

Osmi kongres nastavnika matematike  
PMF, Zagreb, 5. srpnja 2018.



# Vjerojatnost i statistika – zašto, kada, kako?

Sanja Loparić, prof.  
Tehnička škola Čakovec



„If the understanding of number, measure, and probability is judged crucial in the pursuit of science, then instruction in these subjects should begin as intellectually honestly and as early as possible in a manner consistent with the child's forms of thought.“ (Bruner, 1960.)

**MATHEMATICS**

**is**

**GEOMETRY**  
**PROBABILITY**  
**ALGEBRA**  
**STATISTICS**  
**CALCULUS**



# THE MAP OF MATHEMATICS

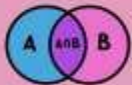
## FOUNDATIONS

### FUNDAMENTAL RULES

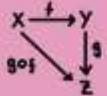
#### MATHEMATICAL LOGIC

$$P \Rightarrow Q$$

#### SET THEORY



CONSISTENT SET OF AXIOMS?  
GÖDEL INCOMPLETENESS THEOREMS  
NOPE



#### CATEGORY THEORY

## THEORY OF COMPUTATION



$P \neq NP?$

COMPLEXITY THEORY



#### PARTITION THEORY



#### ORDER THEORY



#### PERMUTATION GROUP

## COMBINATORICS



#### GRAPH THEORY



#### GROUP THEORY



## CARDINAL NUMBERS

$\aleph_0$  ALFPH NULL

PRIME NUMBERS  
3, 11, 47, 907

INFINITY  
 $\infty$

#### NUMBER THEORY

## OCTONION

$\pi$

EXPONENTIAL  
 $e$

REAL NUMBERS  
 $-4\pi, \sqrt{2}, e$

INTEGERS  
 $\dots -2, -1, 0, 1, 2 \dots$

ARITHMETIC  
 $+ - \times \div$

## LINEAR ALGEBRA



MATRICES  
 $\begin{pmatrix} 6 & 7 \\ -3 & 2 \end{pmatrix}$

VECTORS  
 $\vec{x}$

## STRUCTURES

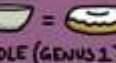
## SPACES

#### GEOMETRY



## PURE MATHEMATICS

#### TOPOLOGY



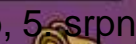
MAHABHARAT SET



#### FRactal GEOMETRY



#### CHAOS THEORY



#### DYNAMICAL SYSTEMS

FLUID FLOW



ECOSYSTEMS



#### VECTOR CALCULUS

## CHANGES

#### CALCULUS

DIFFERENTIAL  
GRADIENT =  $\frac{dy}{dx}$

INTEGRAL  
AREA =  $\int_a^b f(x) dx$

DIFFERENTIAL EQUATIONS

DIFFERENTIAL EQUATIONS

DIFFERENTIAL EQUATIONS

DIFFERENTIAL EQUATIONS

DIFFERENTIAL EQUATIONS

DIFFERENTIAL EQUATIONS

DIFFERENTIAL EQUATIONS

## ORIGINS

INDIA c. 820  
FIRST ZERO

NEGATIVE NUMBERS  
-8 III

CHINA 200 BCE

GREECE 600-300 BCE

PERMA c. 820

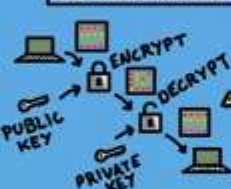
c. 1730 MATHEMATICAL NOTATION

EGYPT FIRST EQUATION 3000 BCE

COUNTING

50,000 BCE

## CRYPTOGRAPHY

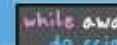


## COMPUTER SCIENCE

TURING MACHINE

while awake:  
do science()  
if self.tired():  
Awake = False  
self.repair\_brain()

