

**‘Negative’ World of Metamaterials  
or  
How Ambiguity of Square Root Reversed  
Electromagnetics**

**“Negativan” svijet metamaterijala  
ili  
kako je dvoznačnost kvadratnog korijena okrenula  
elektromagnetizam naglavačke**

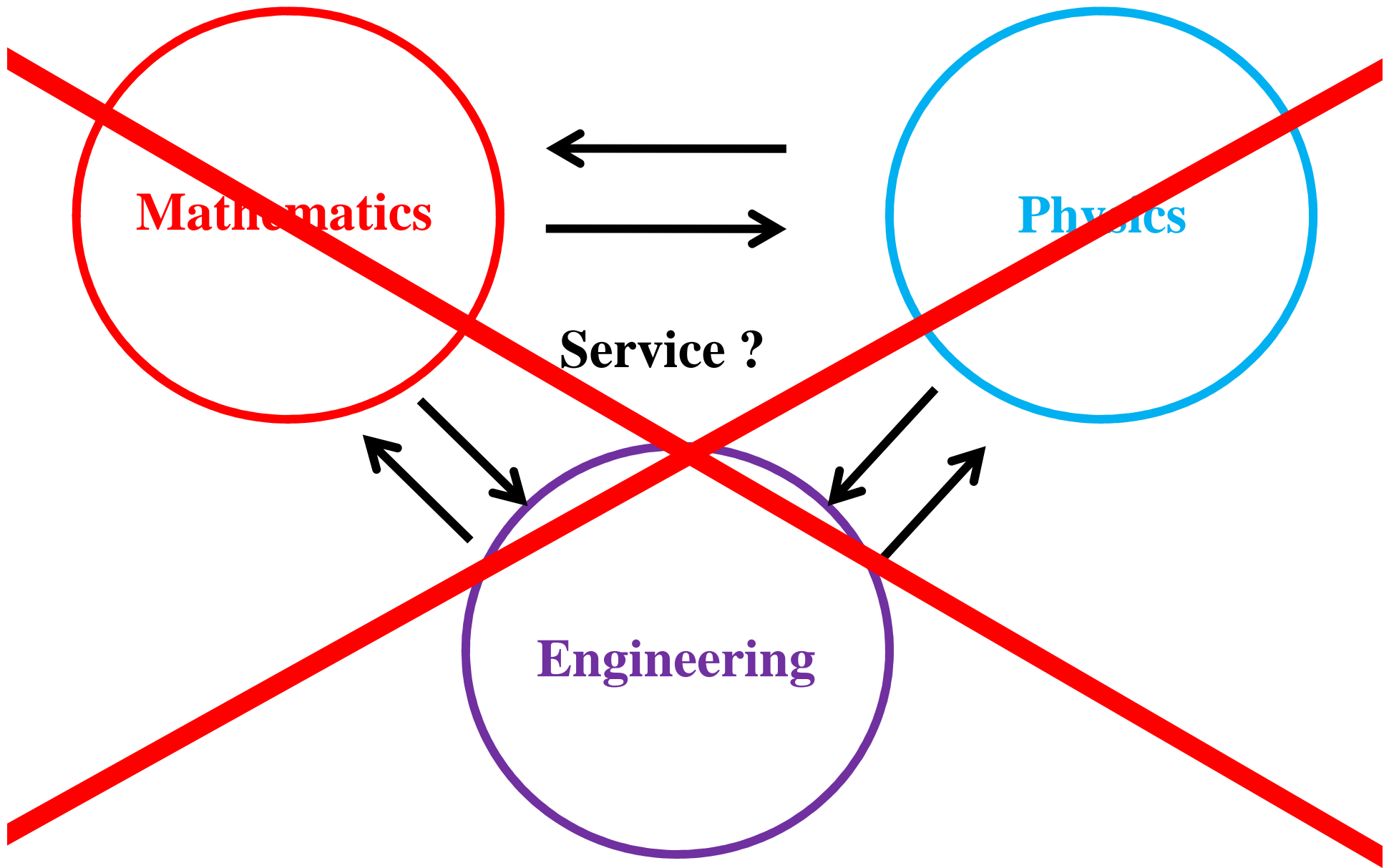
Silvio Hrabar

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# Outline

- Mathematics versus Physics and Engineering
- Electromagnetics of Classical (“Positive”) Materials
- Are “Negative” Materials Possible?
- New Physical Phenomena - Fundamental research
- Potential Engineering Applications – Applied Research
- Conclusions

# **Mathematics versus Physics and Engineering**



**What is the difference among engineers ,  
physicists and mathematicians ?**

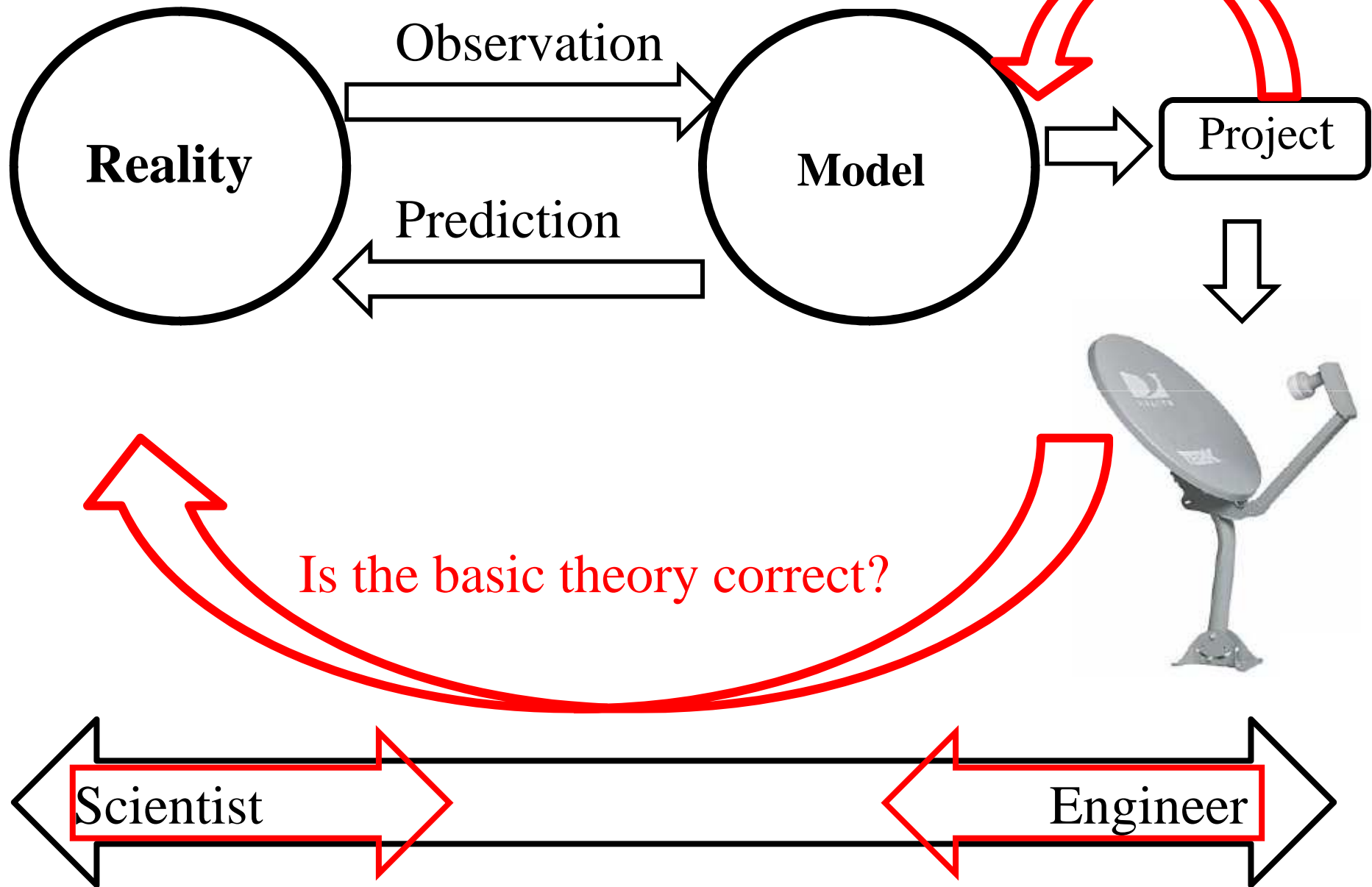
Theodor von Karman

(1881-1963)

Scientists look at things that are and ask  
“why”;

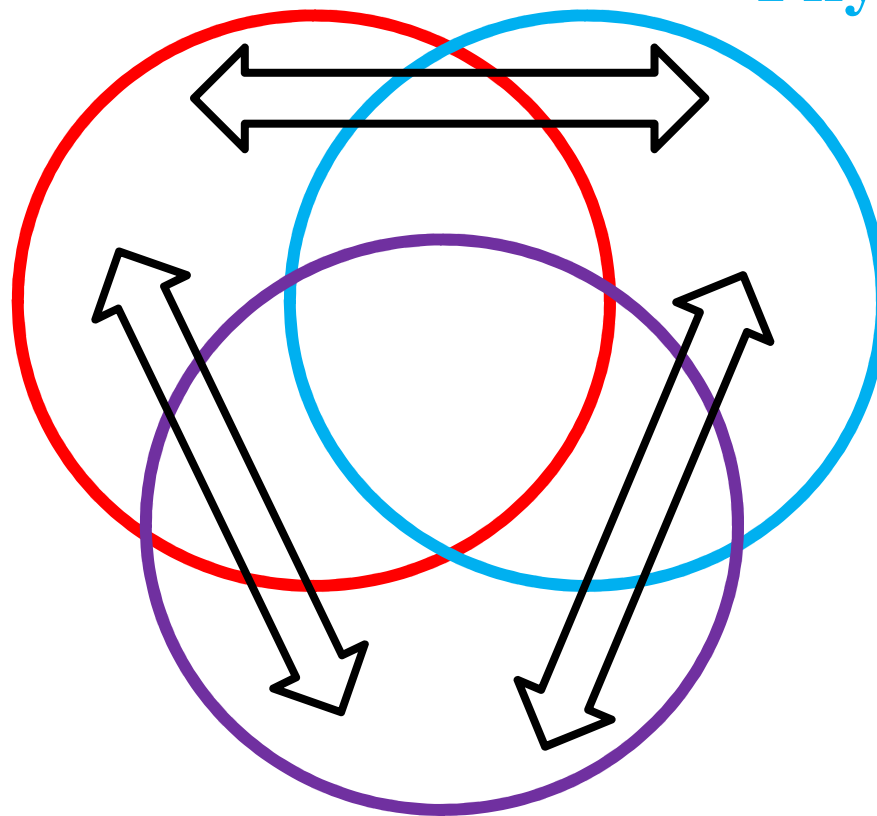
Engineers dream of things that never  
were and ask “why not”

Model gives non-physical results – Why ?



**Mathematics**

**Physics**



**Engineering**

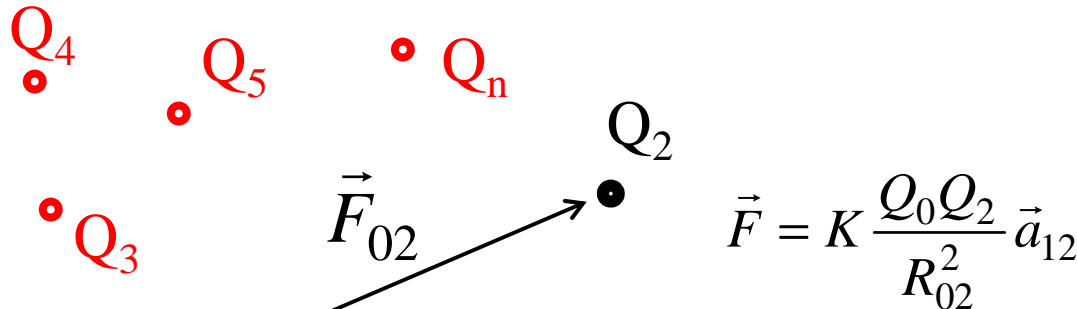


# **Electromagnetics of Classical (“Positive”) Materials**

# The Story of Electricity

•Ancient Greece – Why does amber attract papyrus ?

•Charles Augustin de Coulomb (1736-1806)  
and concept of **Electric field**



$$\vec{F} = K \frac{Q_0 Q_2}{R_{02}^2} \vec{a}_{12} + K \frac{Q_0 Q_3}{R_{03}^2} \vec{a}_{13} + \dots K \frac{Q_0 Q_n}{R_{0n}^2} \vec{a}_{1n}$$

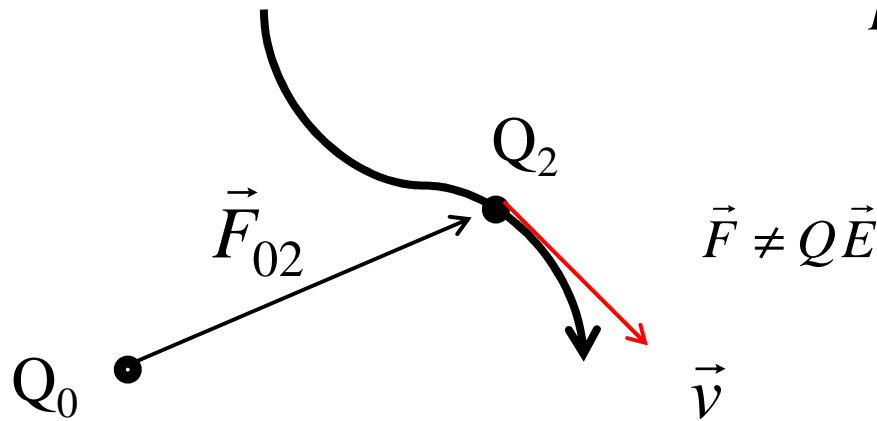
$$\vec{F} = Q_0 \left[ K \left( \frac{Q_2}{R_{02}^2} \vec{a}_{12} + \frac{Q_3}{R_{03}^2} \vec{a}_{13} \dots \frac{Q_n}{R_{0n}^2} \vec{a}_{1n} \right) \right] = Q_0 \vec{E}$$

$$\vec{E} = \lim_{Q_0 \rightarrow 0} \frac{\vec{F}}{Q_0}$$

# The Story of Magnetism

- Ancient Greece – a strange rock from island of Magnesia

## Hendrik Antoon Lorentz

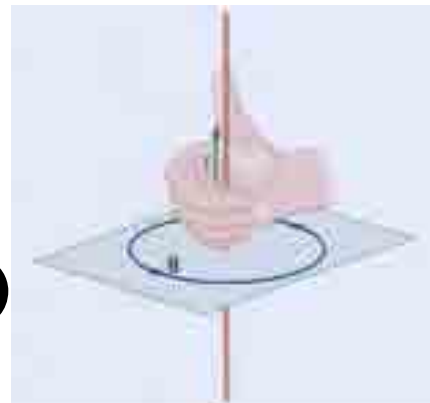


$$\vec{F} = Q_0 \vec{E} + f(\vec{v}, Q_0, R_{02})$$
$$= Q_0 (\vec{E} + \vec{v} \times \vec{B}) \quad \vec{E} \perp \vec{B}.$$



Concept of **B field**

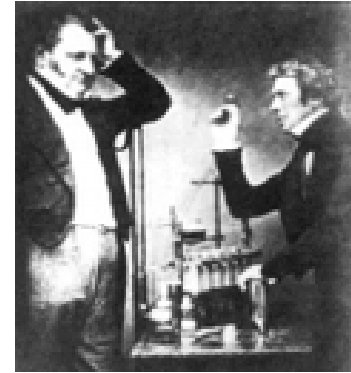
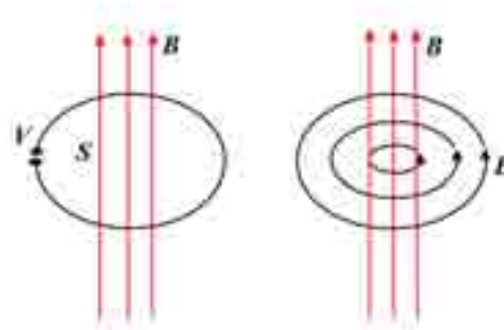
## Andre Marie Ampere (1775-1836)



# Wedding of Electricity and Magnetism – Electromagnetism (Electromagnetics)

- Michael Faraday (1791-1867)

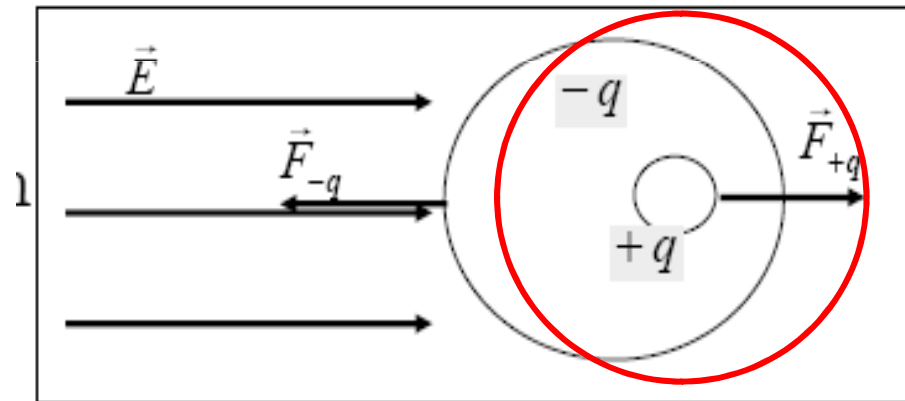
$$V = -\frac{\partial \vec{B}}{\partial t}$$



Are the E and B Vectors  
Enough?

Yes in principle, but what  
about the influence of  
matter?

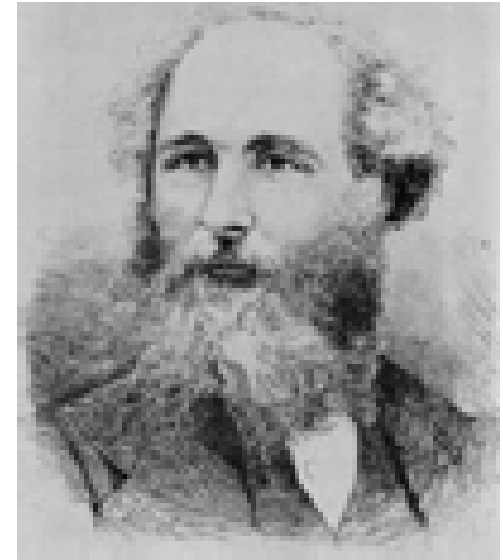
...atomic dimensions are on  
order of  $10^{-11}$  m!



$$\vec{D} = \epsilon \cdot \vec{E} \quad \text{Electric flux density}$$

$$\vec{H} = \frac{1}{\mu} \cdot \vec{B} \quad \text{Magnetic field}$$

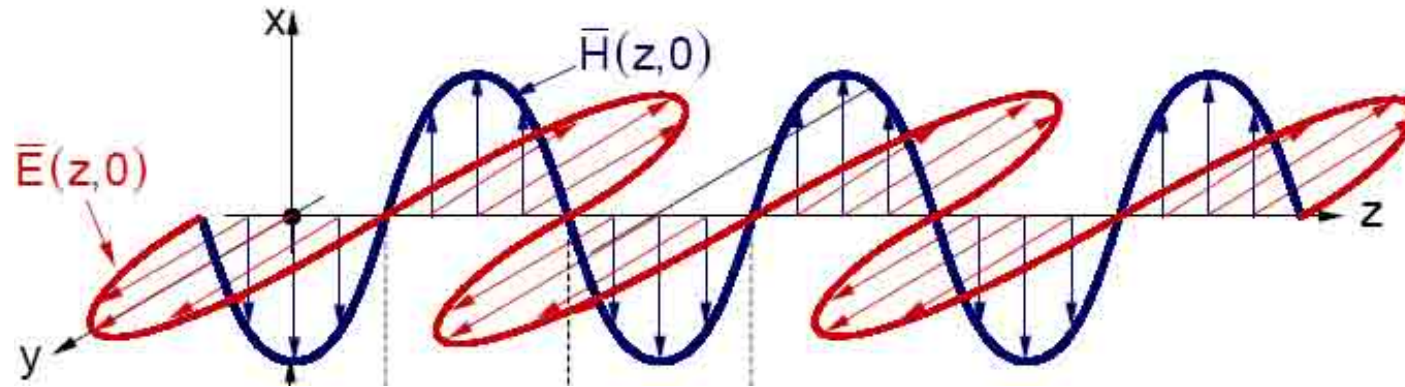
James Clerk Maxwell (1831-1879)



$$\nabla \times \vec{H} = \vec{J} + \epsilon \frac{\partial \vec{E}}{\partial t}$$
$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$$

Solution of these equations is a wave!

# What is EM Harmonic Wave ?



$$e(x,t) = A \cos(\omega t - k \cdot z) = \text{Re} \left\{ A e^{j\omega t} e^{jk \cdot \vec{z}} \right\}$$

Wave vector  $\vec{k} = \frac{2\pi}{\lambda} \vec{a}_0$

# From Theory towards Applications...



• **Heinrich Rudolf Hertz (1857-1894)**



• **Nikola Tesla (1856-1943)**

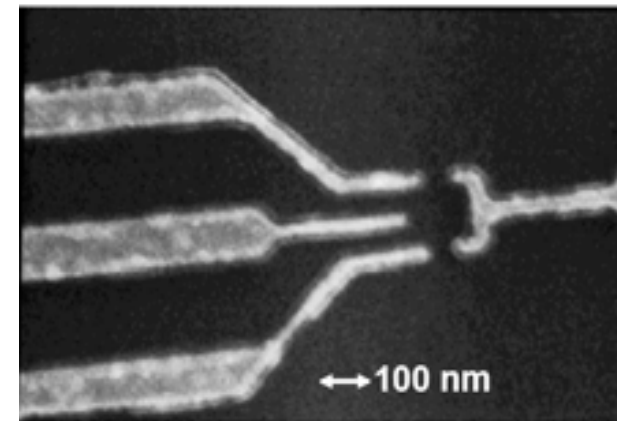
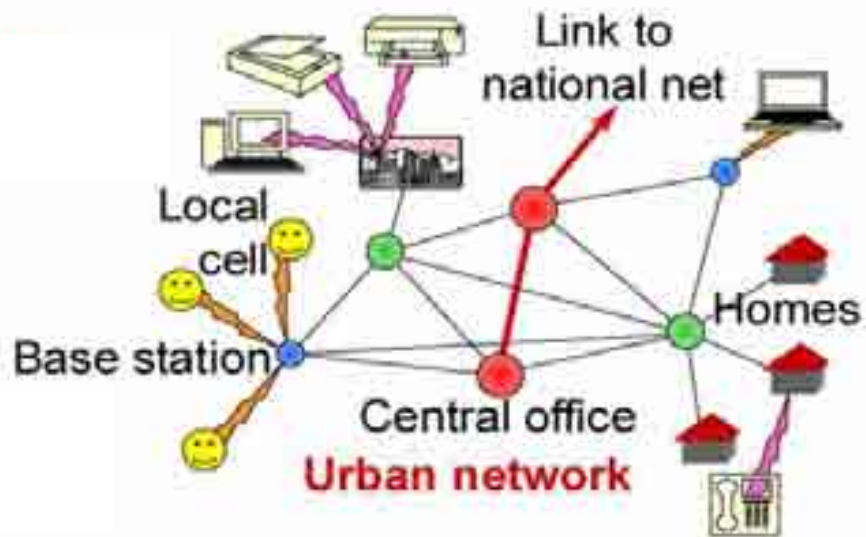


• **Guglielmo Marconi (1874-1937)**

.... and here we are !



