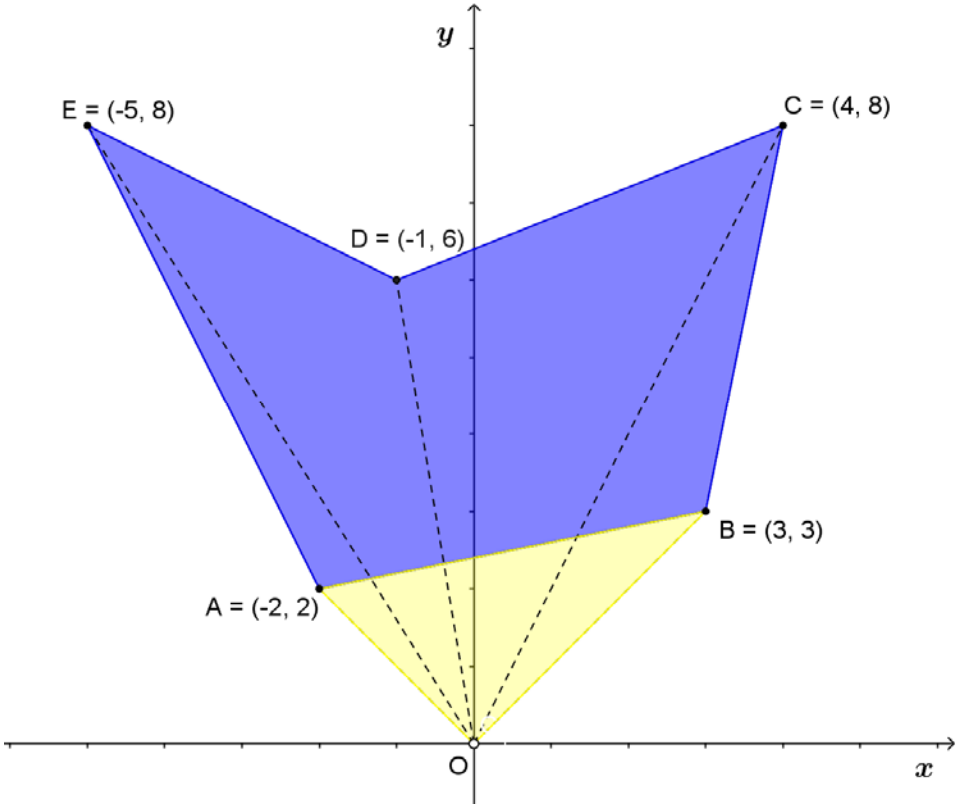
$$= \frac{1}{2}|15-(-11)| = 13$$

Površina je?

https://edutorij.e-skole.hr/share/proxy/alfresco-noauth/edutorij/api/proxy-guest/af9b8682-eef4-478e-9b92-edcba4790886/html/4807_Povrsina_trokuta.html#subtitle2