STATISTICS

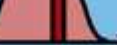


BAYES' RULE  
 $P(A|B) = \frac{P(B|A)P(A)}{P(B)}$

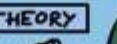
GAME THEORY



MATHEMATICAL PHYSICS



MATHEMATICAL CHEMISTRY



NUMERICAL ANALYSIS



ENGINEERING



CONTROL THEORY

BIOMATHEMATICS



## MACHINE LEARNING



CAT

OPTIMIZATION



MATHEMATICAL FINANCE



ECONOMICS



APPLIED MATHEMATICS

APPLIED MATHEMATICS

APPLIED MATHEMATICS

APPLIED MATHEMATICS

APPLIED MATHEMATICS

APPLIED MATHEMATICS

APPLIED MATHEMATICS

APPLIED MATHEMATICS

APPLIED MATHEMATICS

APPLIED MATHEMATICS

APPLIED MATHEMATICS

APPLIED MATHEMATICS

APPLIED MATHEMATICS

APPLIED MATHEMATICS

APPLIED MATHEMATICS

APPLIED MATHEMATICS

APPLIED MATHEMATICS

APPLIED MATHEMATICS

Sanja Loparić, prof.

BY DOMINIC WALLIMAN © 2017

YOUTUBE: THE MAP OF MATHEMATICS

Zagreb, 5. srpnja 2018.

# Definicije

Statistika kao znanstvena disciplina bavi se razvojem metoda prikupljanja, opisivanja i analiziranja podataka te primjenom tih metoda u procesu donošenja zaključaka na temelju prikupljenih podataka.

Vjerojatnost je pojam povezan s predviđanjem i analizom slučajnih pojava. Pojam vjerojatnosti nastao je u pokušaju brojčanog izražavanja stupnja vjerovanja da će se dogoditi neki zamišljeni događaj.

# Povijest nastave vjerojatnosti i statistike u Hrvatskoj

- HNOS 2006. godine - *Prikazivanje i analiza podataka i Vjerojatnost slučajnog događanja* u sedmom razredu osnovne škole
- NOK 2011. - teme u svim ciklusima obrazovanja – *Podatci*
- CKR 2016. – svi ciklusi –domena: *Podatci, statistika i vjerojatnost*



# PISA i TIMSS

The graph shows the number of apples John picked each day.



On which day did John pick 5 apples?

- (A) Monday
- (B) Tuesday
- (C) Wednesday
- (D) Thursday

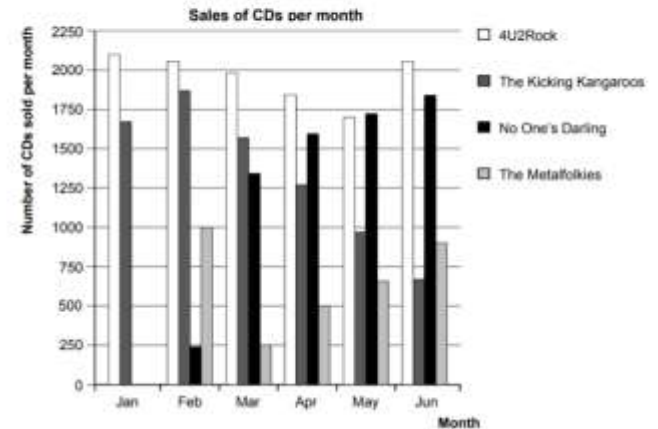
This graph shows the points obtained by 4 drivers in the car racing championship. Montoya is in first place. Alonso is in third place. Draw a bar which shows how many points Alonso has scored.



Zagreb, 5. srpnja 2018.

## CHARTS

In January, the new CDs of the bands 4U2Rock and The Kicking Kangaroos were released. In February, the CDs of the bands No One's Darling and The Metallfolkies followed. The following graph shows the sales of the bands' CDs from January to June.



### Question 1: CHARTS

PM918Q01

How many CDs did the band *The Metallfolkies* sell in April?