300 m



$$\vec{E}, \vec{H}, \epsilon, \mu.$$



Are “Negative” Materials Possible?

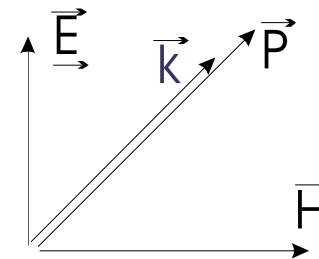
(Are some solutions of Maxwell equations meaningless?)

# What would happen if $\mu$ and $\epsilon$ were negative numbers? (Veselago, 1968)



- A plane wave solution of Maxwell equations:

Forward wave



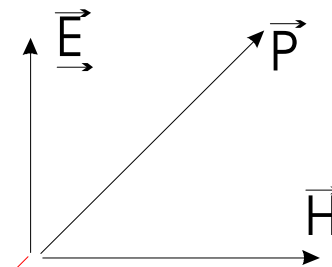
Right-handed material

$$\vec{E} = A e^{-j\vec{k}\vec{r}}$$

$$\vec{k} = \vec{u}_0 \omega \sqrt{\mu\epsilon}$$

$$\vec{P} = \vec{E} \times \vec{H}$$

Backward wave

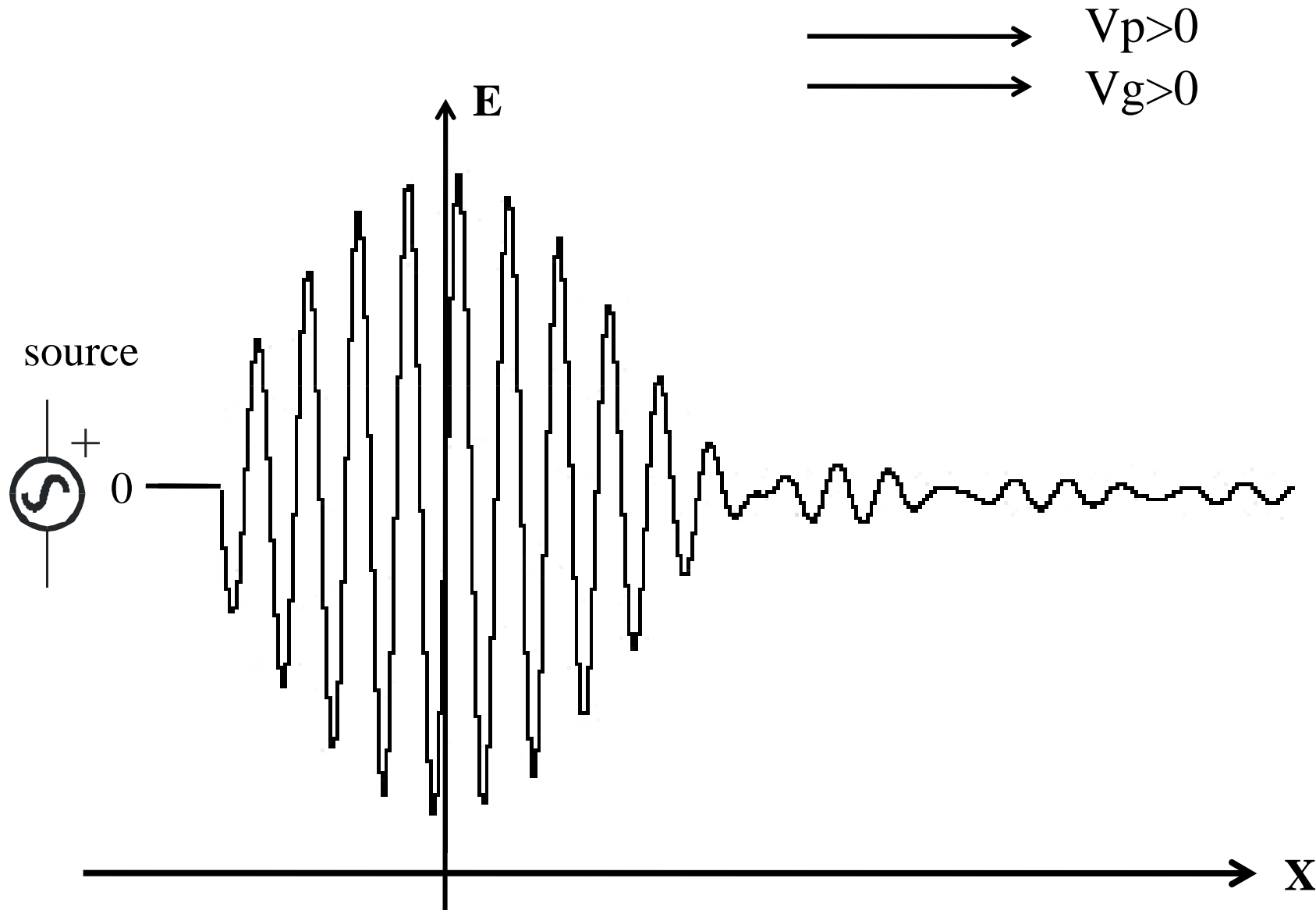


Left-handed material

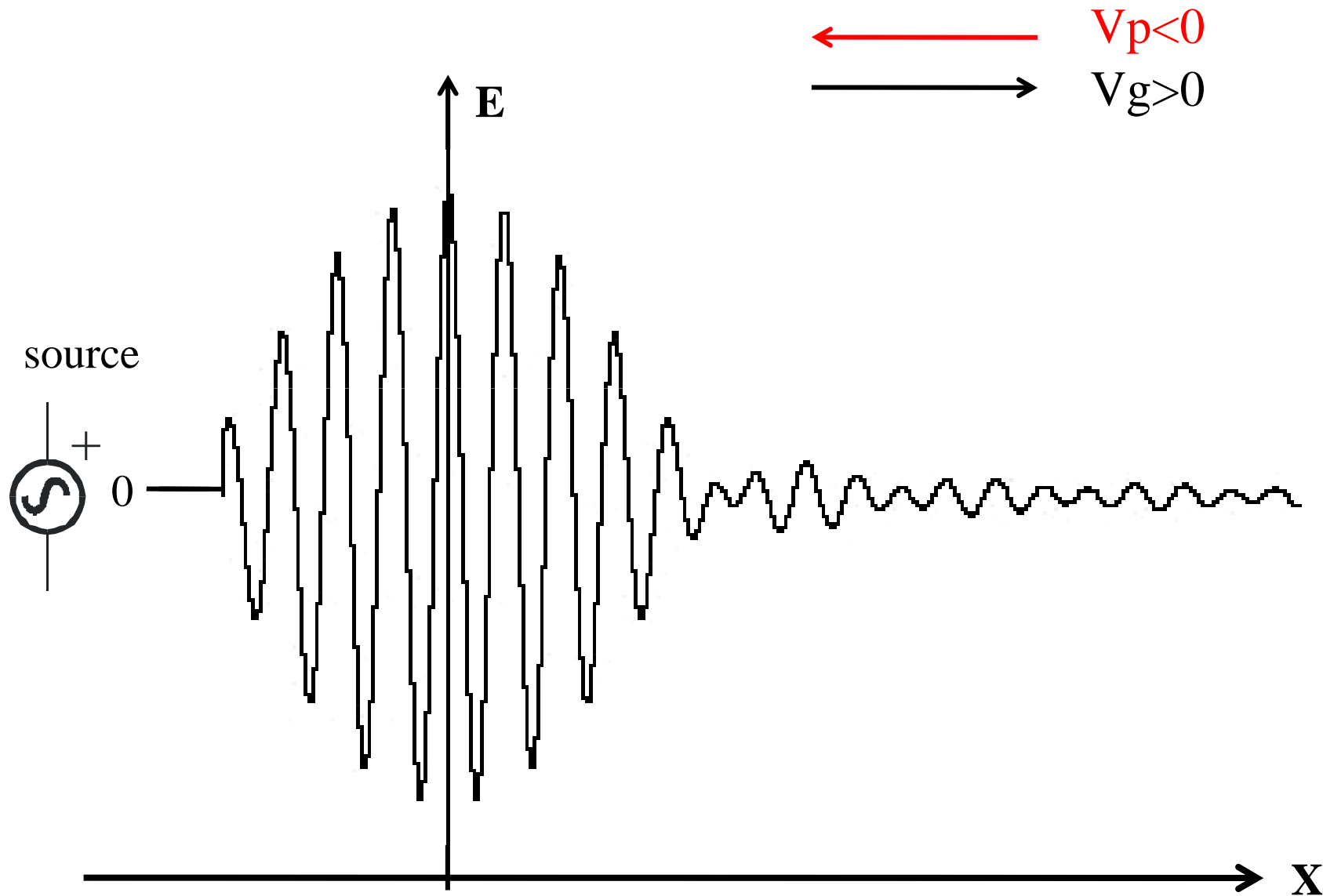
Sign ambiguity

$\vec{k}$

# One-dimensional Wave Propagation in Material with $\mu > 0$ and $\epsilon > 0$ (forward wave propagation)

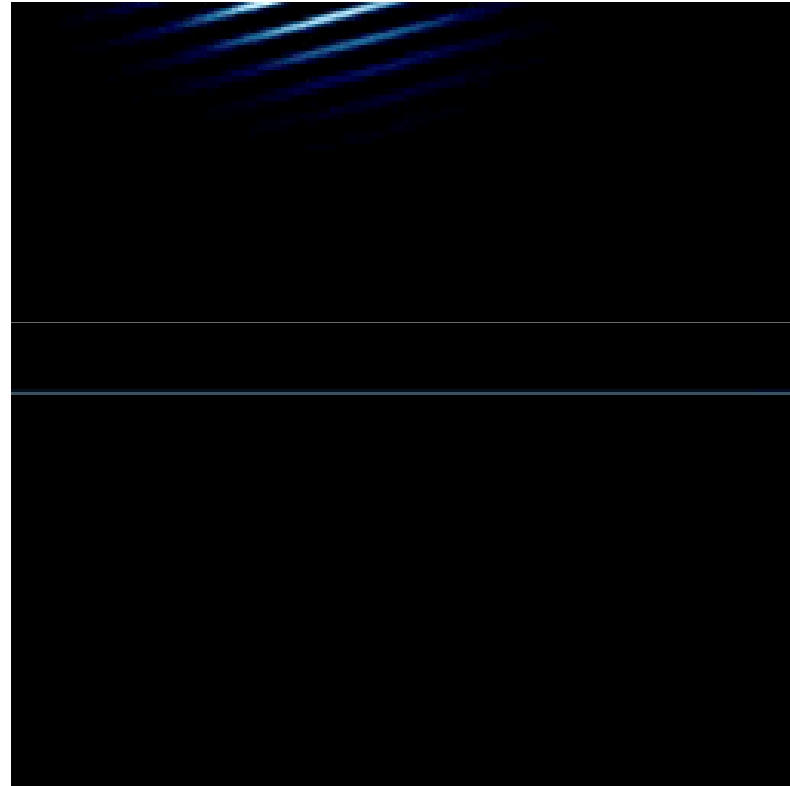
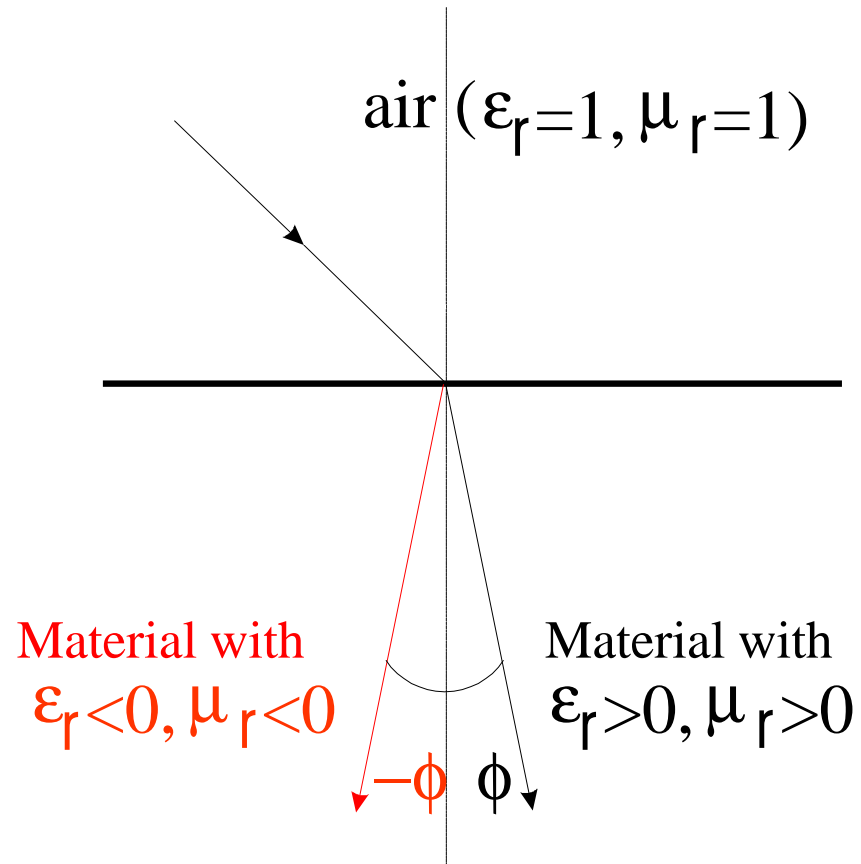


# One-dimensional Wave Propagation in Material with $\mu < 0$ and $\epsilon < 0$ (backward wave propagation)

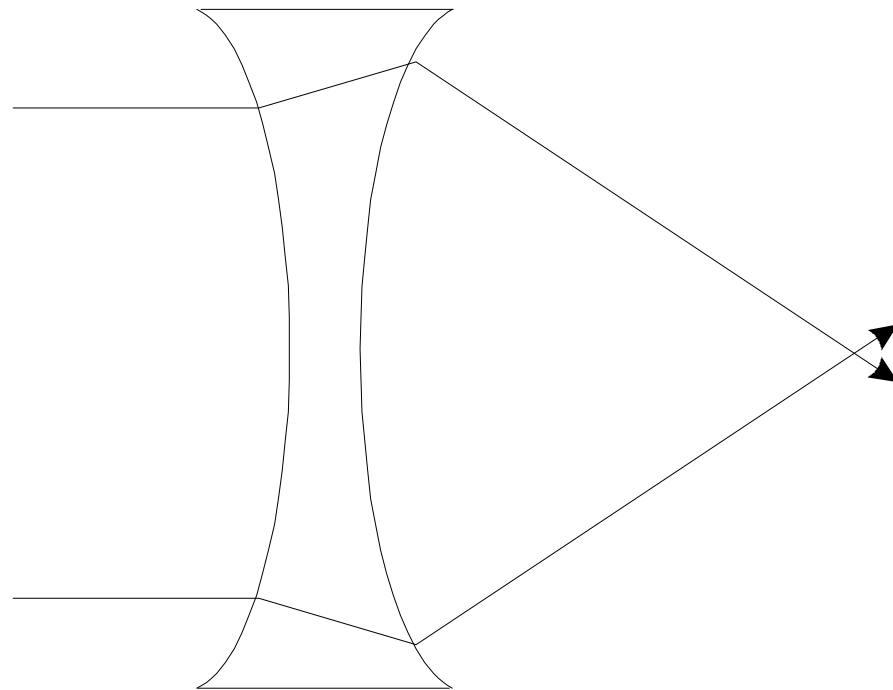
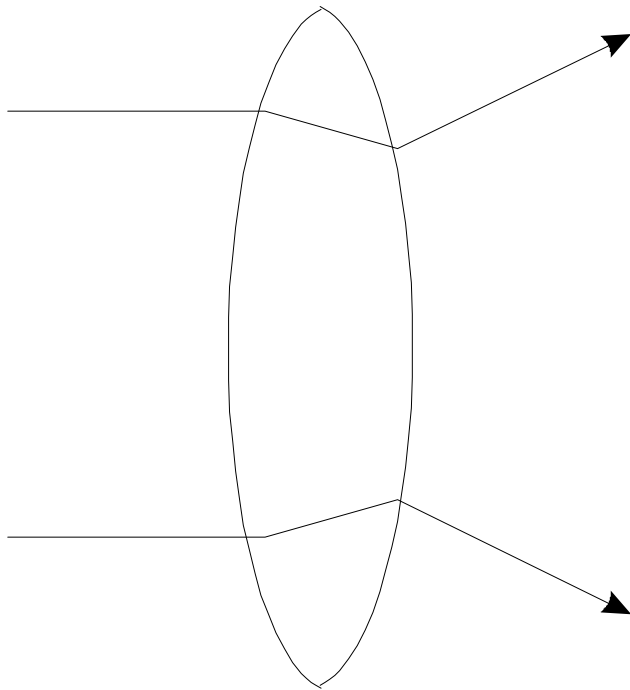


- **$P$  is opposite to  $k$  !**
- This causes ‘reversion’ of many basic electromagnetic phenomena such as Snell law, Doppler effect e.t.c.

# 'Reversion' of Snell Law

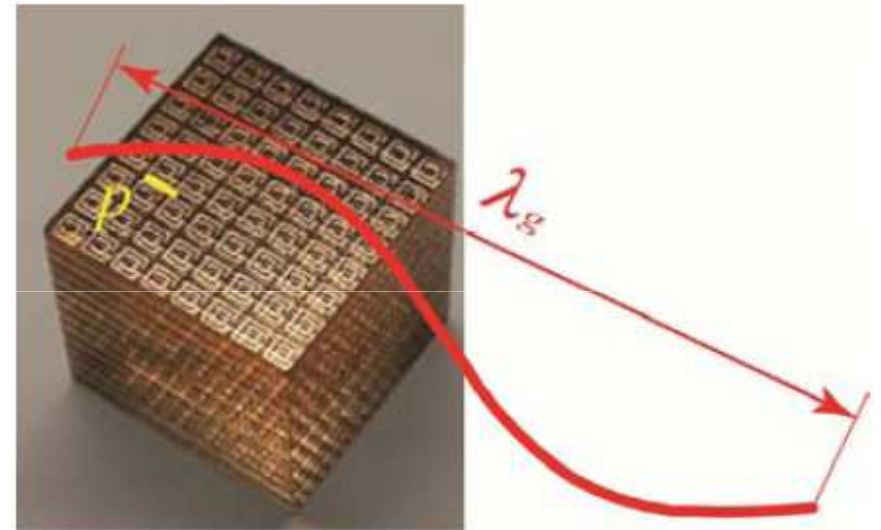
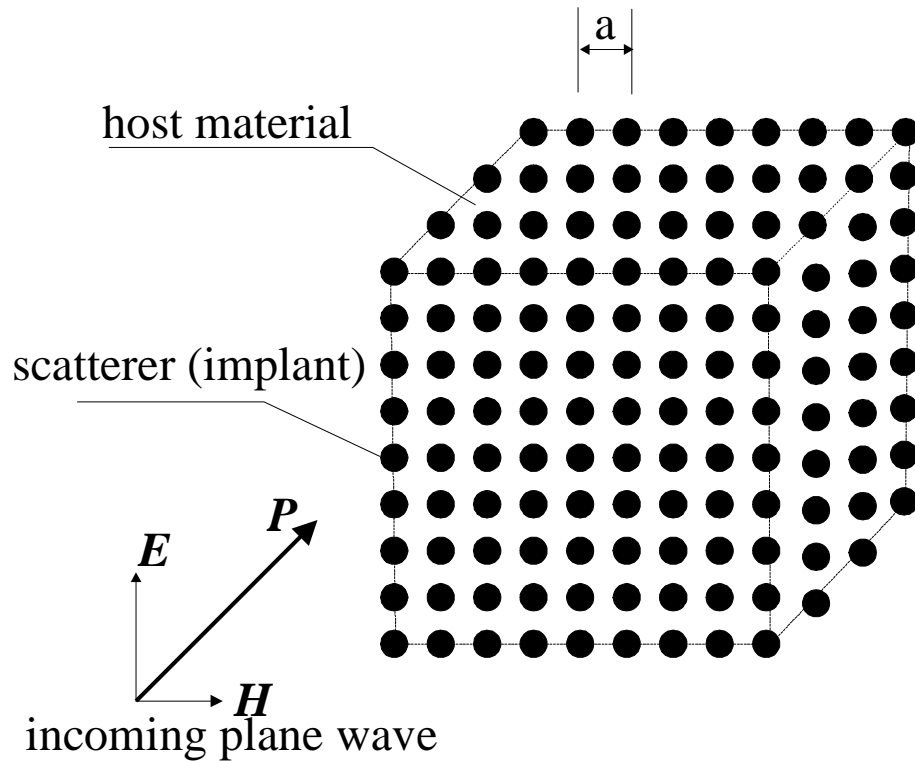