Ako poligon ne sadrži ishodište

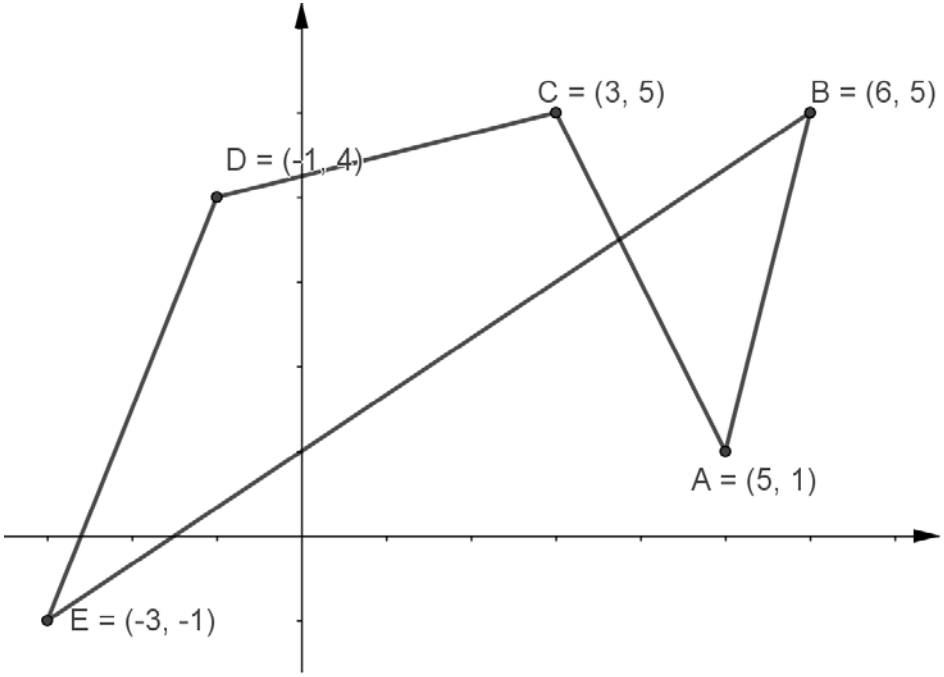


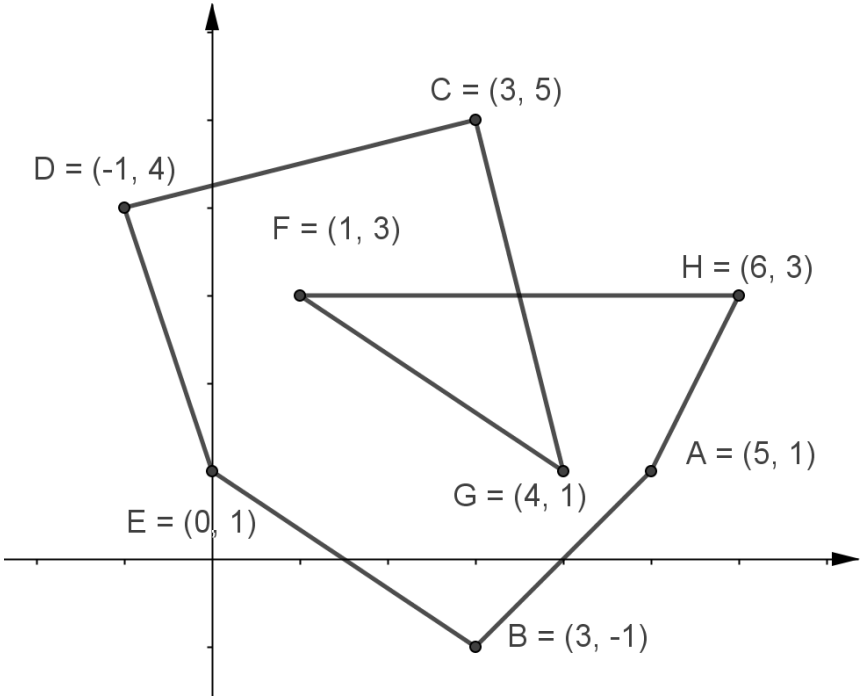


Metoda vezica

$$P = \frac{1}{2} \cdot \left| \begin{array}{cc} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ \vdots & \vdots \\ x_{n-1} & y_{n-1} \\ x_n & y_n \\ x_1 & y_1 \end{array} \right| = \frac{1}{2} \left| (x_1 y_2 + x_2 y_3 + \ldots x_n y_1) - (x_2 y_1 + x_3 y_2 + \ldots x_1 y_n) \right|$$

Ograničenje





Tehnologija

Grafičko računalo

TI-82, TI-83, TI-84

```
Prompt N
ClrList L1,L2
For (C,1,N)
Disp C
Prompt X
X → L1(C)
Prompt Y
Y → L2(C)
End
L1 (N)L2(1) → S
L1 (1)L2 (N) → T
For (C,2,N)
S + L1 (C – 1)L2 (C) → S
T + L1 (C)L2 (C – 1) → T
End
Abs(S – T)/2 → A
Disp "AREA"
Disp A
Stop
End
```