- A 250
- B 500
- C 1000
- D 1270

### Question 2: CHARTS

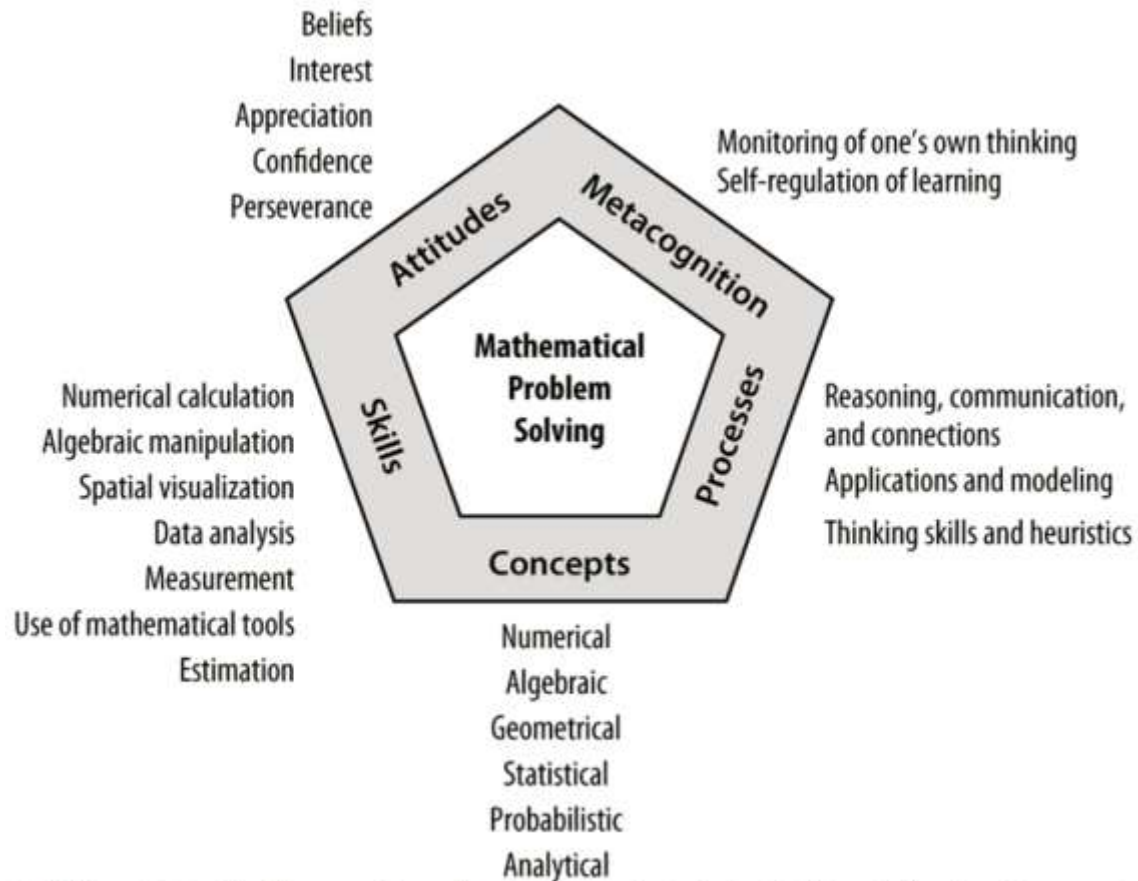
PM918Q02

In which month did the band *No One's Darling* sell more CDs than the band *The Kicking Kangaroos* for the first time?

- A No month
- B March
- C April
- D May

Sanja Loparić, prof.

# SINGAPUR



© Ministry of Education, Singapore (Reproduced with permission by the Ministry of Education, Singapore)



# Podatci, statistika i vjerojatnost

- 1. razred: 7,14% (služiti se podacima i prikazivati ih piktogramima i jednostavnim tablicama)
- 2. razred: 11,11% (koristiti se podacima iz neposredne okoline i odrediti je li neki događaj moguć ili nemoguć)
- 3. razred: 6,67% (služiti različitim prikazima podataka)
- 4. razred: 7,69% (provođenje jednostavnih istraživanja i analiziranje dobivenih podataka te opisivanje vjerojatnosti događanja)

# Učenje vjerojatnosti

- Piagetu – 7 godina prerano za poučavanje vjerojatnosti
- Hodnik, (2011.) - djeca mogu osnovne pojmove iz vjerojatnosti (siguran, moguć, vjerojatan i nemoguć događaj) usvojiti već u 1. razredu osnovne škole
- Andrew, (2010.) je dokazao da su rezultati učenja bolji kod učenika koji su učili vjerojatnost eksperimentiranjem na konkretnim materijalima u odnosu na učenike koji su koristili formule

# Učenje vjerojatnosti

Polaki (2002.) navodi četiri stadija učenja vjerojatnosti:

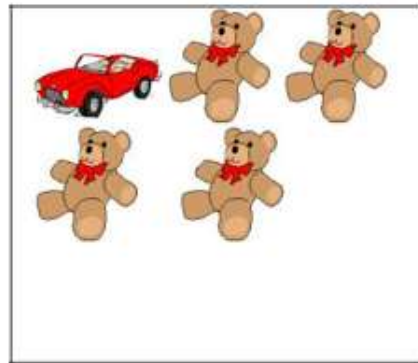
- 1. stadij – subjektivni – kada učenici predviđaju najvjerojatniji i najmanje vjerojatan događaj na temelju njihovih subjektivnih doživljaja
- 2. stadij – predviđaju se najvjerojatniji i najmanje vjerojatan događaj
- 3. stadij – učenici točno predviđaju vjerojatnije događaje i upotrebljavaju brojeve
- 4. stadij – određuje se točna vjerojatnost događaja

# Primjer: kutije s igračkama

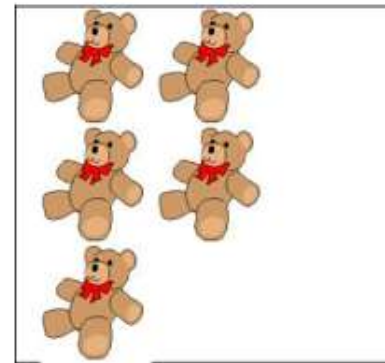
1. kutija



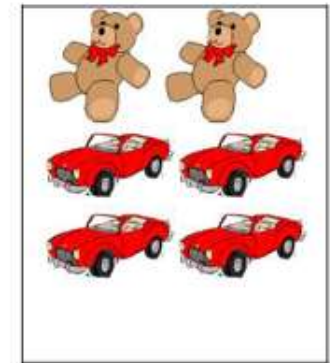
2. kutija



3. kutija



4. kutija



Može li se iz 1. kutije izvući medvjedić? Može li se izvući autić? Jesi li siguran/sigurna? Može li se iz 2. kutije izvući medvjedić? Je li vjerojatnije iz druge kutije izvući medvjedića ili autića? Isprobaj! Kolika je vjerojatnost da izvučemo medvjedića iz svake od navedenih kutija? ...



# Primjer: bomboni



Math: the only place where you have to figure out the ratio of yellow candy to blue candy when all you're thinking about is eating them.

*Cool Math 4 Kids*



Zagreb, 5. srpnja 2018.

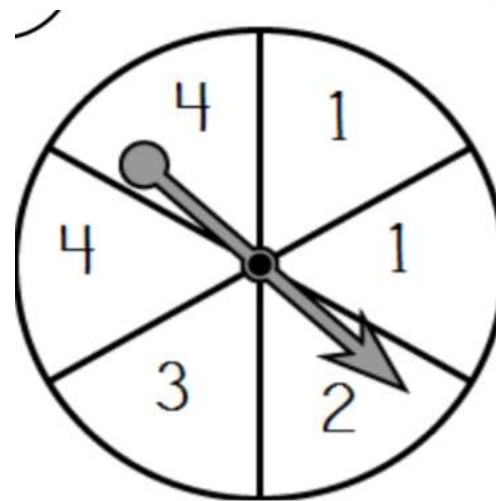
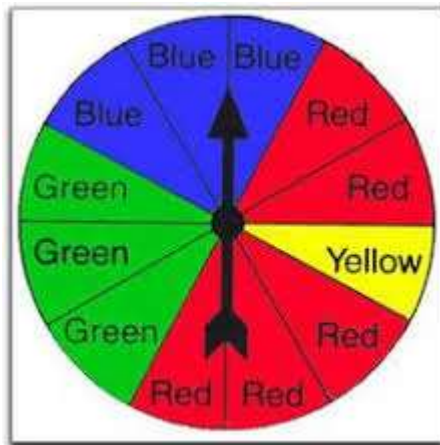
Sanja Loparić, prof.