**Concave and Convex Lenses using material  
with  
 $\mu < 0$  and  $\varepsilon < 0$**



# The Concept of Metamaterial

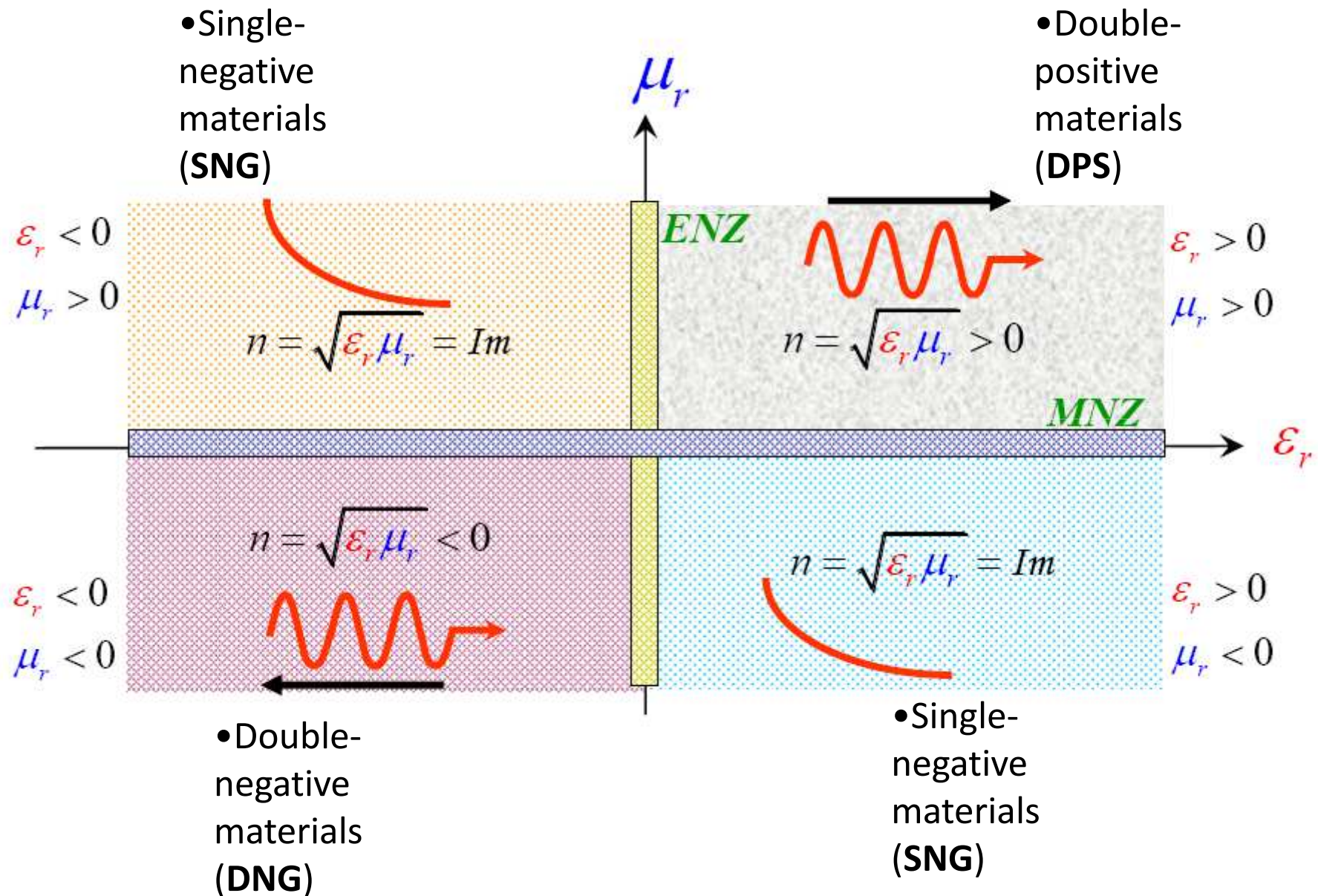
μετα = meta = beyond (Greek)



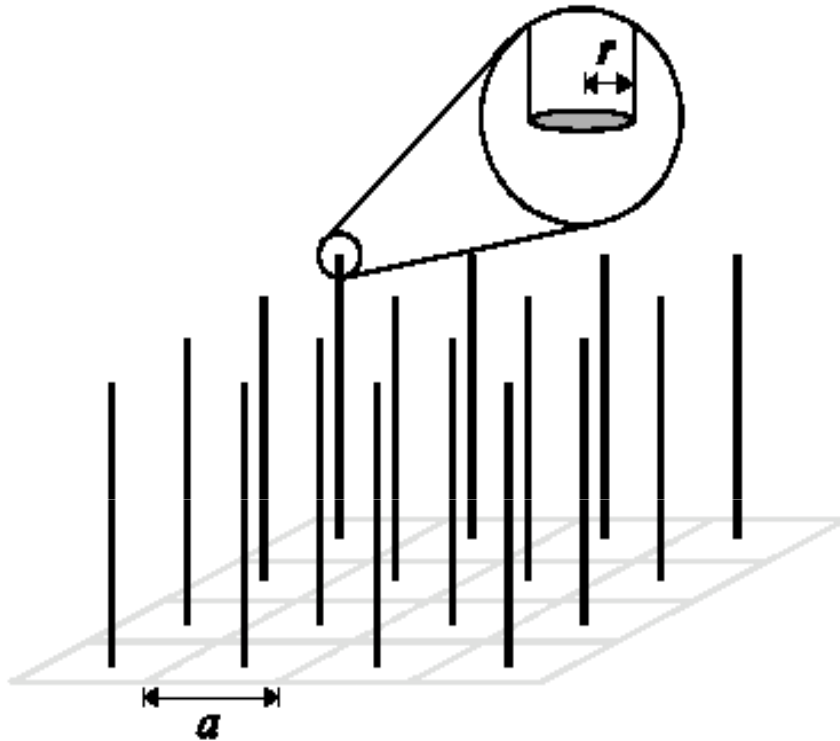
- $a \ll \lambda$ , the structure behaves as a homogenous material with some new  $\mu$  and  $\epsilon \Rightarrow$  metamaterial



# Classification of Metamaterials



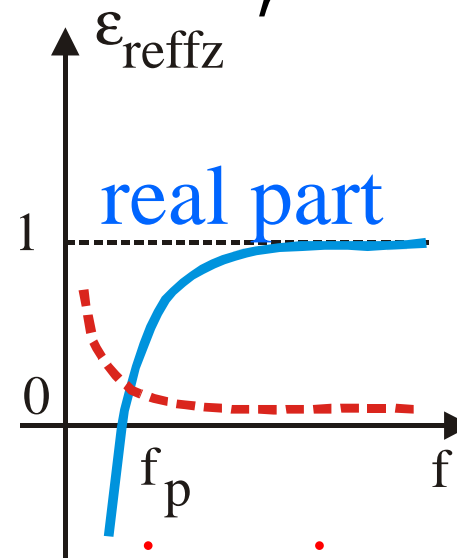
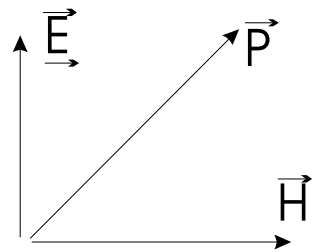
# Thin-wire Plasma-like Structure with $\epsilon < 0$ ( Rotman 1962, Pendry 1998)



$$\epsilon_r = 1 - \left( \frac{\omega_p}{\omega} \right)^2$$

$$a \ll \lambda$$

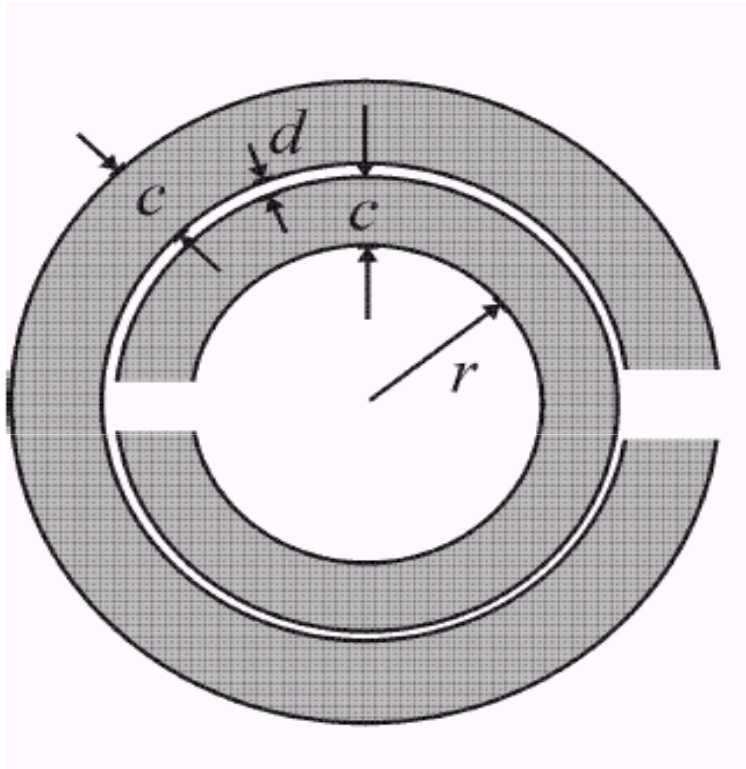
$$\frac{a}{r} \gg 1$$



real part

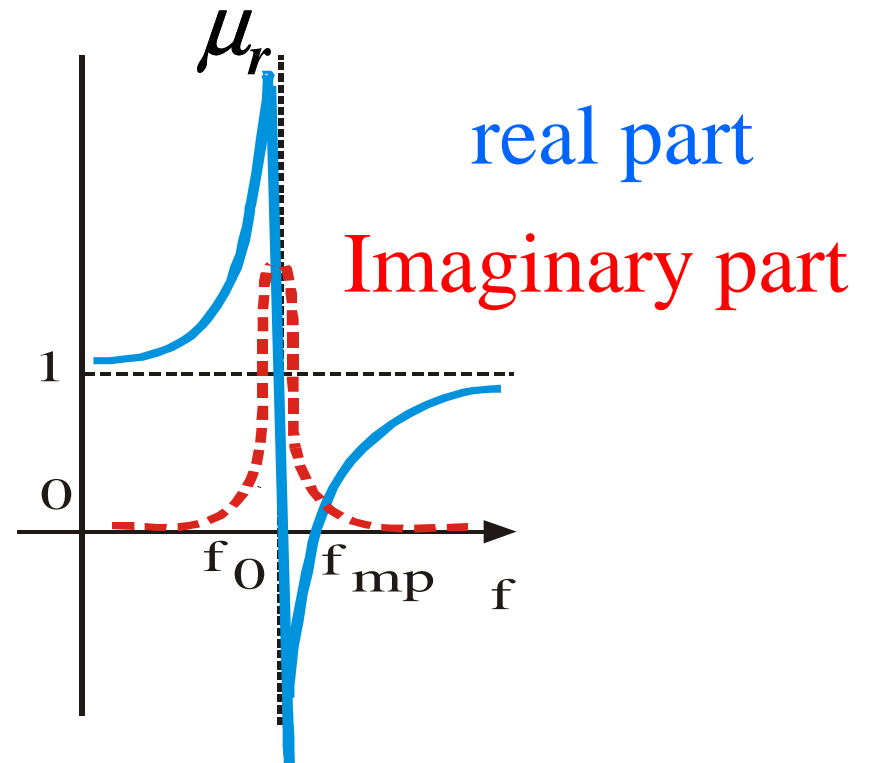
imaginary part

# The 'Split-ring Resonator' Structure with $\mu < 0$ (Pendry 1999)

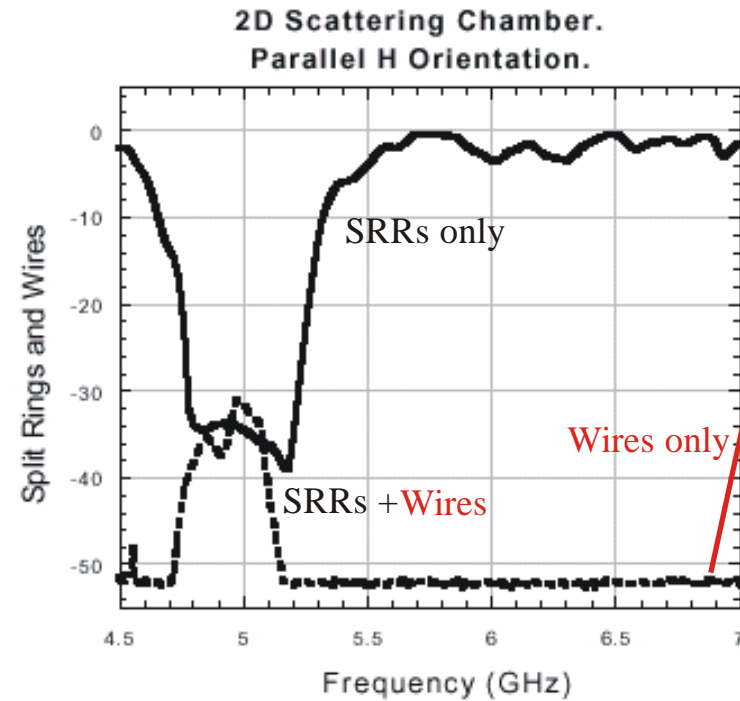
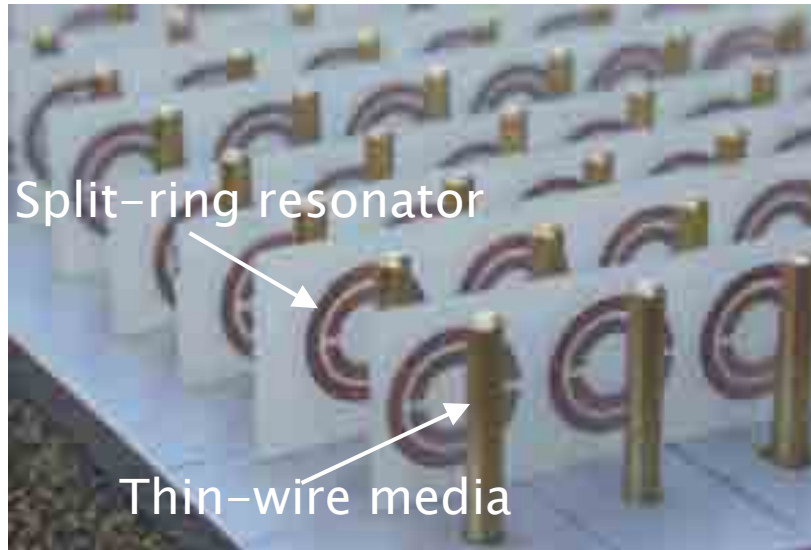


$$r \ll \lambda$$

$$a \ll \lambda$$



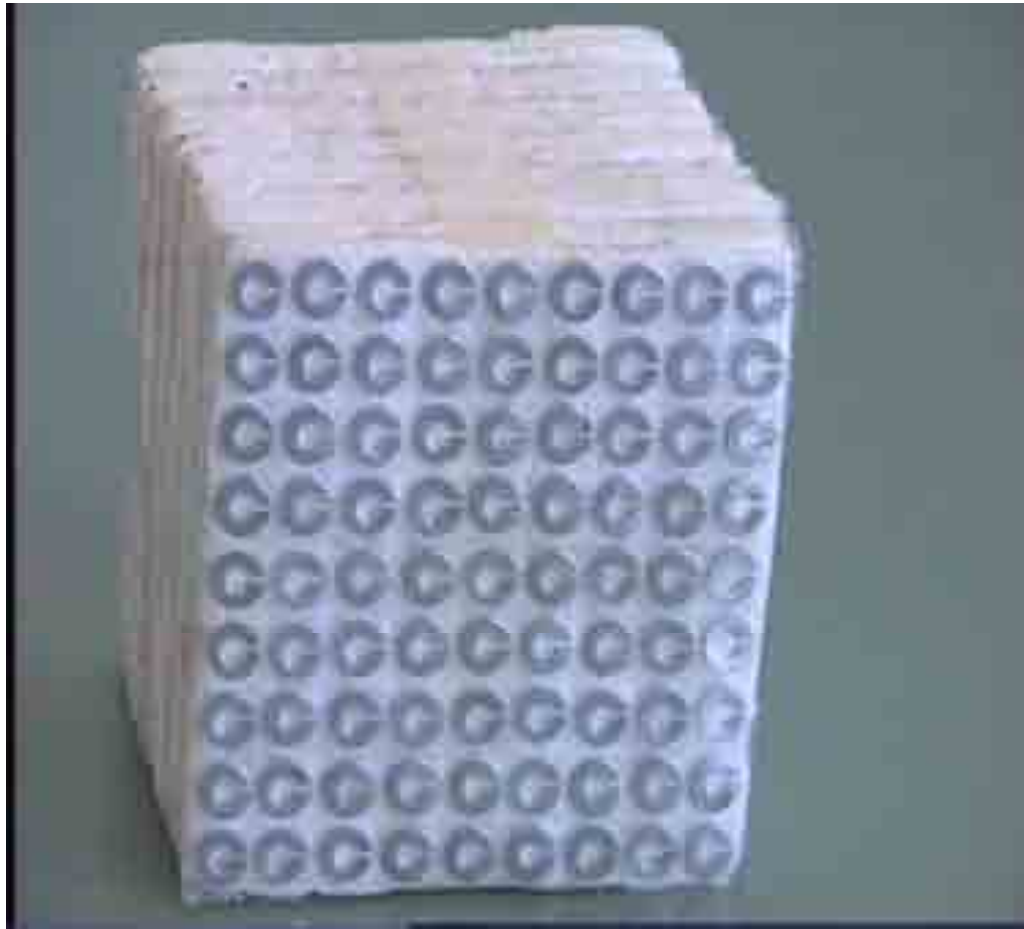
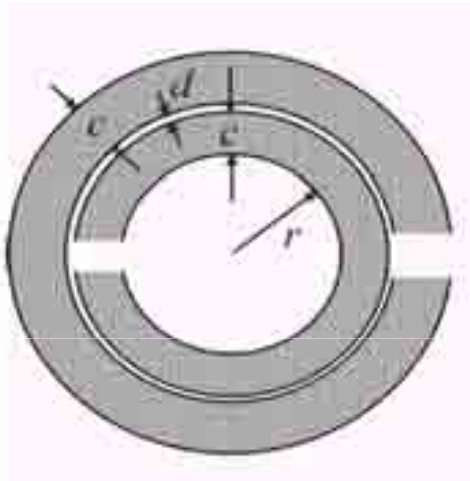
# The first reported Backward-wave Material (Smith et al. 2000)



$$k = \omega \sqrt{\mu \epsilon}$$

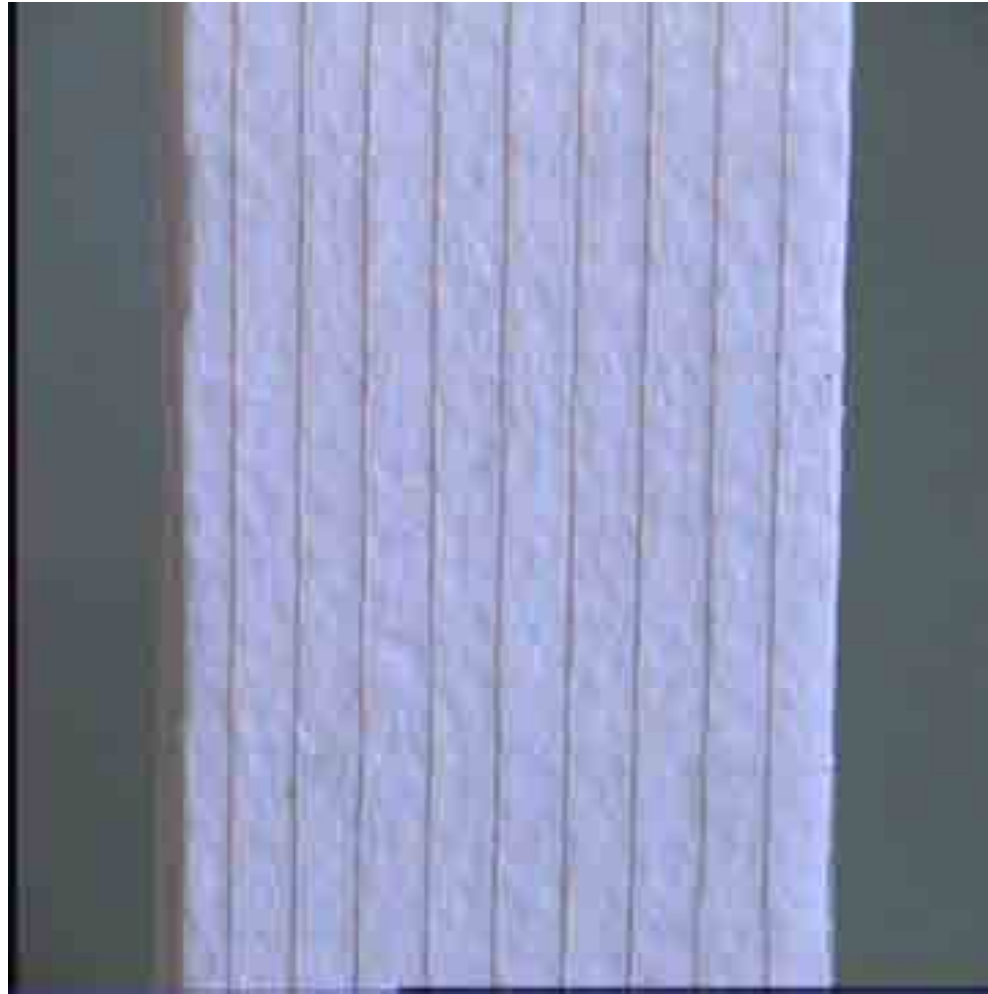
*SRR's and wires* :  $\mu < 0$   $\epsilon < 0$ ,  $k$  becomes positive real number  $\Rightarrow$  backward-wave propagation

# Capacitively loaded loops and thin-wire structure (Hrabar, Barbaric; 2000 –1/3)

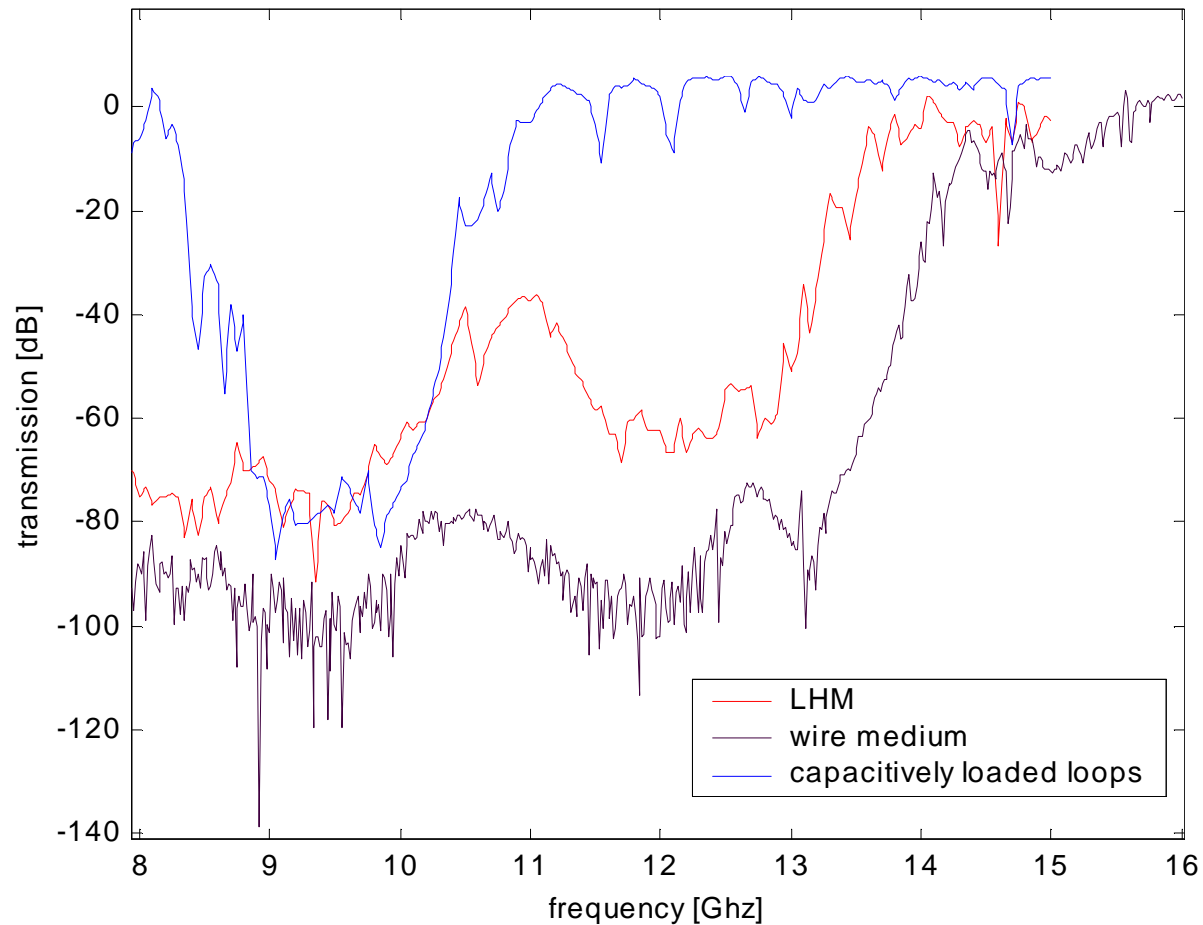




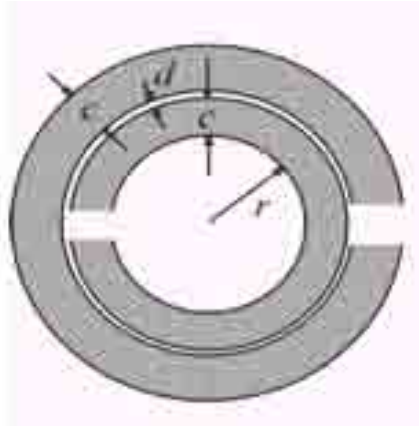
# Capacitively loaded loops and thin-wire structure (Hrabar, Barbaric 2000; -2/3)