Excel

Računanje površine jednostavnih poligona metodom vezica

Točka	X	Y	Računanje			
1	-2	2	-6	6	=C4*D5	=D4*C5
2	3	3	24	12	=C5*D6	=D5*C6
3	4	8	24	-8	=C6*D7	=D6*C7
4	-1	6	-8	-30	=C7*D8	=D7*C8
5	-5	8	-10	-16	=C8*D9	=D8*C9
1	-2	2	24	-36	=SUM(E4:E8)	=SUM(F4:F8)

1. Obično računanje 30

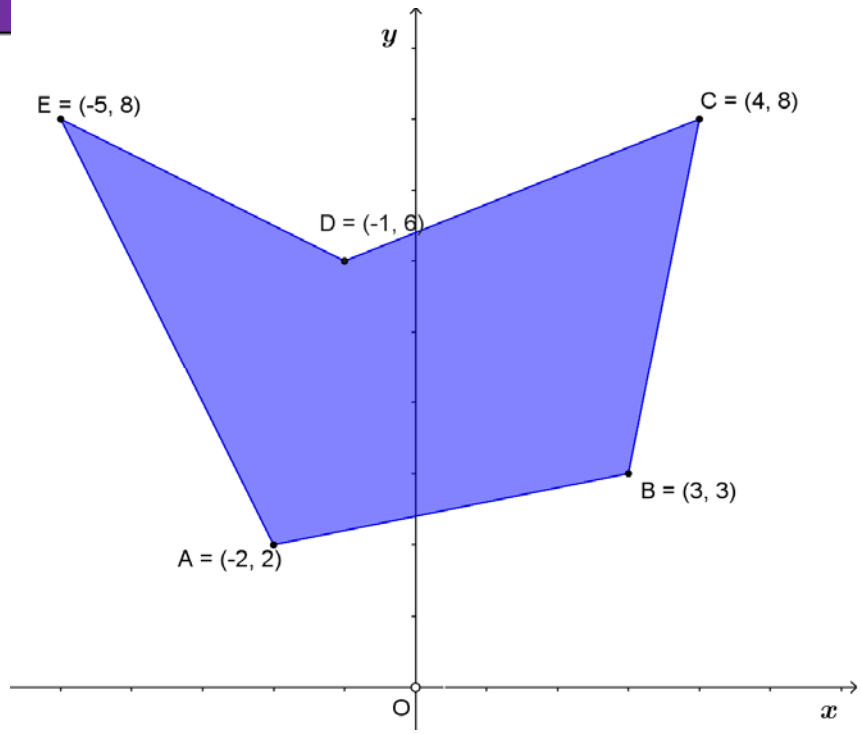
=0.5*ABS(E9-F9)

2. Koristeći SUMPRODUCT 30

=0.5*ABS(SUMPRODUCT(C4:C8;D5:D9)-SUMPRODUCT(D4:D8;C5:C9))