# Primjer: kolo sreće

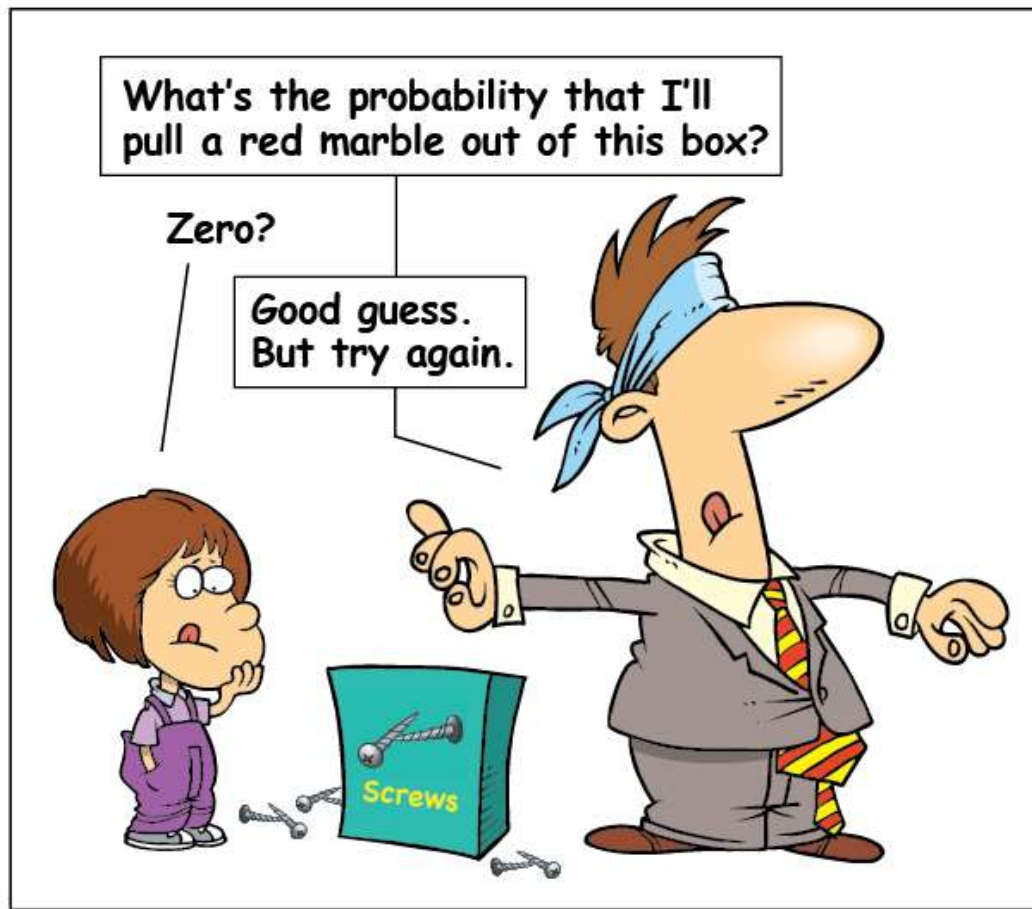
Spinner A



Spinner B



# Primjeri: kocka, novčić, dvije kocke...



# Učenje statistike

Predmatematičke vještine:

- razvrstavanje predmeta prema određenom obilježju
- uspoređivanje
- nizanje i održavanje redoslijeda
- orijentiranje u ravnini (Glasnović Gracin, 2016)