# Capacitively loaded loops and thin-wire structure (Hrabar, Barbaric 2000; -3/3)



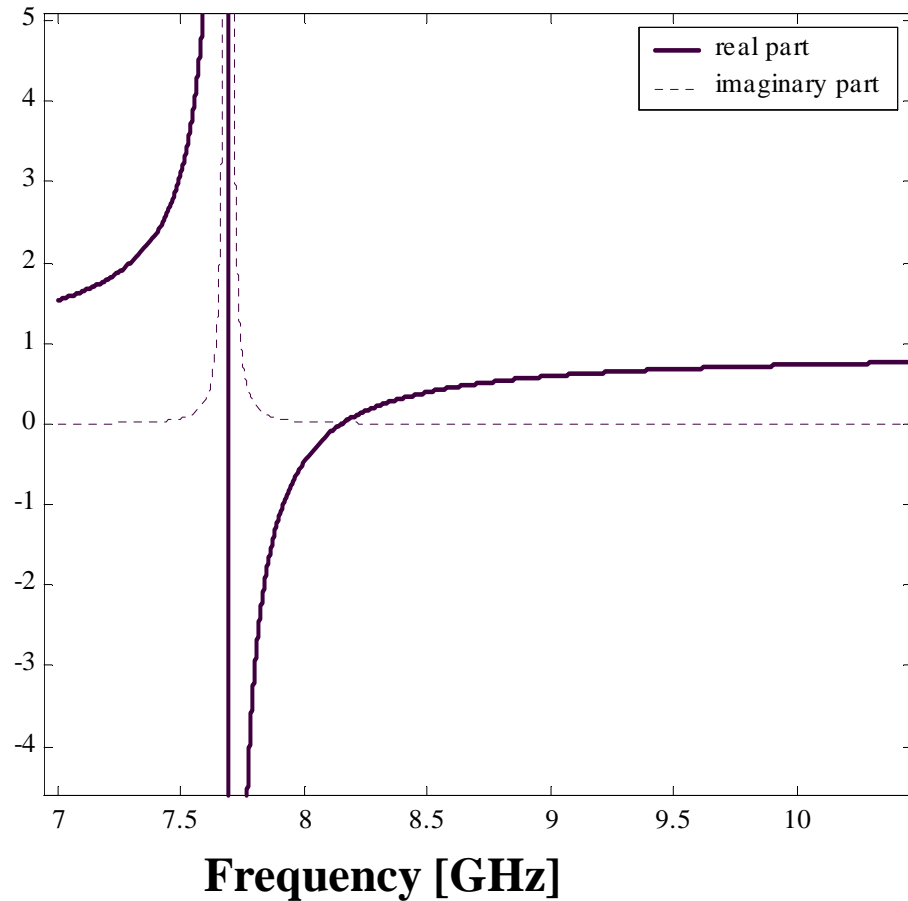
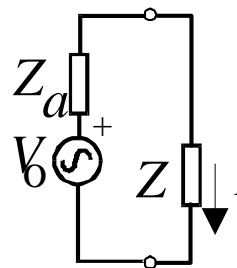
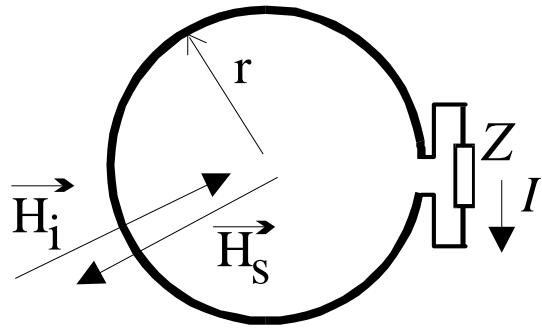
# The SRR behaves as capacitively loaded loop (Hrubar, Bartolic, Eres; 2002)



Calculated  
effective  
permeability

$$H = H_i + H_s$$

$$H = H_i \left( 1 - j \frac{K \omega \mu_0 S}{R + j \omega L - j / \omega C} \right)$$





# Schelkunoff, Friis – 'Antennas, Theory and Practice', 1952

by decreasing the permeability and increasing the dielectric constant.

This is particularly undesirable since the effects of the changes in  $\mu$  and  $\epsilon$  on the index of refraction are opposite. What we need is a way of increasing the permeability rather than decreasing it.

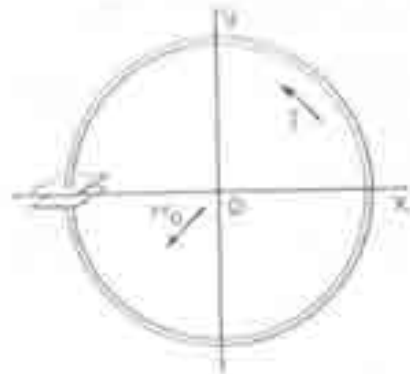


FIG. 19.10 A loop loaded with capacitance.

## 19.10 Methods for increasing the permeability of artificial dielectrics

Consider a loop with a capacitor (Fig. 19.10). Let the impressed magnetic intensity  $H_0$  be in the positive  $z$  direction. The counterclockwise induced current is

$$I = \frac{-j\omega\mu_0 H_0 S}{j\omega L + (1/j\omega C)} = \frac{\omega^2 \mu_0 C S H_0}{1 - \omega^2 LC}, \quad (52)$$

where  $L$  is the inductance of the loop and  $C$  the capacitance in series with it. The moment of the magnetic doublet equivalent to the loop is

$$p_m = \mu_0 I S. \quad (53)$$

Hence, the magnetic polarizability is

$$\chi_m^0 = \frac{\omega^2 \mu_0^2 C S^2}{1 - \omega^2 LC} = \frac{\omega^2 \mu_0 \epsilon_0 (C/\epsilon_0) S^2}{1 - \omega^2 \mu_0 \epsilon_0 (LC/\mu_0 \epsilon_0)} \mu_0. \quad (54)$$

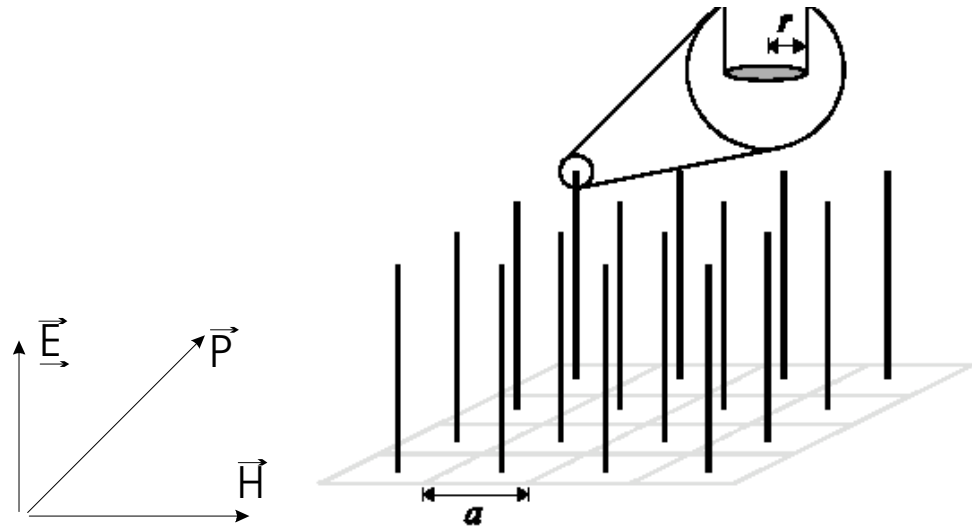
The ratios  $C/\epsilon_0$  and  $L/\mu_0$  depend only on the geometry of the metal object;  $\omega^2 \mu_0 \epsilon_0 = 4\pi^2/\lambda^2$  where  $\lambda$  is the wavelength in free space corresponding to the given frequency.

The capacitance may be supplied by the loop itself (Fig. 19.11) if



FIG. 19.11 A loop approaching resonance.

# Wire Medium as Transmission Line (Hrubar 2003,2006)



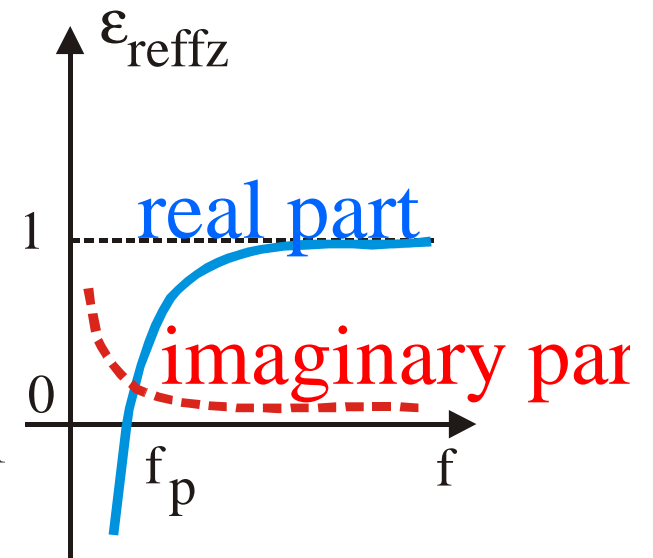
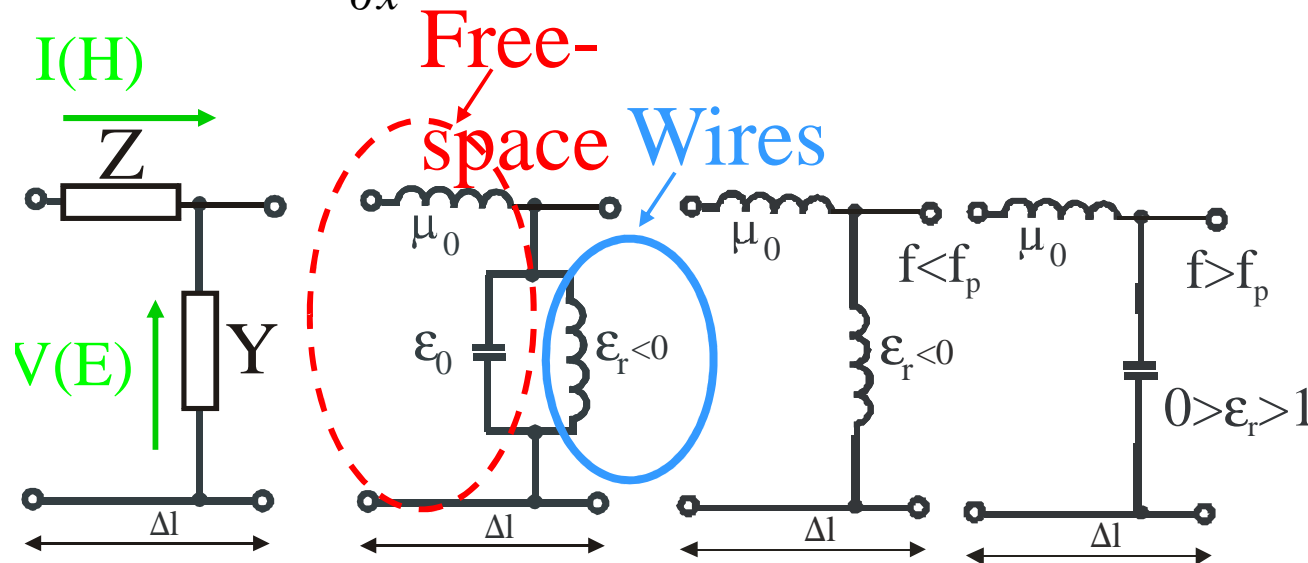
$$\epsilon_r = 1 - \left( \frac{\omega_p}{\omega} \right)^2$$

$$a \ll \lambda$$

$$\frac{a}{r} \gg 1$$

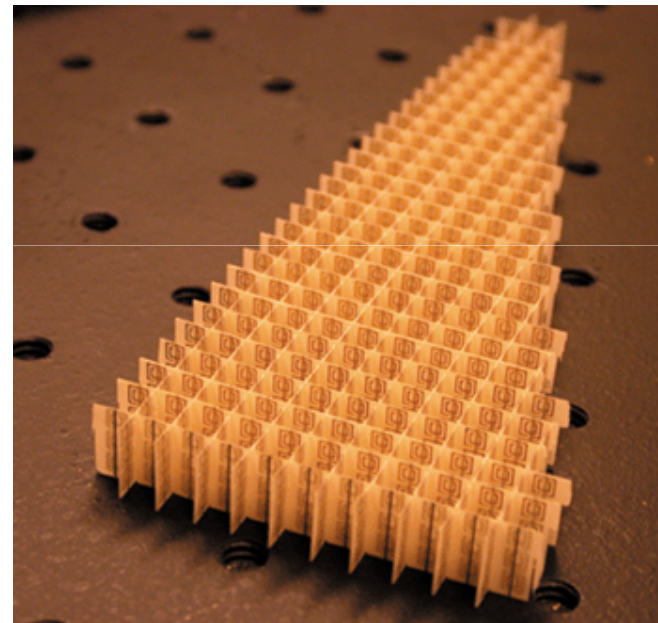
$$\nabla^2 E = -\omega^2 \mu \epsilon E = (j\omega\sqrt{\mu})(j\omega\sqrt{\epsilon})E$$

$$\frac{\partial^2 V}{\partial x^2} = -ZY V = (j\omega X_{series})(j\omega Y_{shunt})V$$

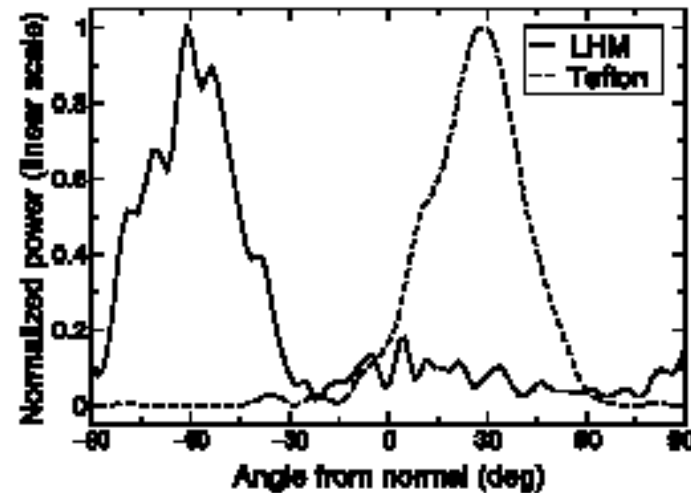
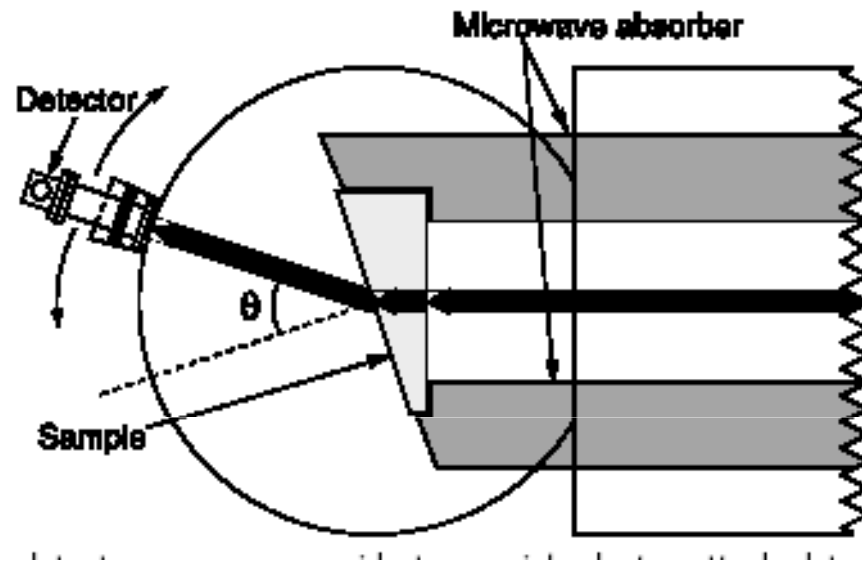


# **New Physical Phenomena - Fundamental research**

# Experimental Verification of Negative Refraction (Shelby et al. 2001)



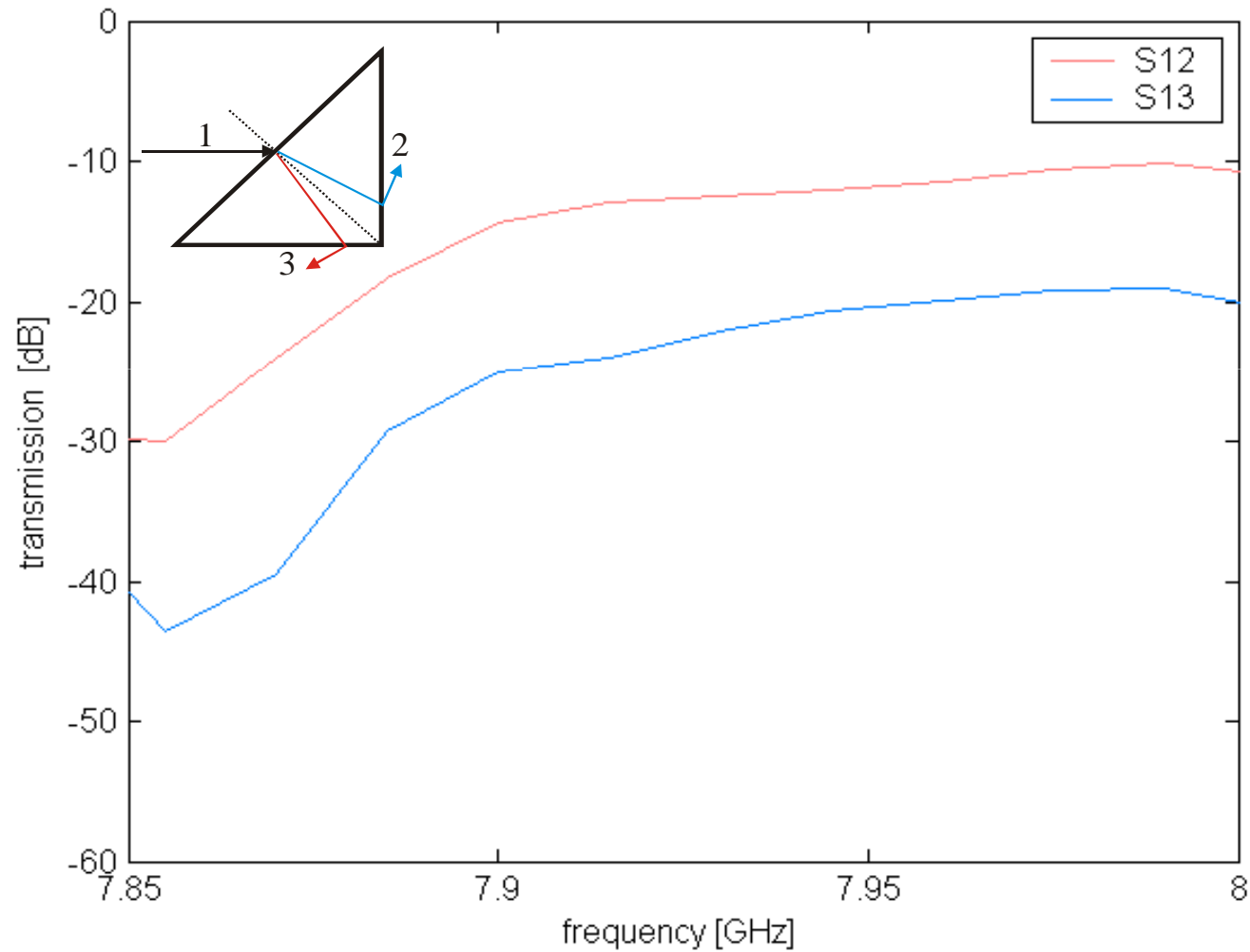
# Experimental Verification of Negative Refraction (Shelby et al. 2001)



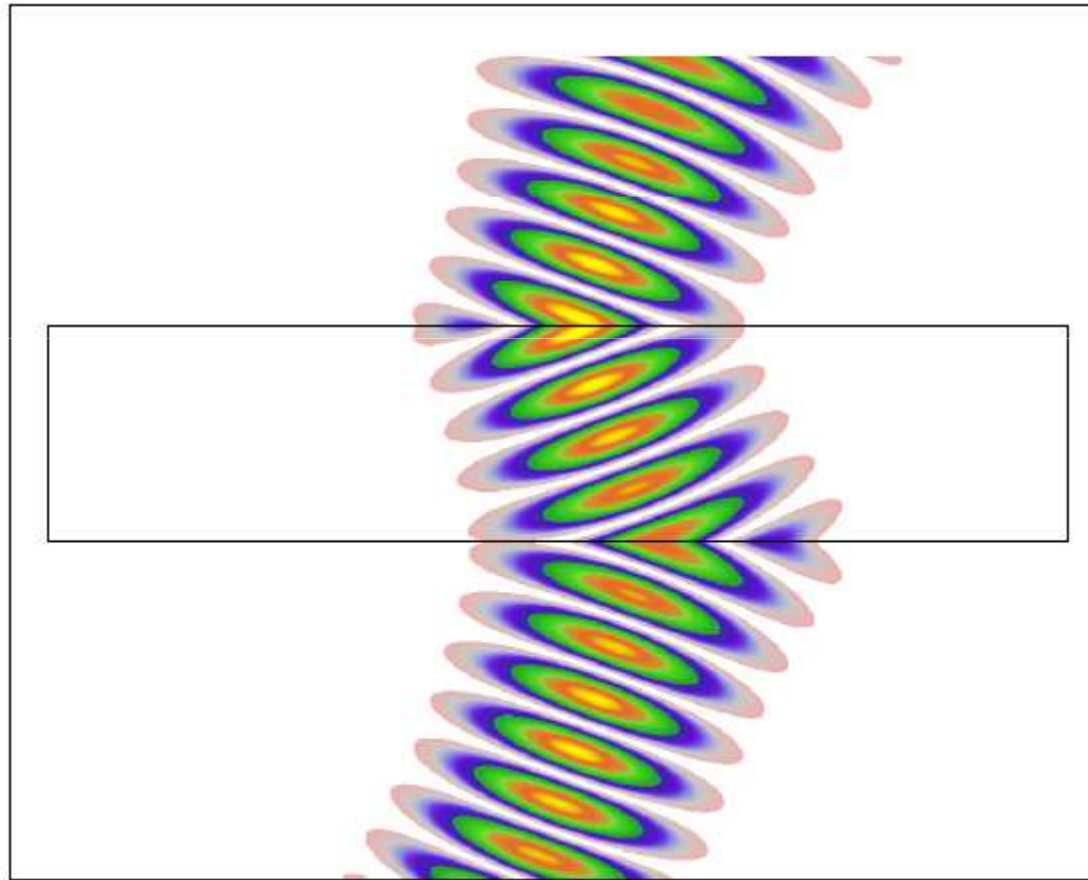
# Experimental Verification of Negative Refraction (Hrabar, Bartolic; 2003-1/2)