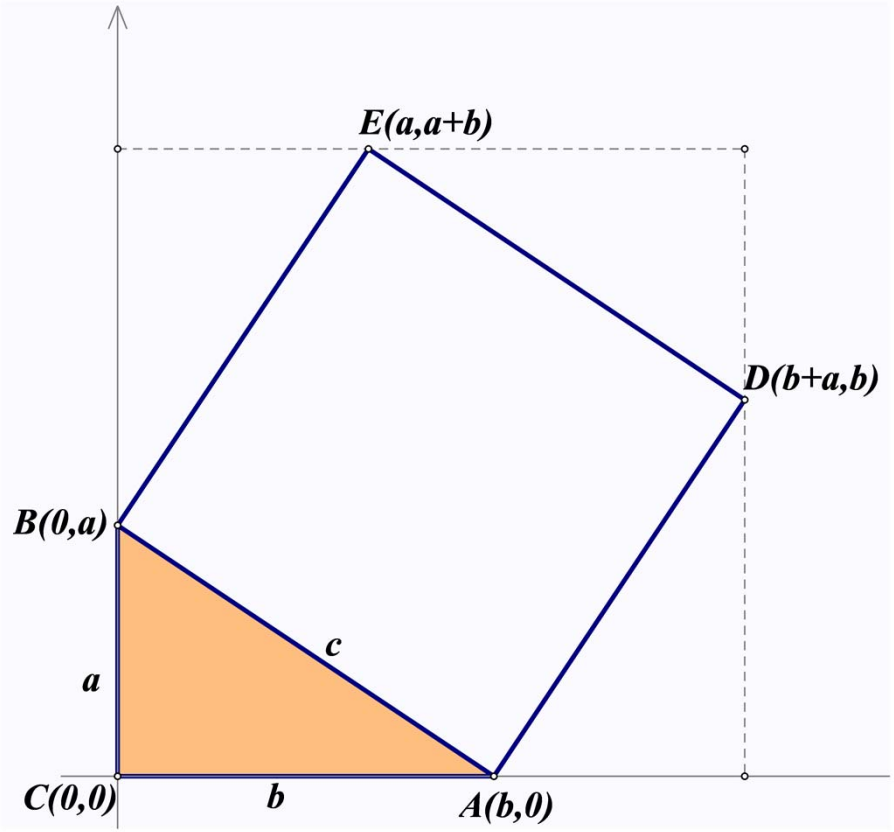
3. Koristeći Array Formula 30 Na kraju CTRL + SHIFT + ENTER

=0.5*ABS(SUM(C4:C8*D5:D9-D4:D8*C5:C9))



https://www.myengineeringworld.net/2014/06/polygon-area-shoelace-algorithm-Excel.html

Pitagorin poučak



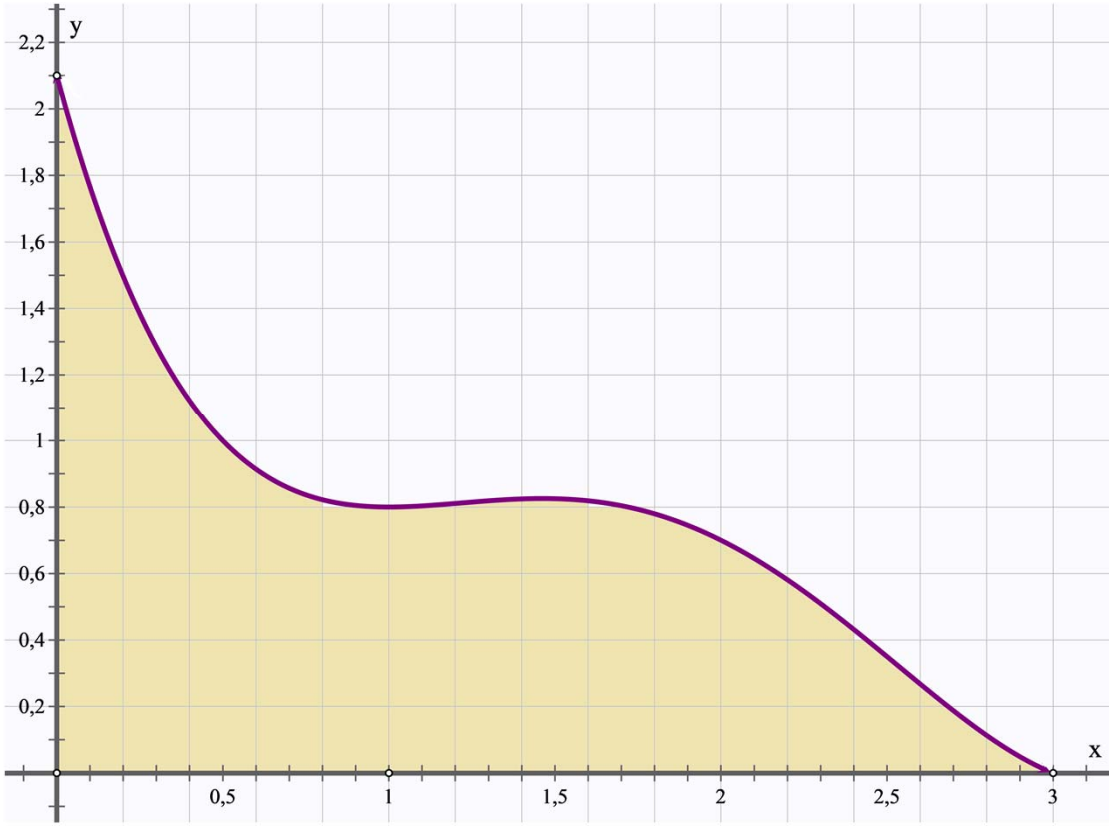
$$c^2 = \frac{1}{2}\begin{vmatrix} b & 0 \\ a+b & b \\ a & a+b \\ 0 & a \\ b & 0 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} - & \\ b^2 & 0 \\ + & + \\ (a+b)^2 & ab \\ + & + \\ a^2 & 0 \\ + & + \\ 0 & ab \end{vmatrix}$$