# Primjer: smješkić

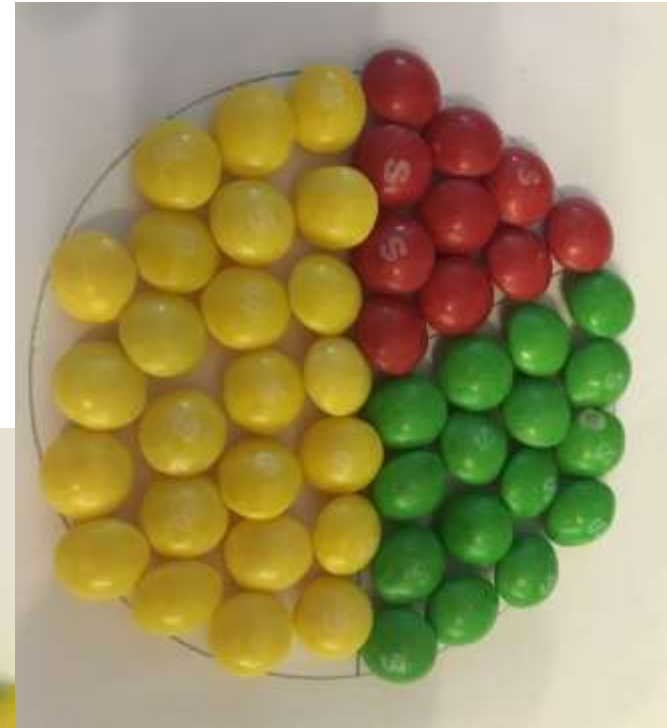
2. b razred

| Ime   | Ponedjeljak | Utorak | Srijeda | Četvrtak | Petak |
|-------|-------------|--------|---------|----------|-------|
| Ivan  |             |        |         |          |       |
| Mia   |             |        |         |          |       |
| Marko |             |        |         |          |       |
| Stipe |             |        |         |          |       |
| Ivona |             |        |         |          |       |
| Ema   |             |        |         |          |       |

2. b razred

| Ime   |  |  |  |  |  |
|-------|--|--|--|--|--|
| Ivan  |  |  |  |  |  |
| Mia   |  |  |  |  |  |
| Marko |  |  |  |  |  |
| Stipe |  |  |  |  |  |
| Ivona |  |  |  |  |  |
| Ema   |  |  |  |  |  |

# Primjer: bomboni



Zagreb, 5. srpnja 2018.

Sanja Loparić, prof.

# Primjer: automobili

One  represents 5 cars

|           |  |
|-----------|--|
| Monday    |   |
| Tuesday   |  |
| Wednesday |  |
| Thursday  |  |



# Digitalne igre

<https://www.free-training-tutorial.com/probability-games.html>

## BASIC PROBABILITY PRACTICE

How likely is it that the spinner will



- certain
- probable
- unlikely
- impossible

## PULLING OBJECTS FROM BAG

See Certain, Likely, or Impossible Pulls from a Bag



- certain.
- impossible.
- likely.
- unlikely.
- equally likely.

?

Check

Harcourt

## PROBABILITY FAIR



## PROBABILITY CIRCUS

Choose the spinner that matches the probability.



The probability of the pointer landing on purple is  $\frac{2}{9}$ .



## BALL PICKING MACHINE

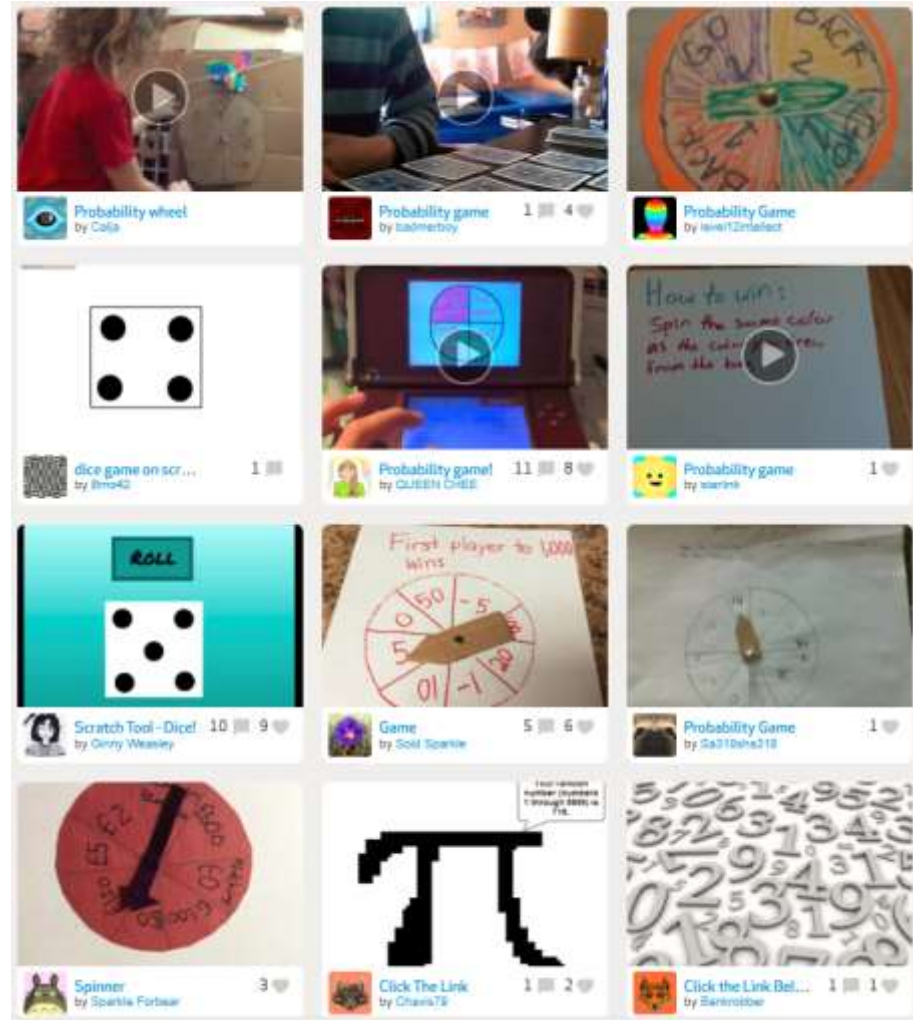
Drag the pulsating blue and red balls on to the correct positions on the probability line and click Done.