# Experimental Verification of Negative Refraction (Hrabar, Bartolic; 2003-2/2)

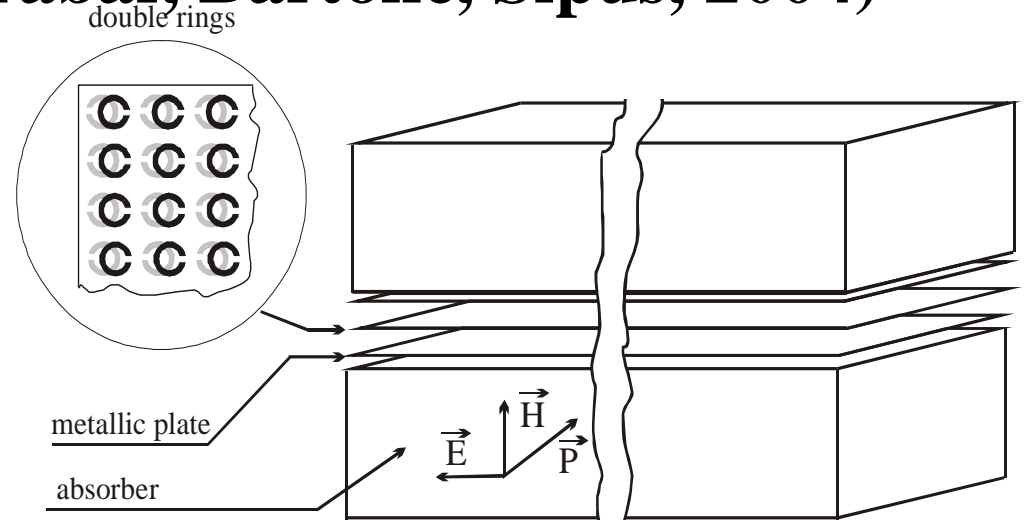
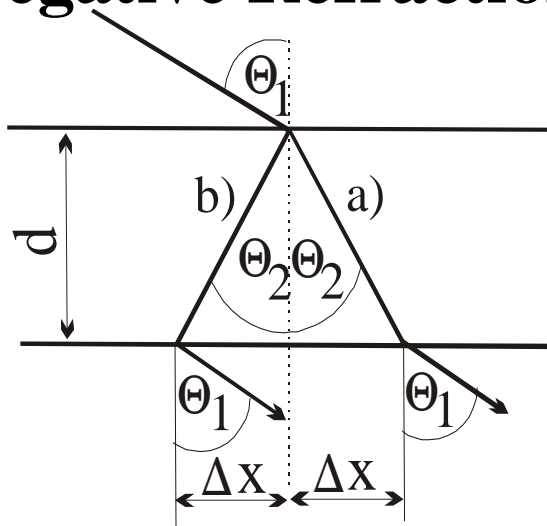


Simulation of Negative Refraction at Plan-parallel Slab with  $n=-1$   
(Ziolkowski; 2003)

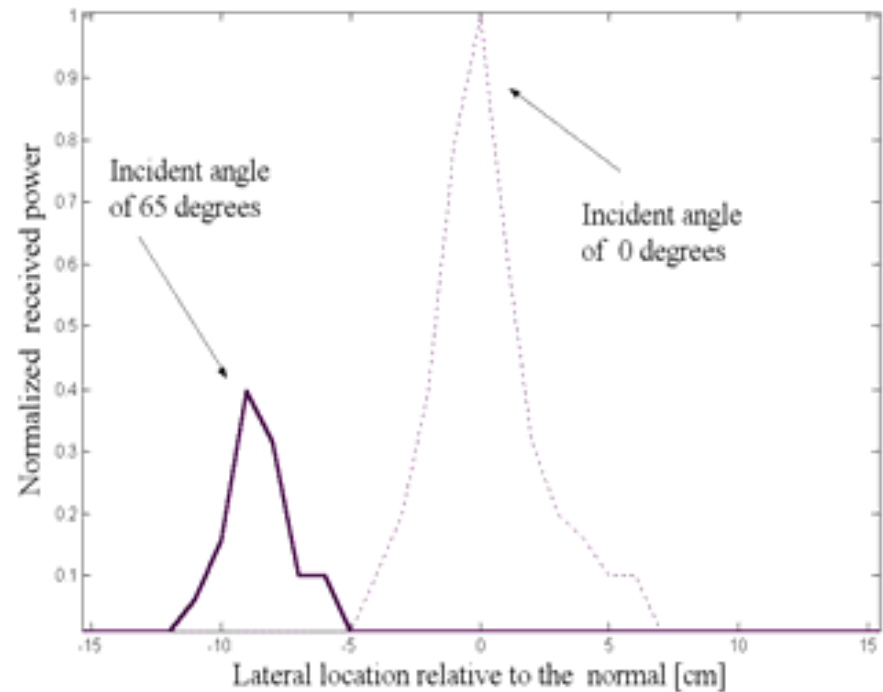




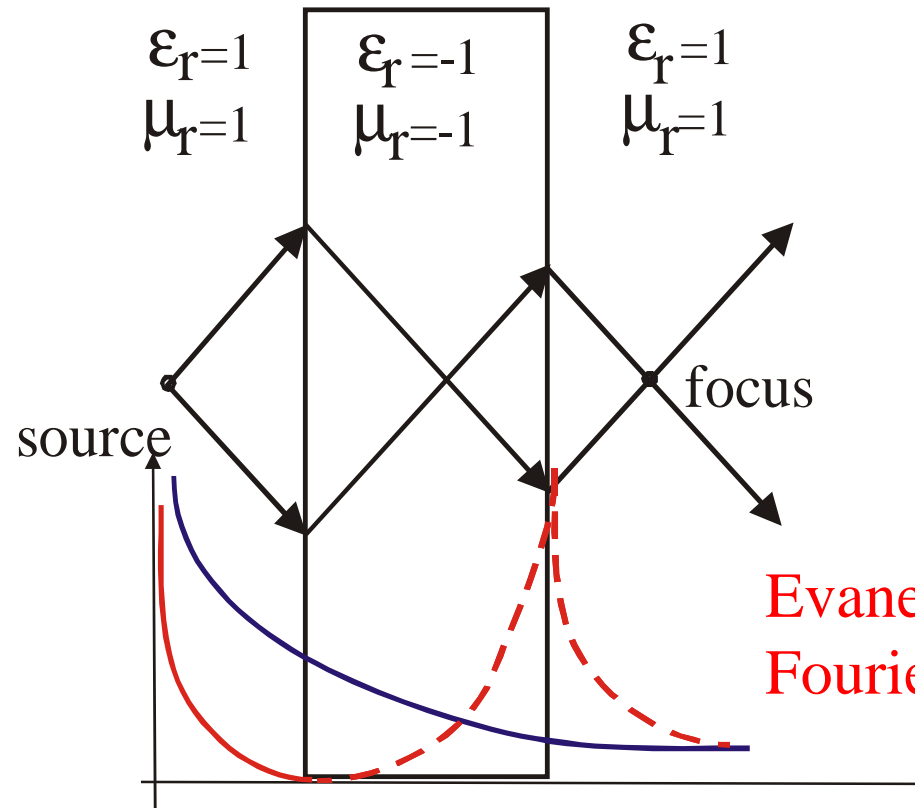
# Verification of BW propagation by measurement of Negative Refraction (Hrabar, Bartolic, Sipus, 2004)



- a) A case of 'forward-wave' slab ( $\epsilon > 0, \mu > 0$ )
- b) A case of 'backward-wave' slab ( $\epsilon < 0, \mu < 0$ )



# Super Lens' with no Resolution Limit (Pendry, 2000)



Propagating part of  
Fourier spectrum

Evanescent part of  
Fourier spectrum

$$E(x, y, z, t) = \sum_{k_x, k_y} A(k_x, k_y) e^{i(k_z z + k_y y + k_x x - \omega t)}$$

$$k_z = \sqrt{\frac{\omega^2}{c^2} - (k_x^2 + k_y^2)}$$

$$k_x^2 + k_y^2 < \frac{\omega^2}{c^2}$$

$$k'_z = -\sqrt{\frac{\omega^2}{c^2} - (k_x^2 + k_y^2)}$$

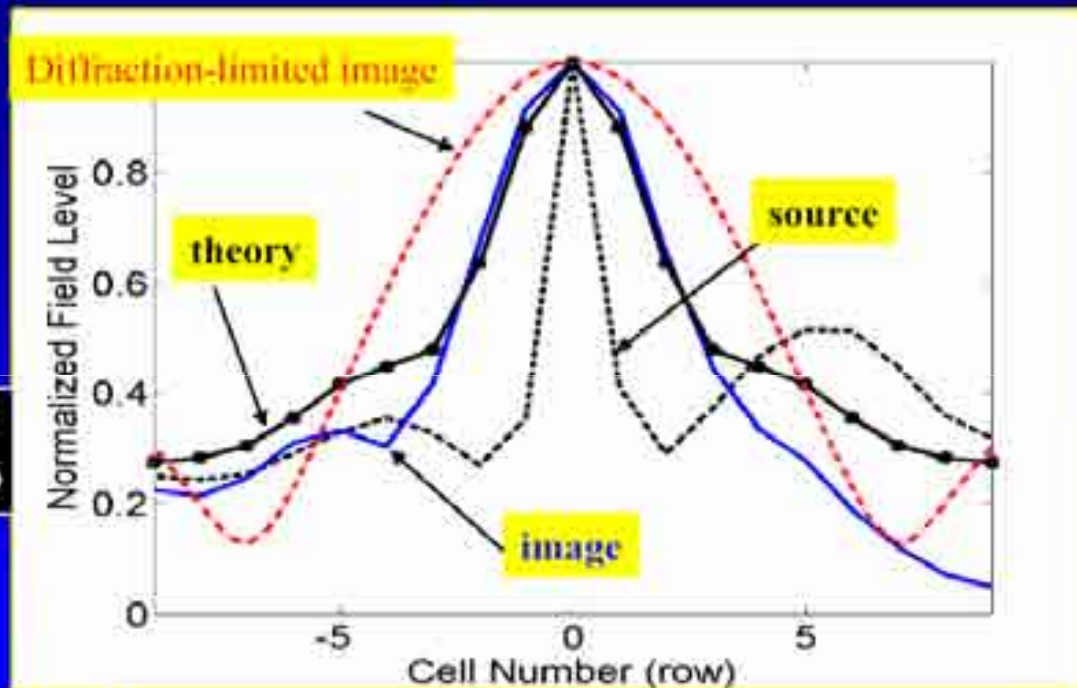
## Experimental verification (Grbic et al. 2004 -1/2)



Figure taken from G. Eleftheriades (University of Toronto) presentation slides

## Experimental verification (Grbic et al. 2004 – 2/2)

### Measured Beamwidth at Focal Plane



Low Loss:  
 $\text{Im}(n)=0.06$

Diffraction-limited peak-null beamwidth:  $\lambda/2$   
Measured peak-null beamwidth:  $\lambda/6$



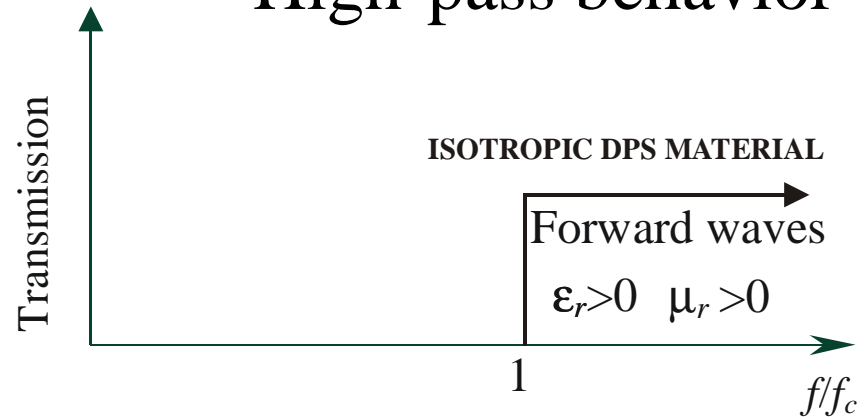
Figure taken from G. Eleftheriades (University of Toronto) presentation slides

# **Potential Engineering Applications**

## **I – Guiding of EM energy**

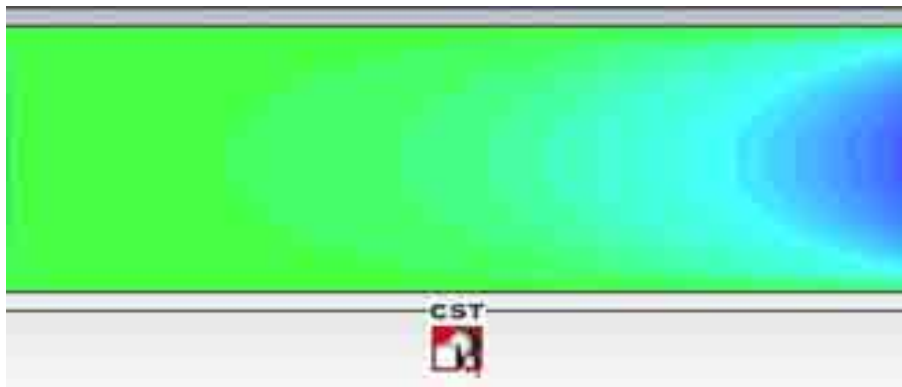
# Classical rectangular waveguide– Electrical field distribution (TE case)

- High-pass behavior

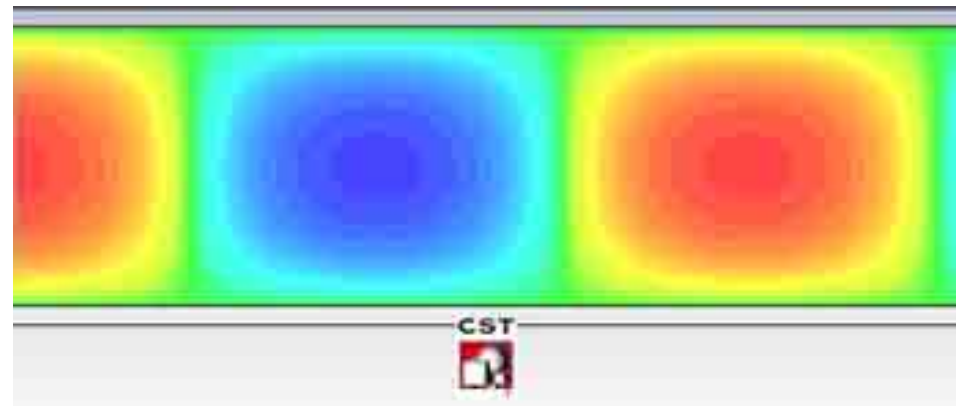


•  $f < f_c$ , Evanescent **E** field

•  $f > f_c$ , Propagating **E** field

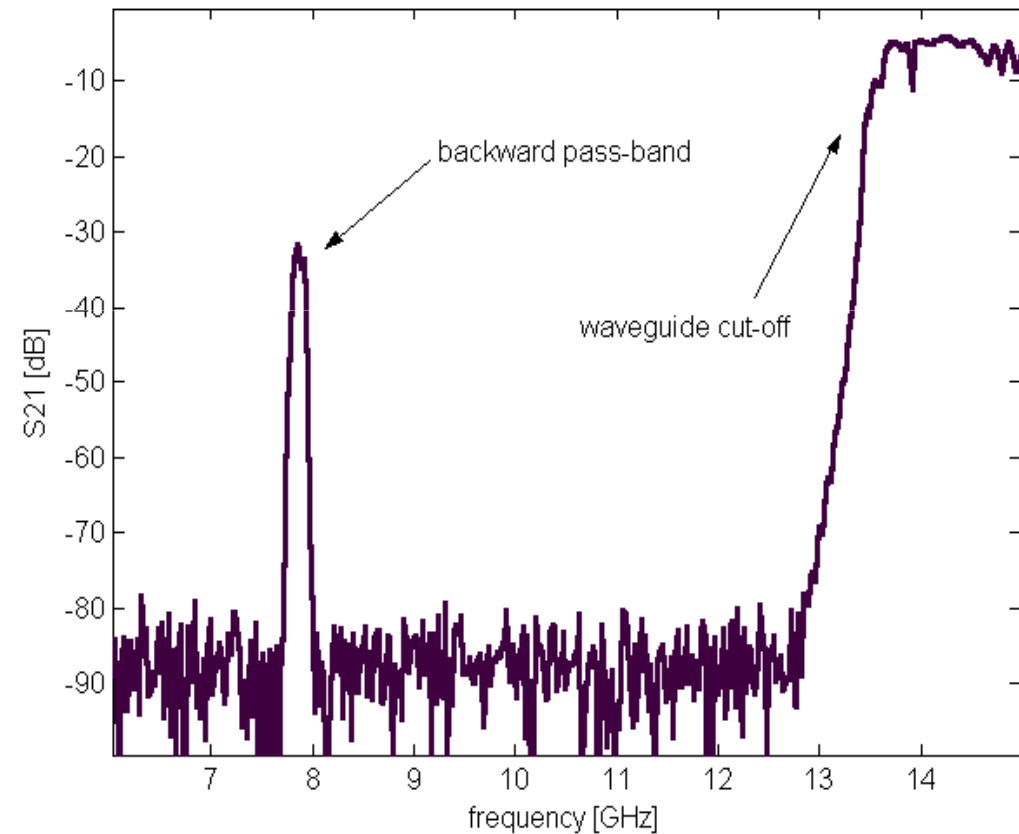
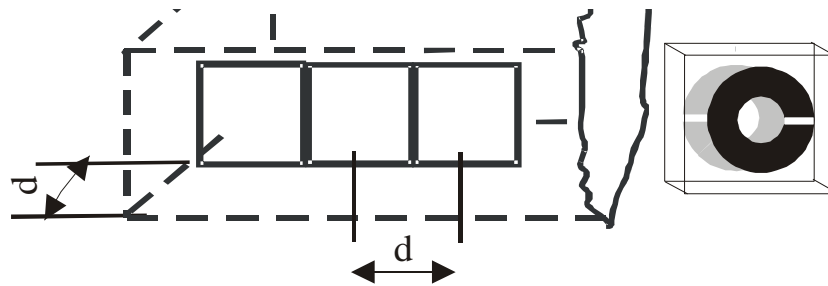