$$= \frac{1}{2}\left|(b^2 + (a+b)^2 + a^2) - 2ab\right| = a^2 + b^2$$

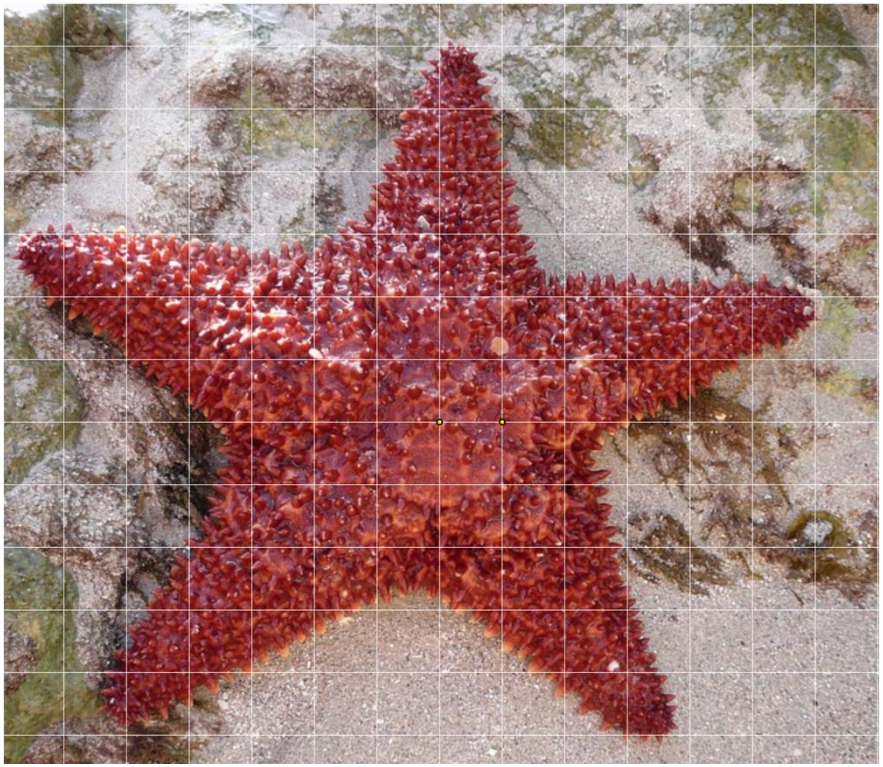
https://www.cut-the-knot.org/pythagoras/proof91.shtml

Za one koji žele više…..

Izračunajte približno osjenčanu površinu:



Ili...



Hvala!

- https://edutorij.e-skole.hr/share/proxy/alfresco-noauth/edutorij/api/proxy-guest/af9b8682-eef4-478e-9b92-edcba4790886/index.html
- https://www.researchgate.net/publication/315737612_Shoelace_formula_Connecting_the_area_of_a_polygon_and_vector_cross_product
- https://youtu.be/0KjG8Pg6LGk
- https://www.quora.com/Is-there-a-way-to-prove-the-shoelace-algorithm-without-using-calculus-or-matrices

acopic@mioc.hr