# Video

<https://diy.org/skills/mathematician/challenges/2133/design-a-probability-game>



Zagreb, 5. srpnja 2018.

Sanja Loparić, prof.

# Video

Introduction to  
Probability by Ms.  
Marks and Snail



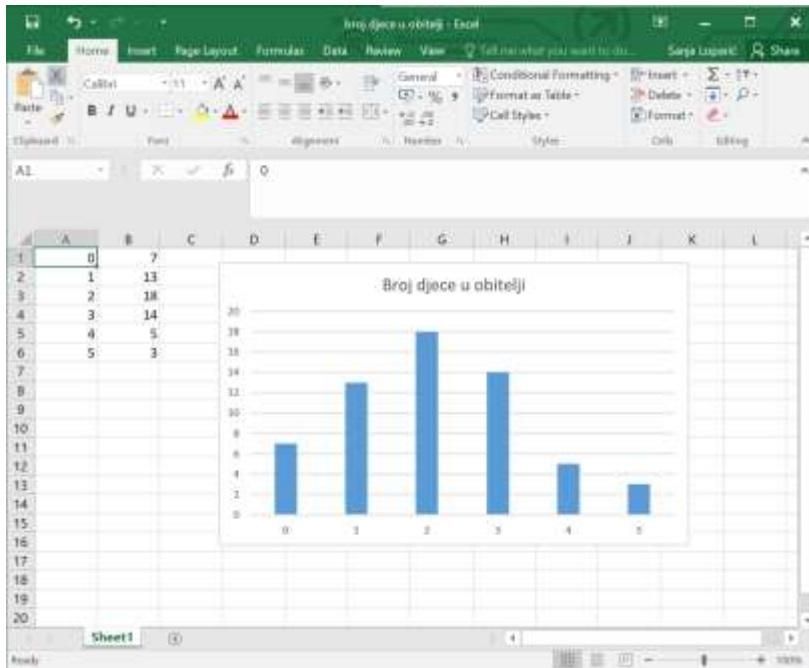
## 1. Equal Probability

**There are 2 pens in a bag: one red, and one blue.**  
**Since** there is one of each, the likelihood of picking a red or a blue pen is equal, therefore the probability is equal. There is 1 blue pen out of 2 pens, or  $1/2$  probability.