← **•P, k**



← **•P, k**

# Miniaturized Waveguide based on Anisotropic Meta-material (Hrabar, Bartolic, Sipus; 2003)



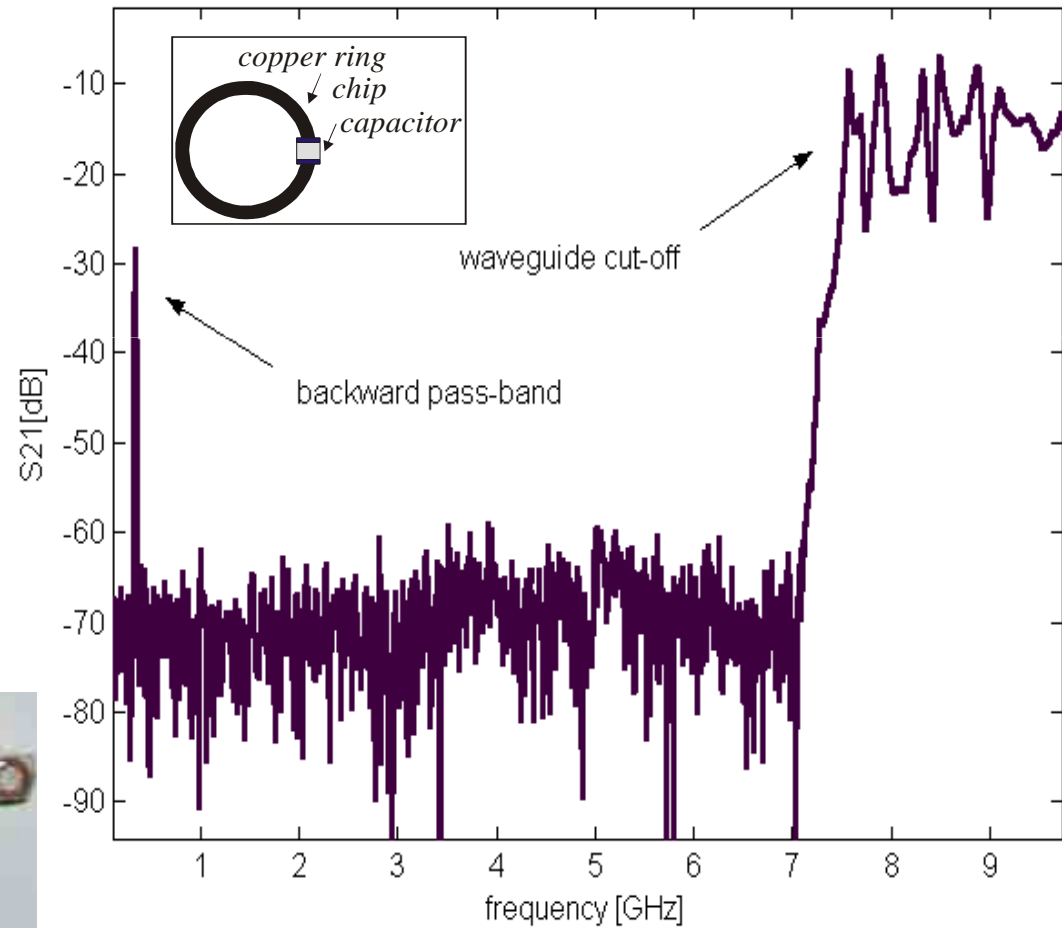


# Miniaturized Waveguide based on Negative Permeability Metamaterial (transversal dimension reduced down to 25%)





# Miniaturized waveguide ( $a=15$ mm) filled with capacitively loaded rings – the transversal width reduced down to 4% !

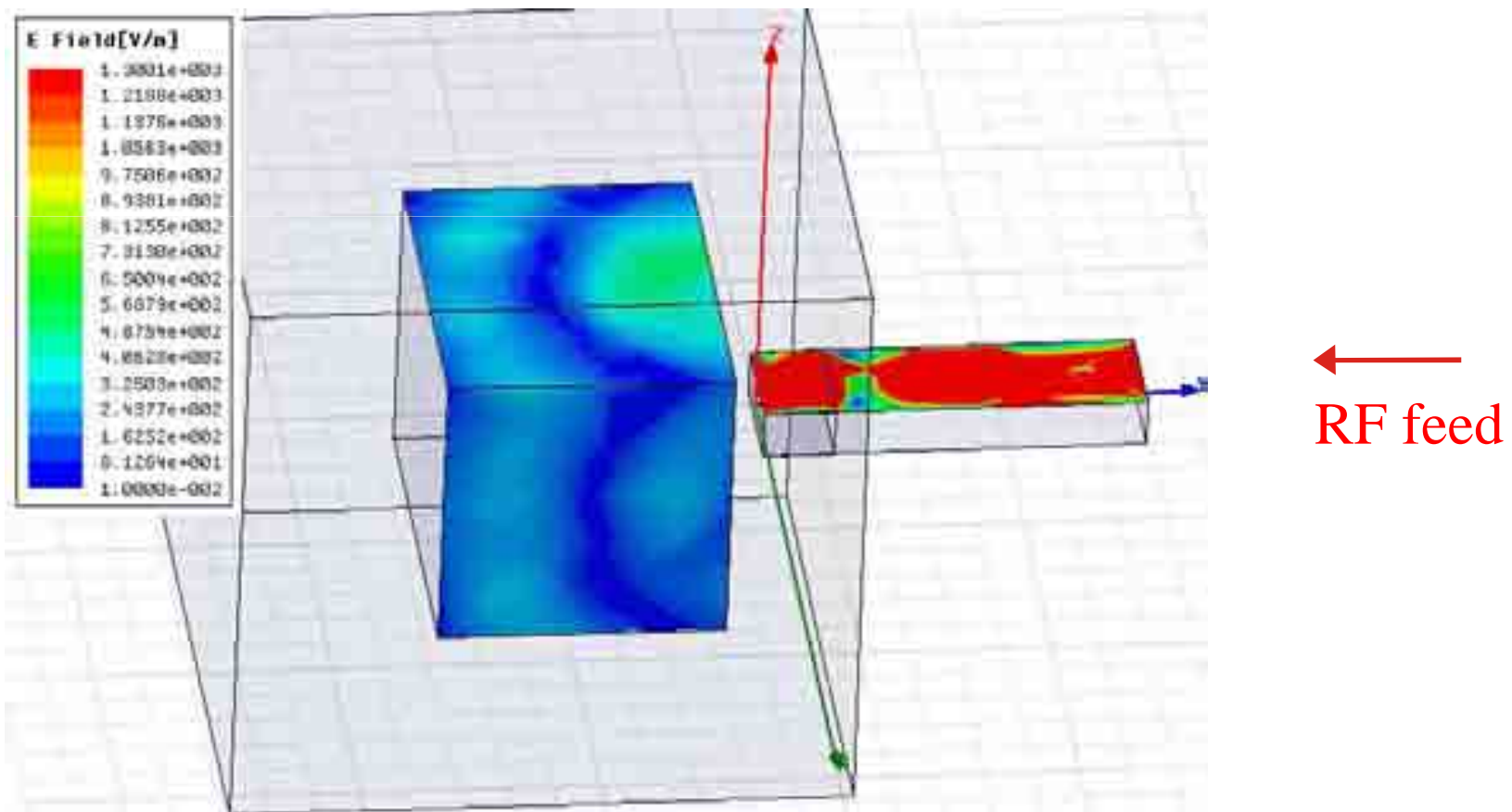


# **Potential Engineering Applications**

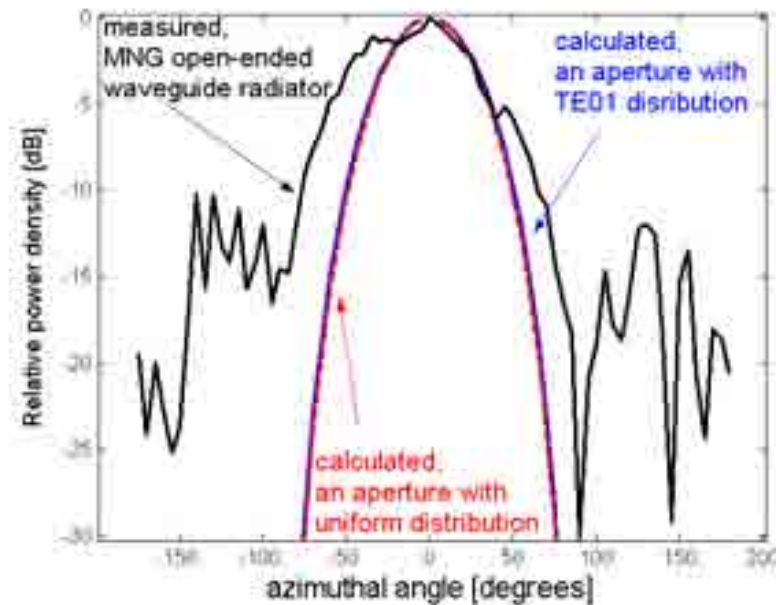
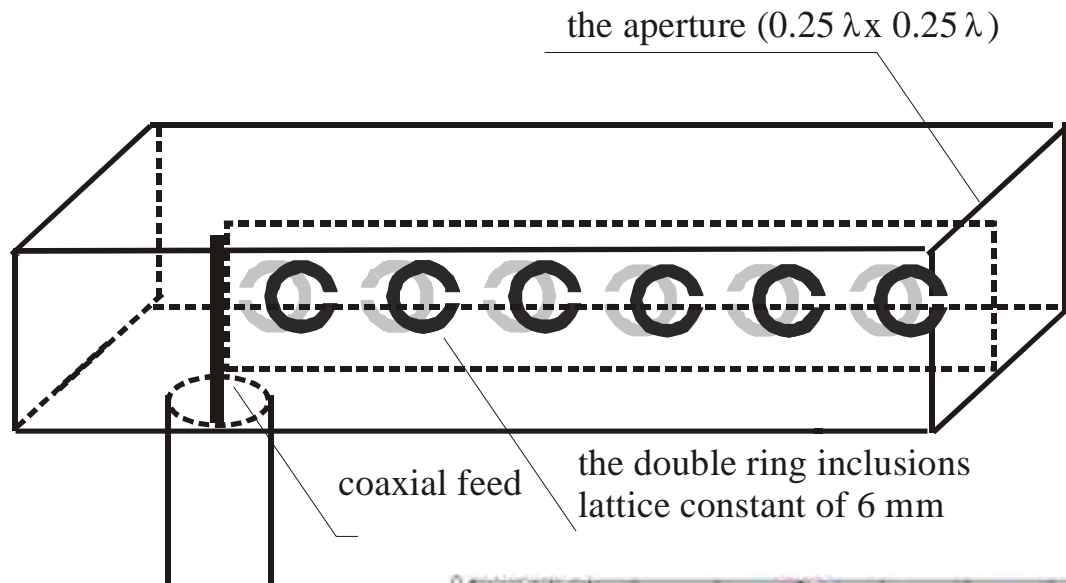
## **II – Antennas**

Does an open-ended SNG-filled miniaturized waveguide radiate into a free space ? (Hrabar, Zivkovic; 2005)

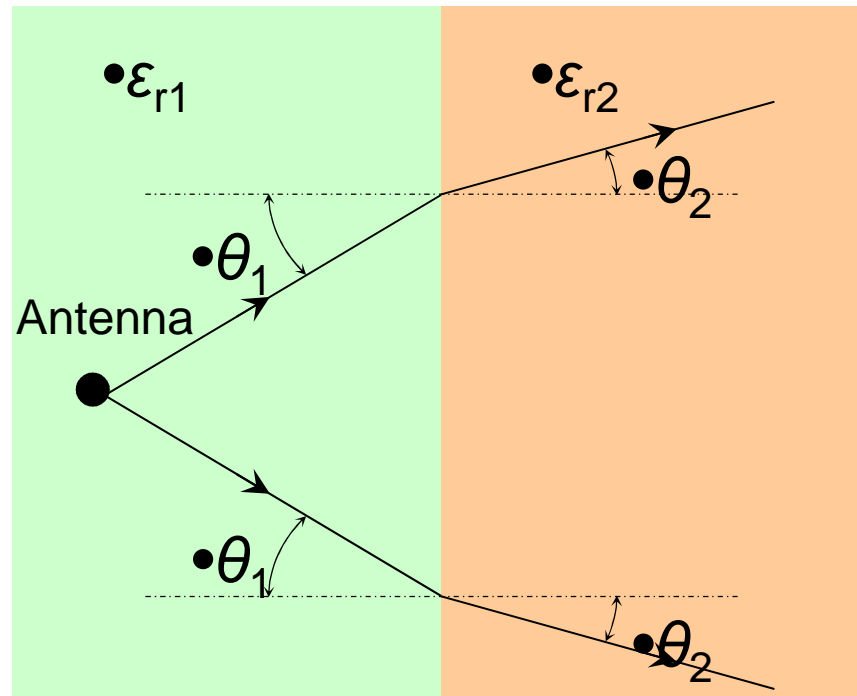
The simulated radiation (BW mode,  $k < 0$ )



# Experimental Miniaturized Open-ended MNG Waveguide Radiator (Hrabar, Jankovic 2005)



# Phenomenon of ultra-refraction



$$\frac{\sin \theta_1}{\sin \theta_2} = \frac{\sqrt{\epsilon_{r2}}}{\sqrt{\epsilon_{r1}}}$$

- If medium 2 is air

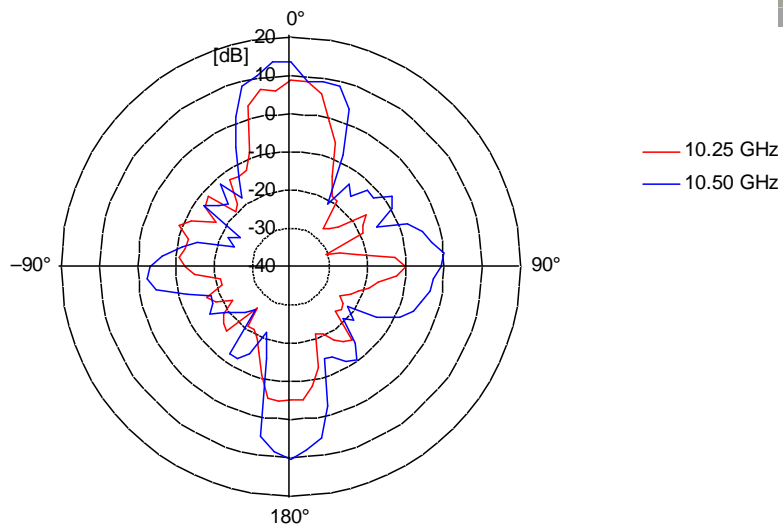
$$\theta_2 = \sin^{-1} \left( \sqrt{\epsilon_{r1}} \sin(\theta_1) \right)$$

if  $\epsilon_r = 0 \Rightarrow \theta_2 = 0$ .

## EXPERIMENTAL INVESTIGATION OF RADIATION PROPERTIES OF AN ANTENNA EMBEDDED IN LOW PERMITTIVITY THIN-WIRE-BASED METAMATERIAL

Davor Bonefačić, Silvio Hrabar, and Dražen Kvakan  
Department of Wireless Communications  
Faculty of Electrical Engineering and Computing  
University of Zagreb  
Unska 3  
Zagreb  
HR-10 000, Croatia

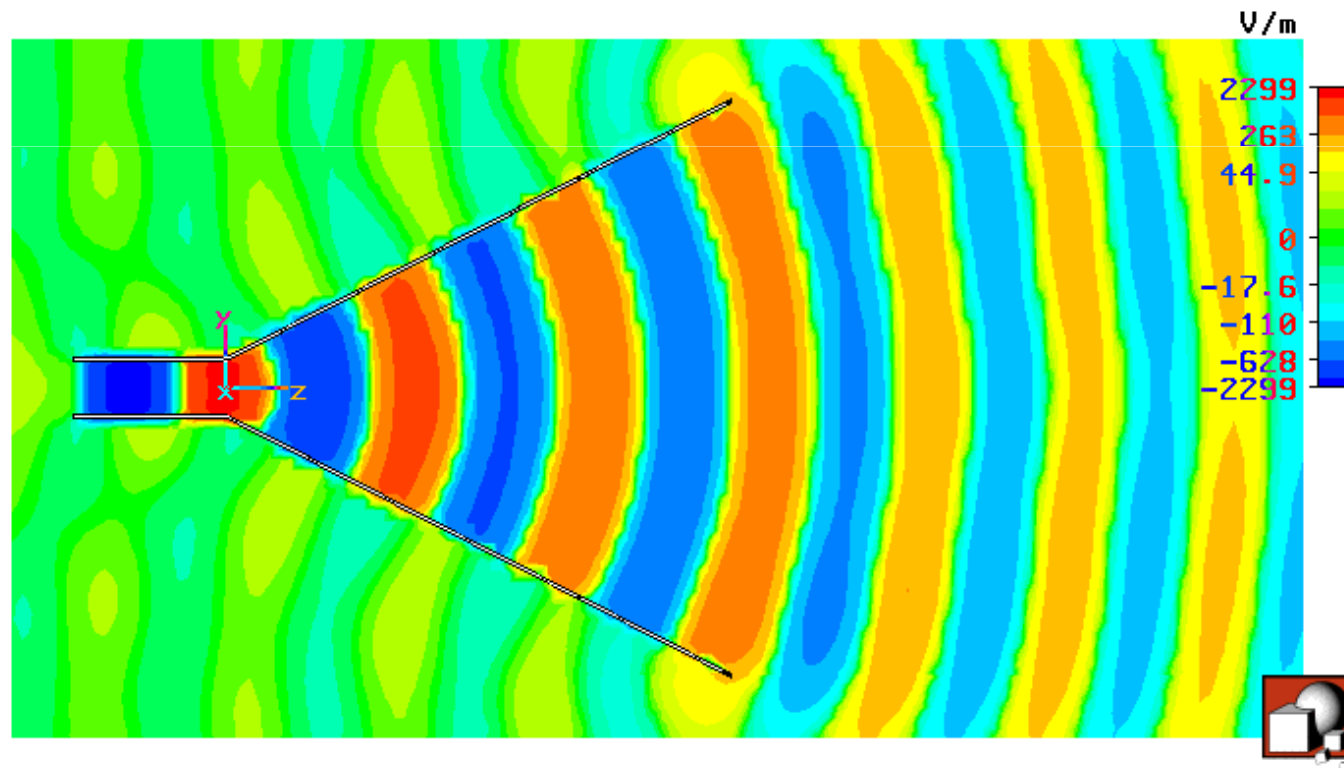
Received 12 June 2006



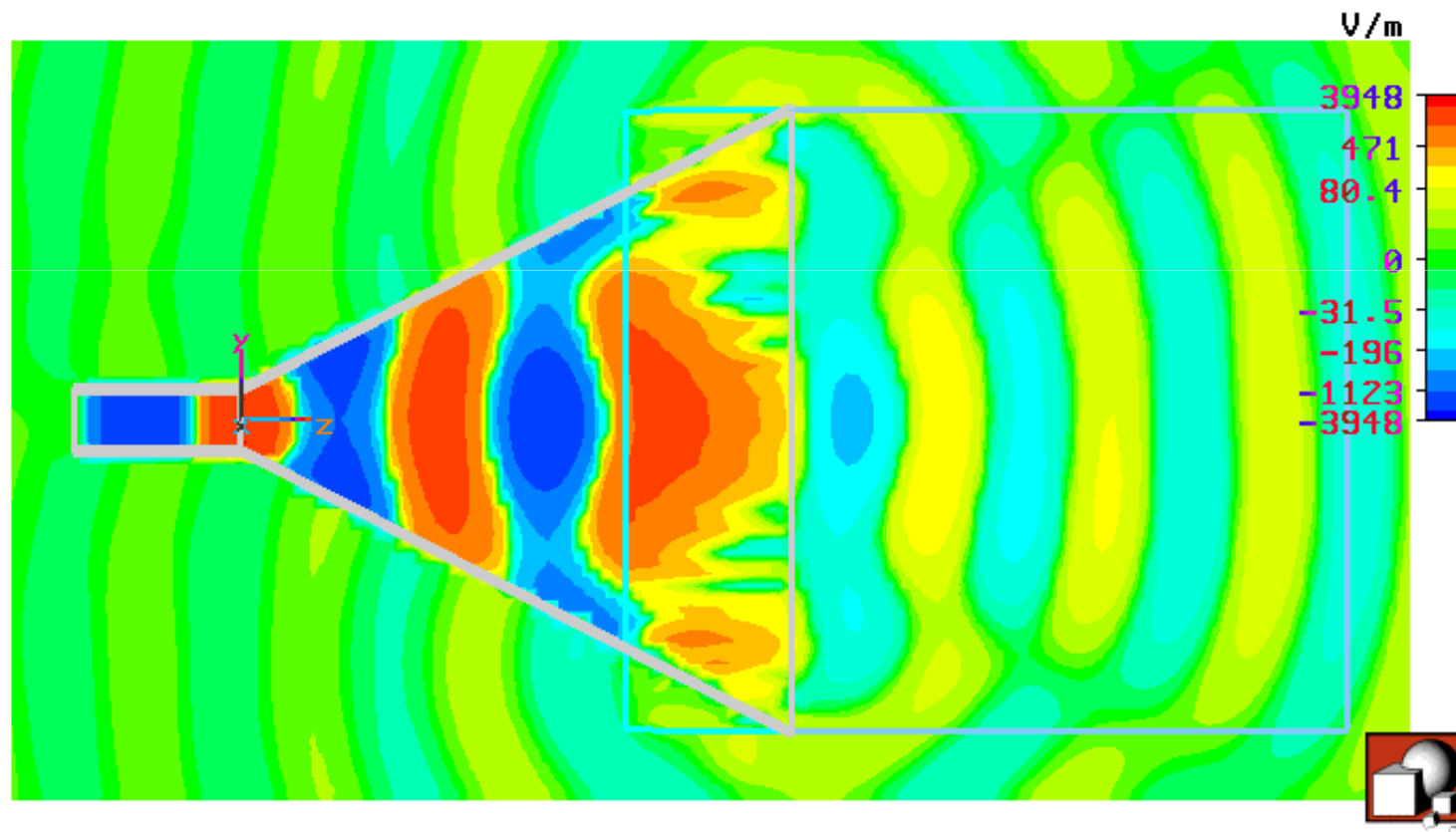
•Figures copied from the original article

# Shortened Horn with ENZ Metamaterial (Hrabar, Bonafacic, Muha, 2008)

Simulated magnitude of the y-component of electric field of the shortened horn

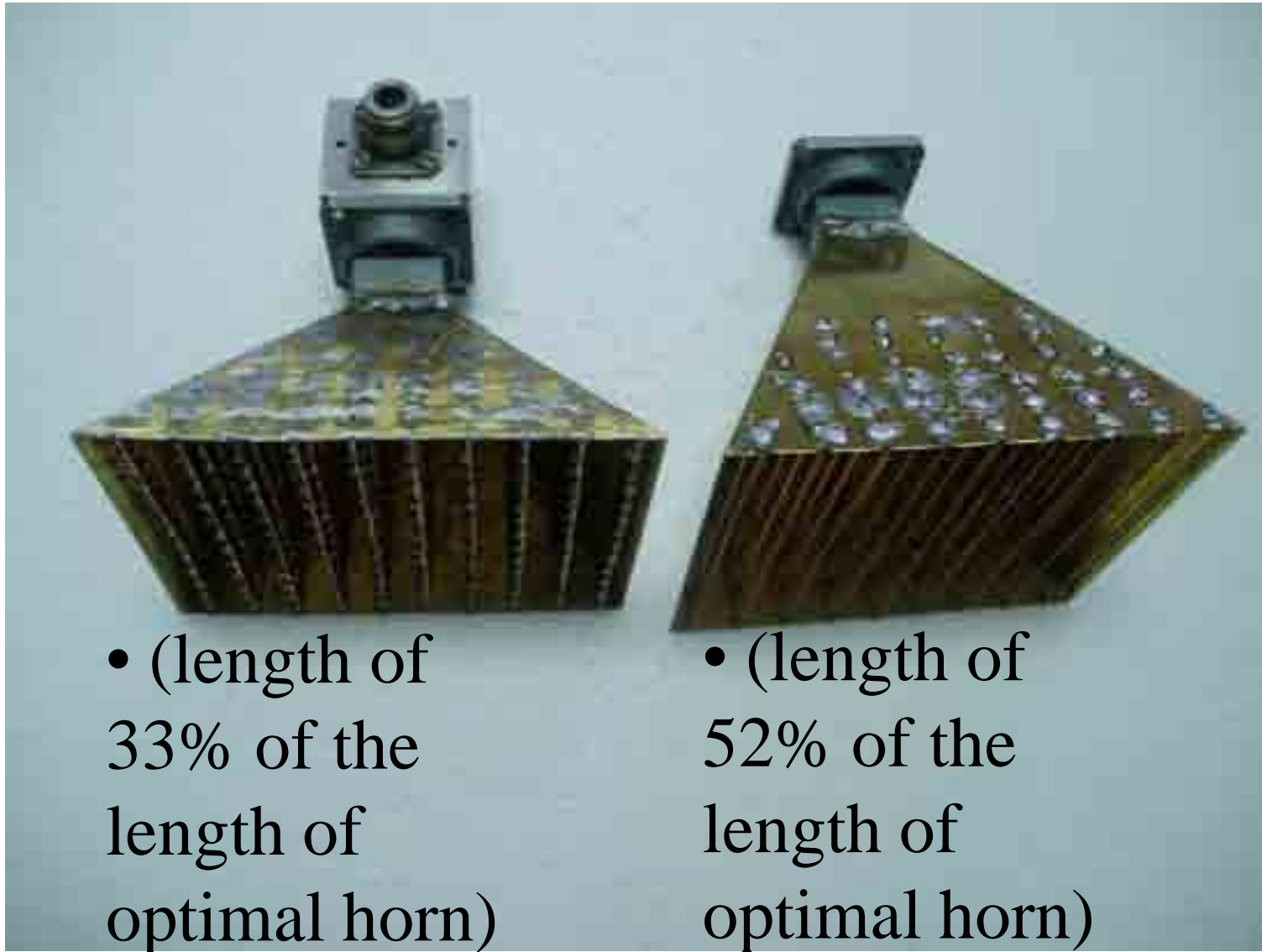


- Simulated magnitude of the y-component of electric field of the shortened horn with ENZ slab





- Comparison of prototyped shortened horns



- (length of 33% of the length of optimal horn)

- (length of 52% of the length of optimal horn)

NEWS

## Metamaterials Arrive in Cellphones

LG Chocolate BL40 is first cellphone to use a metamaterials antenna



Image: LG

BY SASWATO R. DAS //  
OCTOBER 2009

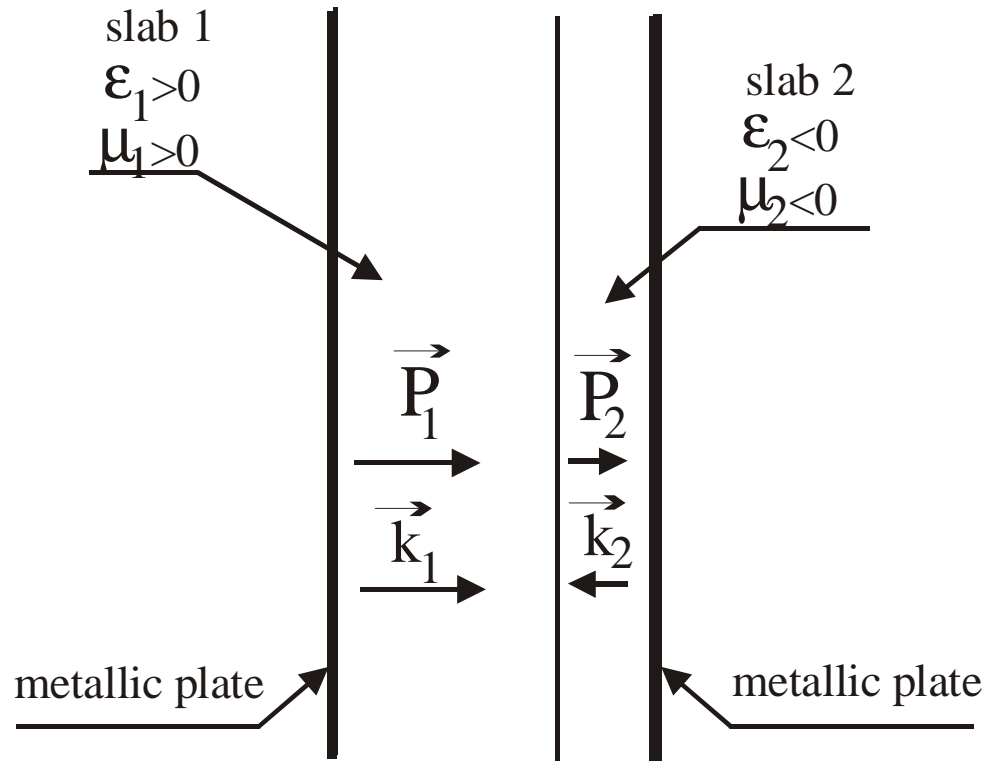
28 October 2009—The quest to build more powerful multiband mobile handsets has gotten a boost from a relatively new class of materials. Called metamaterials, they are specifically engineered to have properties that do not occur naturally, such as the ability to bend light the wrong way. For manufacturers of mobile devices, recent advances in metamaterials promise a way to shrink size while still retaining

multiband functionality.

# **Potential Engineering Applications**

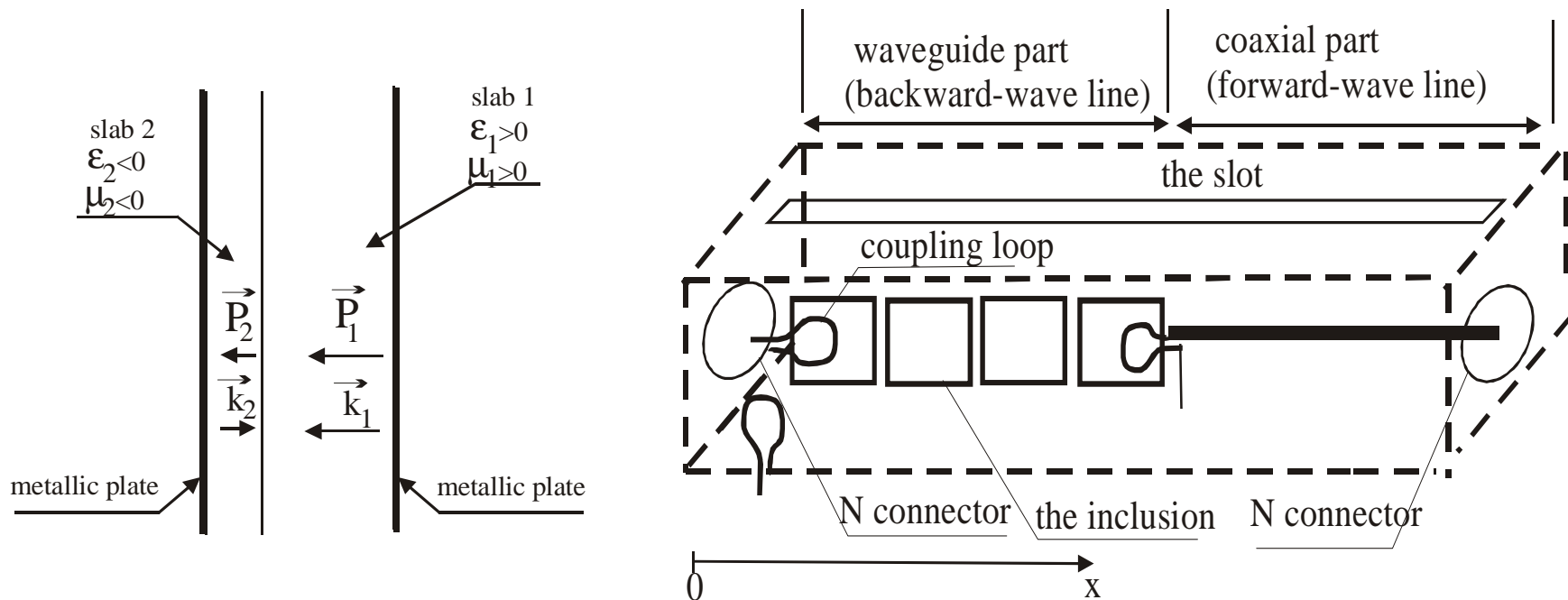
## **III – Resonators**

# Sub-wavelength Resonator (Engheta, 2001)



- The resonance frequency depends on  $d_2/d_1$  instead of  $d_2 + d_1$  which would hold in ordinary materials

# Experimental verification of Engheta's resonator (Hrabar et al., 2004)



( $f=350$  MHz)

overall length

$\sim \lambda/30$

# **Potential Engineering Applications**

## **IV – Going optical!**

# Going Optical - Approach I

## Scaling Down the Size of Inclusions

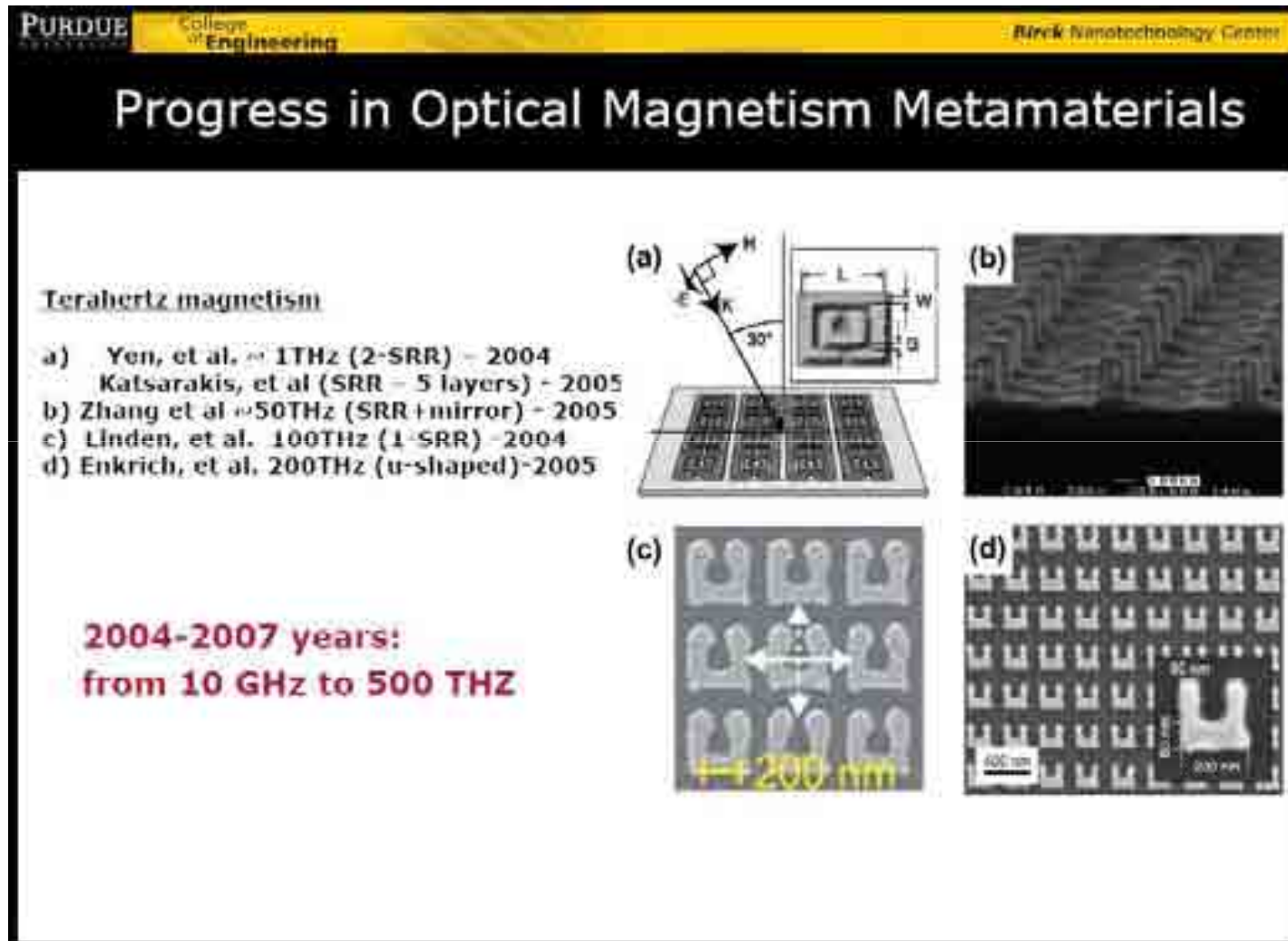


Figure taken from V. Shalaev (Purdue University )  
presentation slides



# Going Optical – Approach II

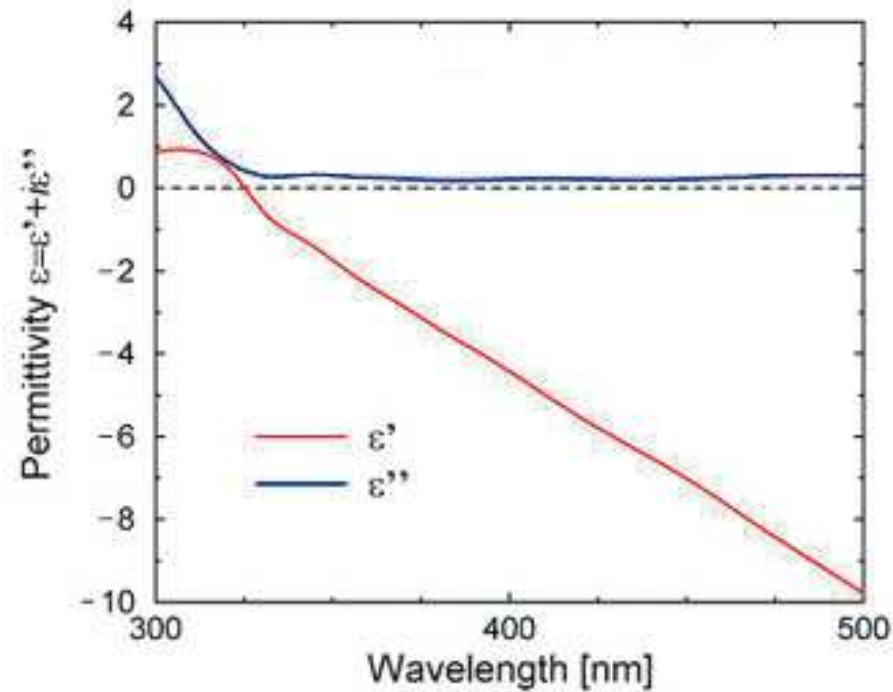
## Make Use of Surface Plasmons



The Lycurgus Cup (glass;  
British Museum; 4<sup>th</sup>  
century A. D.)



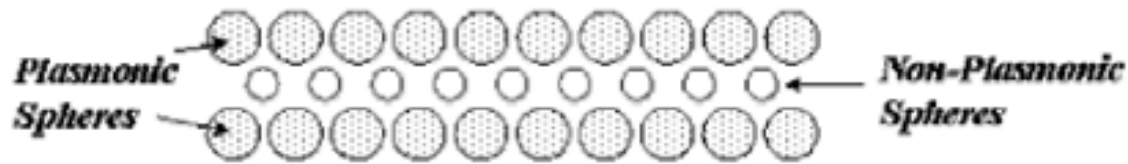
# Going Optical – A ‘Soft’ Approach Make Use of Surface Plasmons



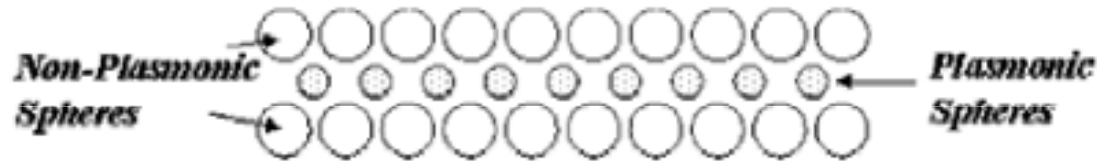
Permittivity of silver

# From Plasmonic Spheres to Nano-circuit Elements in Optical Domain (Alu, Engheta; 2005)

## Nano-transmission-lines



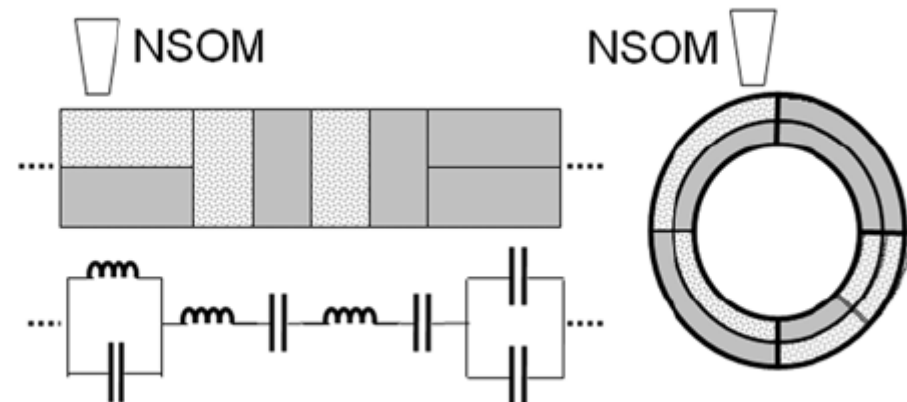
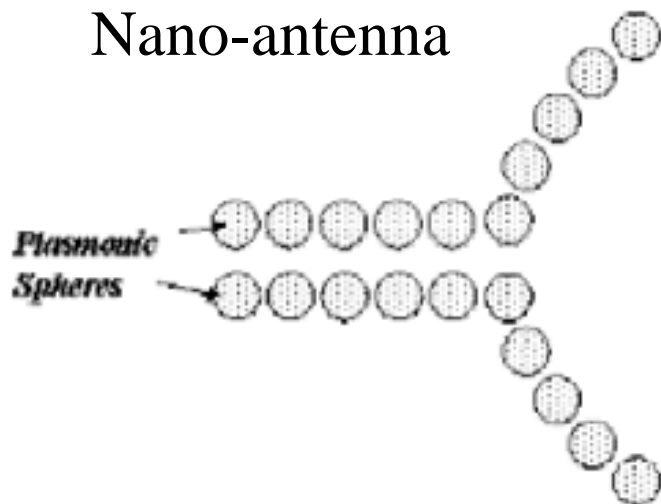
Forward-wave



Backward-wave

## Nano-circuit

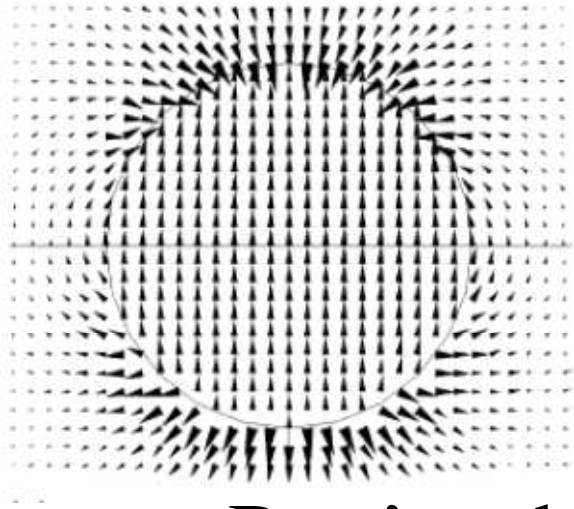
### Nano-antenna



# New Idea - Scaled RF Replicas of Plasmonic Structures

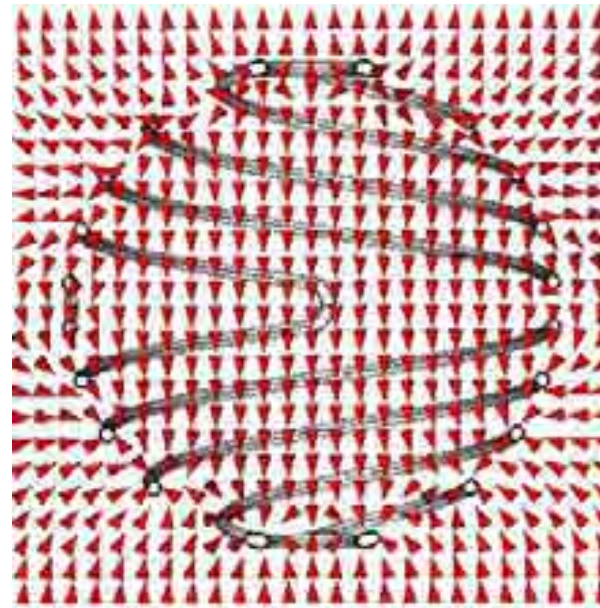
# Prototyping of 'RF replicas' of plasmonic spheres

(Hrbar, Eres; Kumric, 2007)

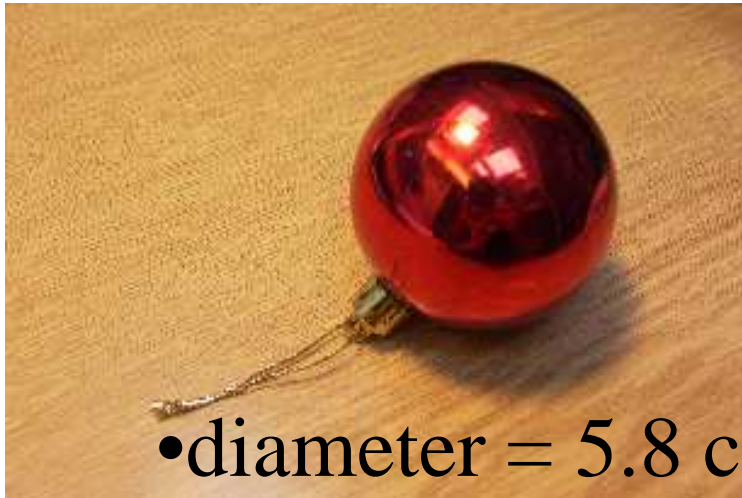


Simulated E-field  
distribution of ideal  
plasmonic nanosphere

Best's spherical resonator



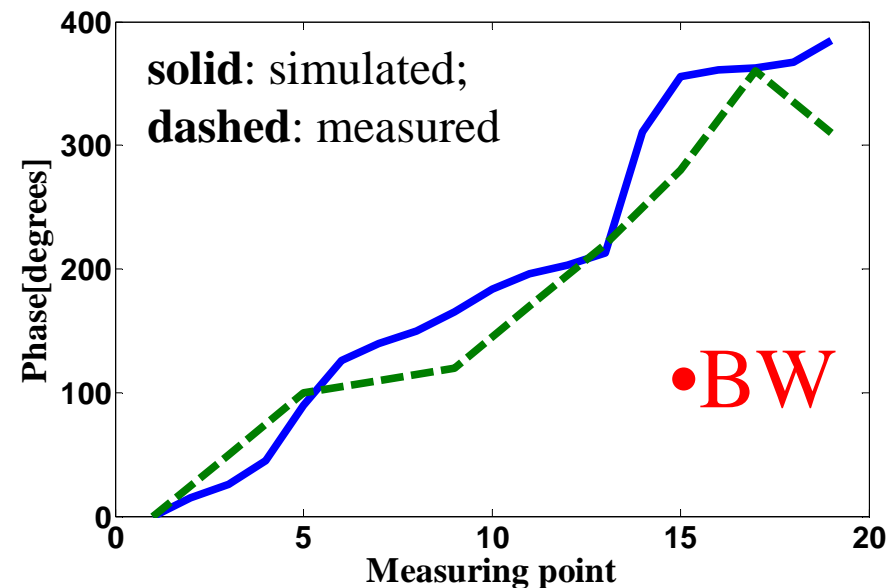
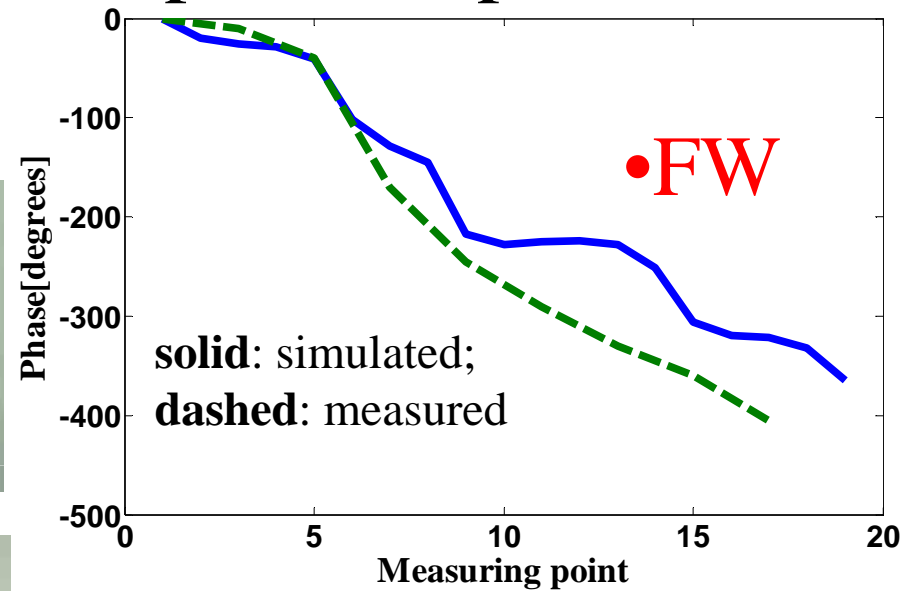
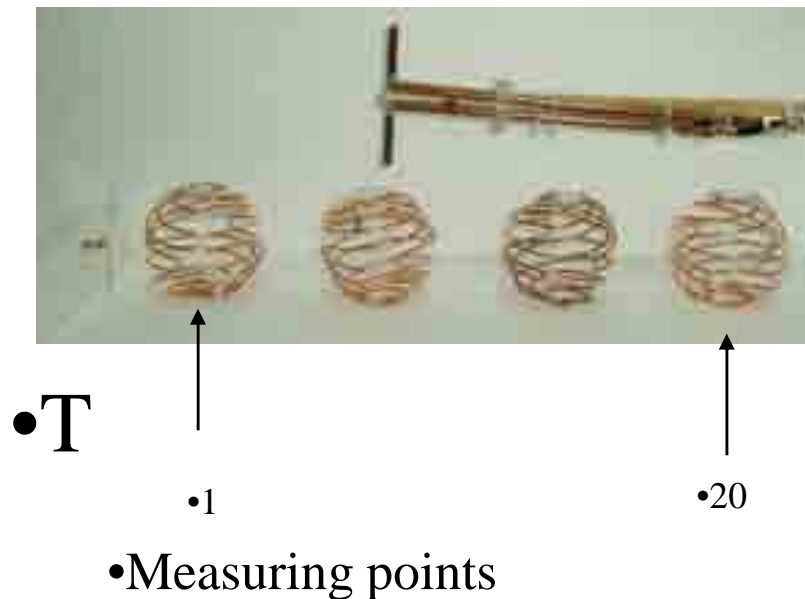
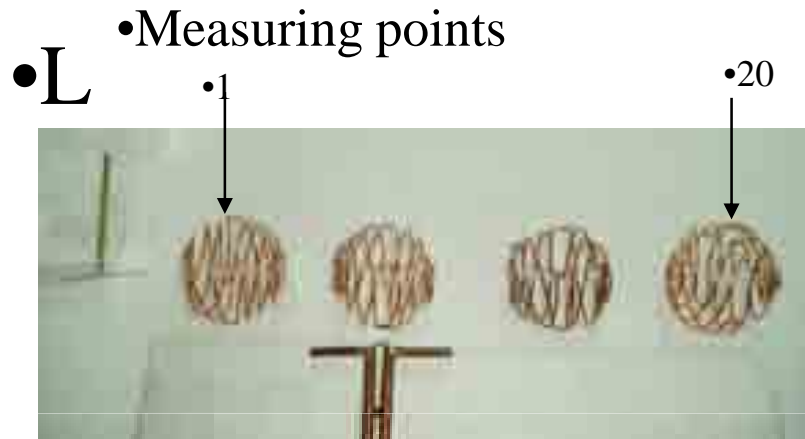
# Prototyping of “Plasmonic” spheres