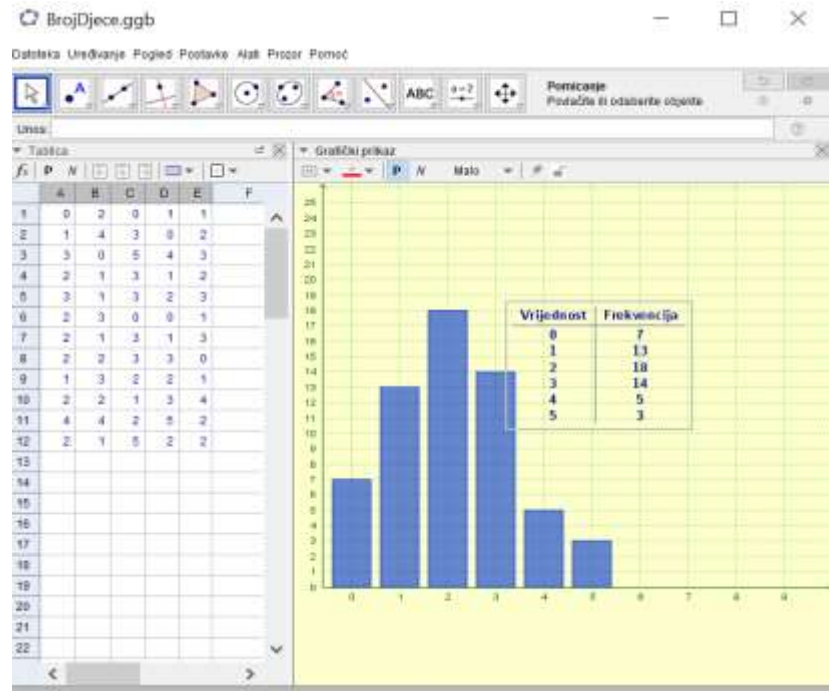


# Digitalni alati

## Excel



## Geogebra



Zagreb, 5. srpnja 2018.

Sanja Loparić, prof.



# Državni zavod za statistiku

[www.dzs.hr](http://www.dzs.hr)

[Mala škola statistike](#)



Mala škola statistike

Državni zavod za statistiku - 3 / 25



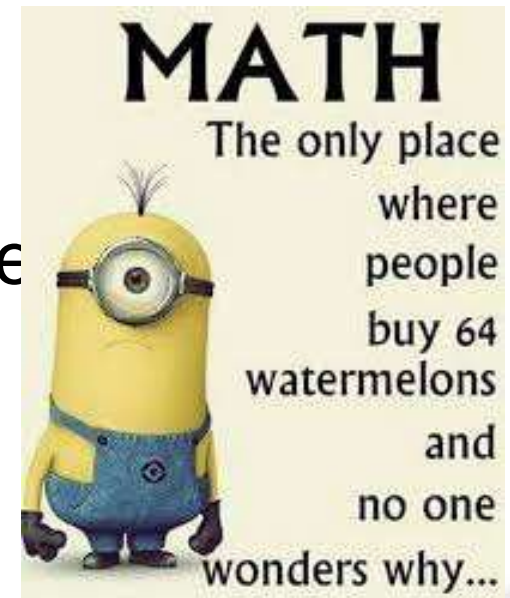
- ▶ **PROCJENA** 1:09  
Mala škola statistike – Procjena stanovništva  
Državni zavod za statistiku
- 4 **MALA ŠKOLA** 1:04  
Mala škola statistike – Metapodaci  
Državni zavod za statistiku
- 5 **STATISTIKA ZA SIROMAŠTVO** 1:59  
Mala škola statistike – Granica rizika od siromaštva  
Državni zavod za statistiku
- 6 **MEĐIJAN** 1:30  
Mala škola statistike – Medijan i prosjek  
Državni zavod za statistiku
- 7 **MALA ŠKOLA STATISTIKE** 1:57  
Mala škola statistike - Desezoniranje  
Državni zavod za statistiku
- 8 **MALA ŠKOLA STATISTIKE**  
Mala škola statistike - Uzorak  
Državni zavod za statistiku

Sanja Loparić, prof.



# Važnost nastave vjerojatnosti i statistike

- prikupiti podataka, organizirati, interpretirati, uspoređivati podataka, donositi zaključke, predviđanja
- potiče logičko zaključivanje
- kritička analiza
- ispitivanje točnosti podataka
- upotreba konkretnih primjere i ekspe
- verbalizacija zaključaka



# Pitanja?



Zagreb, 5. srpnja 2018.

Sanja Loparić, prof.

# Zahvaljujem na pažnji!

Doviđenja!



Zagreb, 5. srpnja 2018.

Sanja Loparić, prof.