•diameter = 5.8 cm

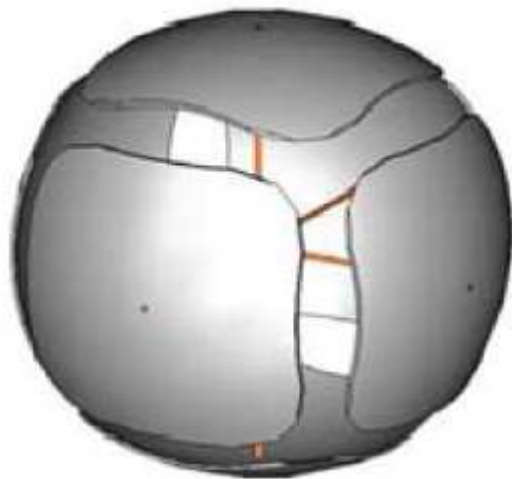
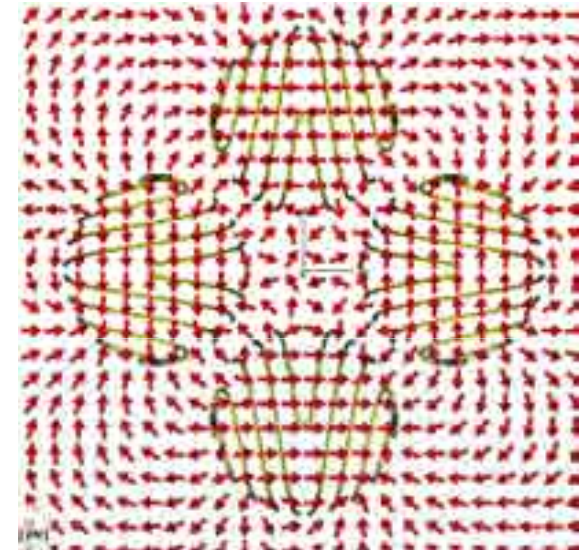
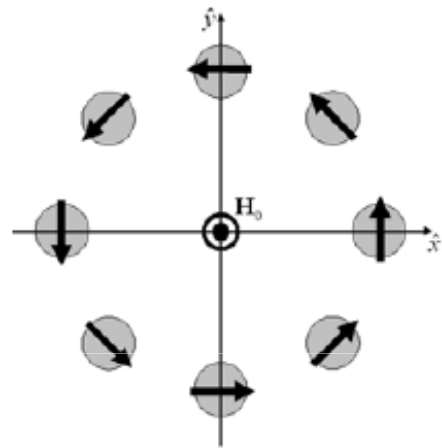


# Measurement of E-field phase distribution along four-sphere chains (RF replicas of plasmonic WG)



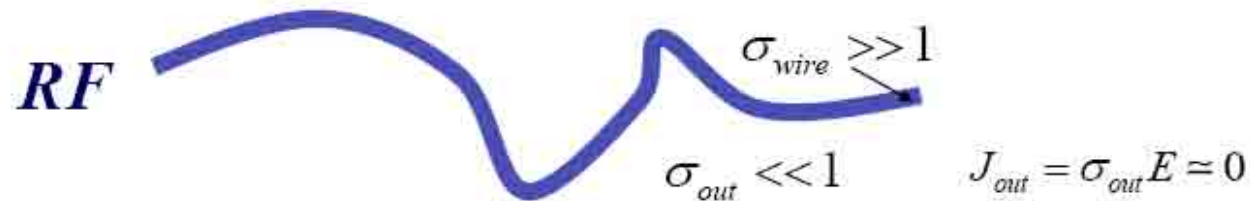


# Prototyping of 'RF analog of 'plasmonic circular cluster ' (Hrabar, Muha, Zaluski, Mlakar, 2010)



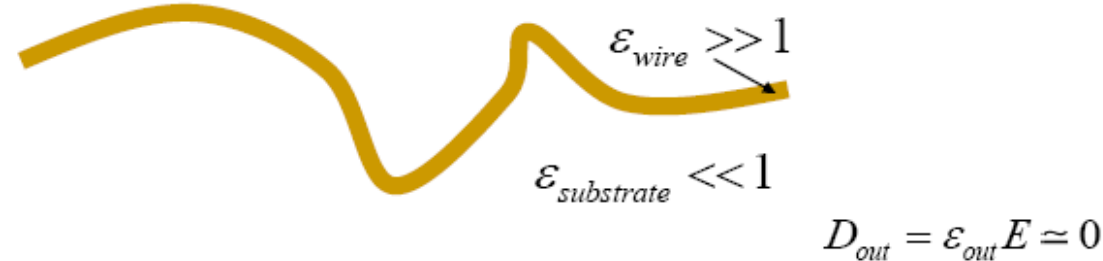


## Current Confinement?



$$D = \epsilon E$$

## Nano-Optics ?



*Alu, Engheta, Optics Express, 15, 13773 (2007)*

Figure taken from N. Engheta (University of Pennsylvania)  
presentation slides



## Grooves in ENZ

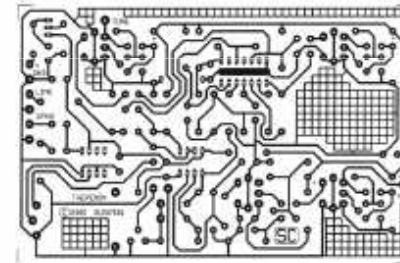
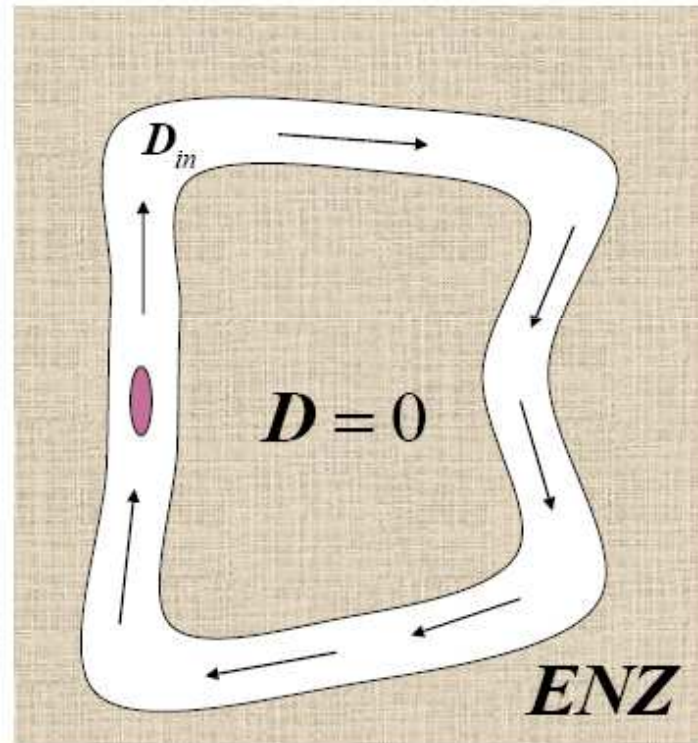


Figure taken from N. Engheta (University of Pennsylvania) presentation slides

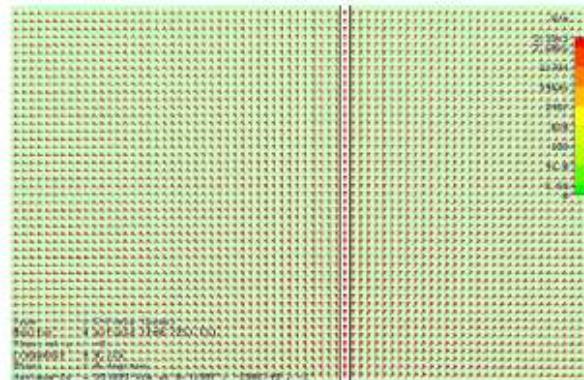
# Numerical Simulations



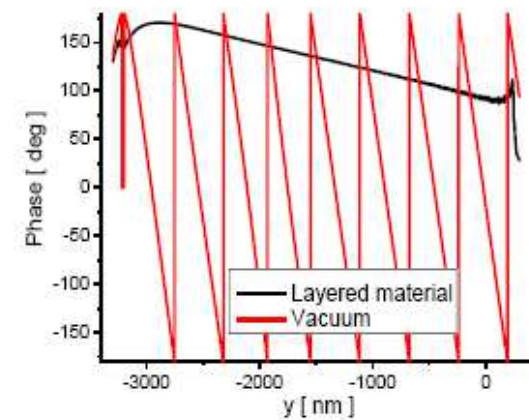
$\text{Si}_3\text{N}_4$

Silver

$f = 725 \text{ THz}$



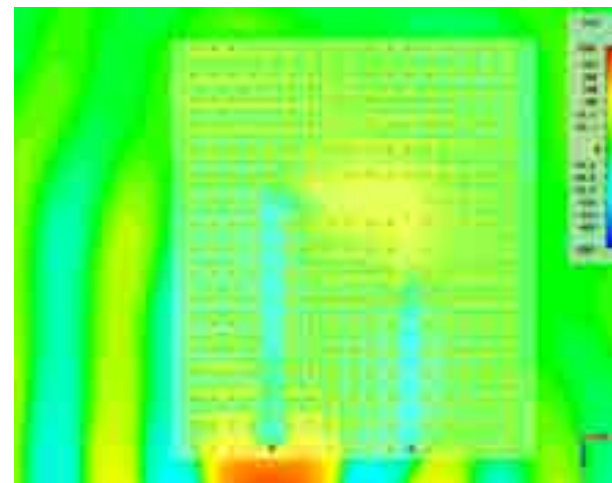
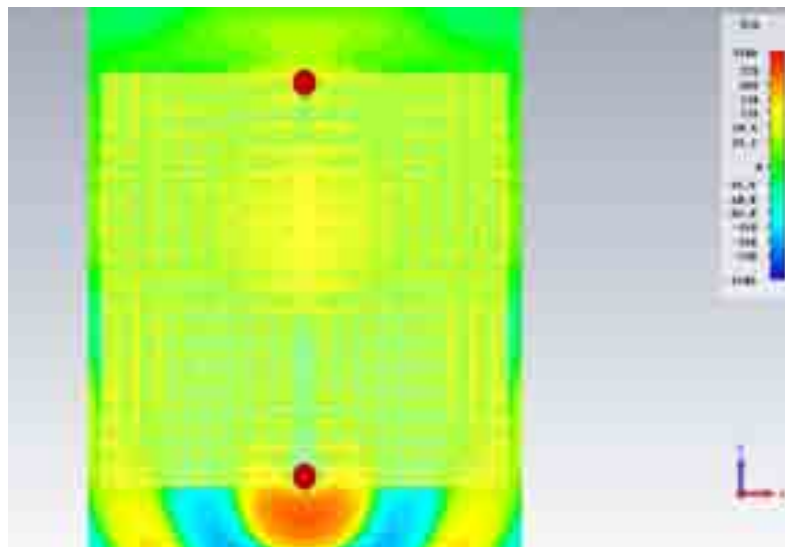
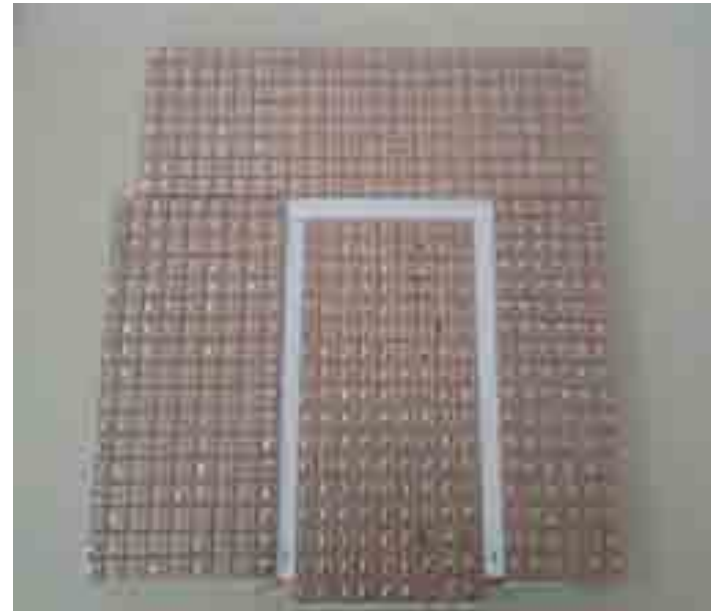
## Phase of $E_y$



Alu and Engheta, *Phys. Rev. Lett.*, 2009

Figure taken from N. Engheta (University of Pennsylvania) presentation slides

# Scaled prototype of optical D-dot wire (Muha, Hrabar et al. 2011)



# Near-field Superlens

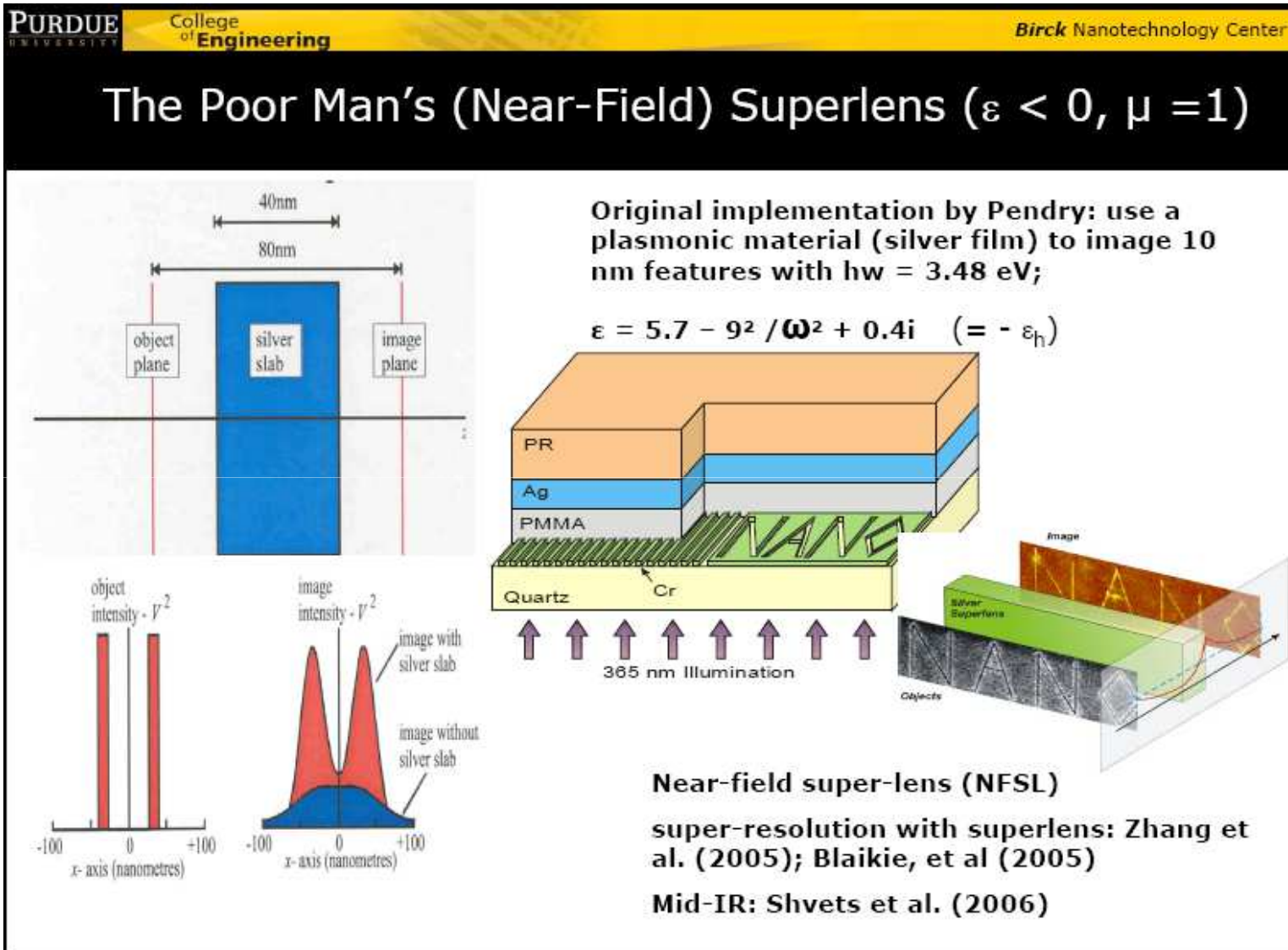


Figure taken from V. Shalaev (Purdue University )  
presentation slides



# Experimentally achieved images

N. Fang, H. Lee, C. Sun, X. Zhang, *Science* **308**, 534, (2005).

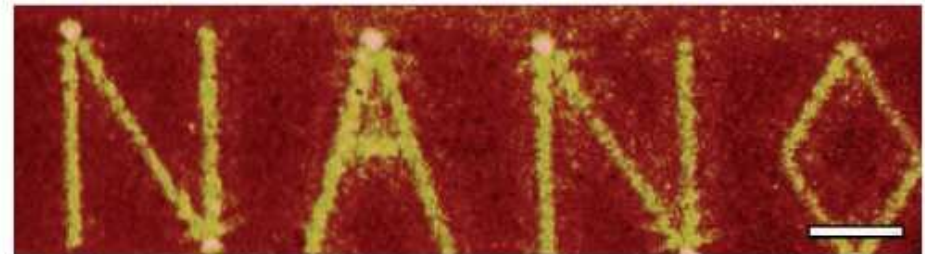
written object (**top**)



optical image (**center**)



optical image without  
super- lens



# **Potential Engineering Applications**

## **V – Is it feasible to be invisible?**

# Invisibility: An Ancient Dream

*Perseus' helmet  
(Greek mythology)*



*Tarnhelm of invisibility  
(Norse mythology)*



*Cloaking devices  
(Star Trek, USA)*



*Ring of Gyges  
("The Republic", Plato)*



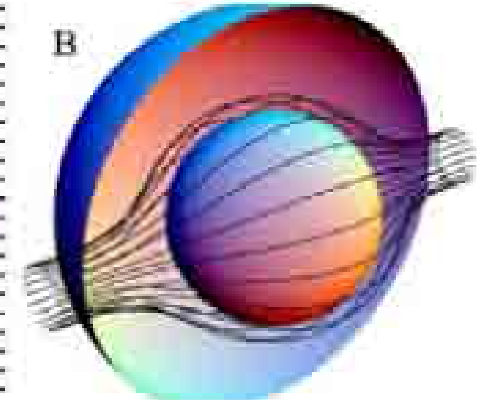
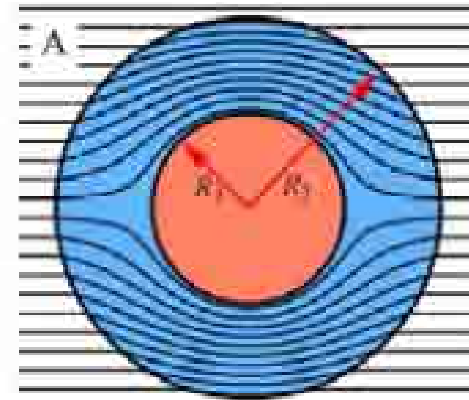
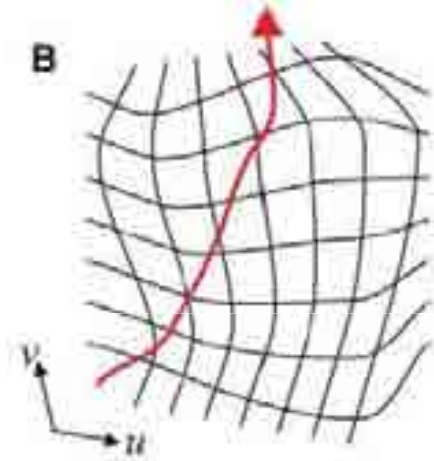
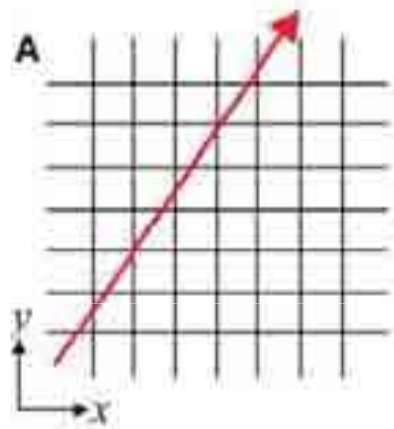
*The 12 Dancing Princesses  
(Brothers Grimm, Germany)*



*Harry Potter's cloak  
(J. K. Rowling, UK)*

Figure taken from V. Shalaev (Purdue University )  
presentation slides

# 'Transformation Electromagnetics' (Dollin 1961, Pendry 2006)

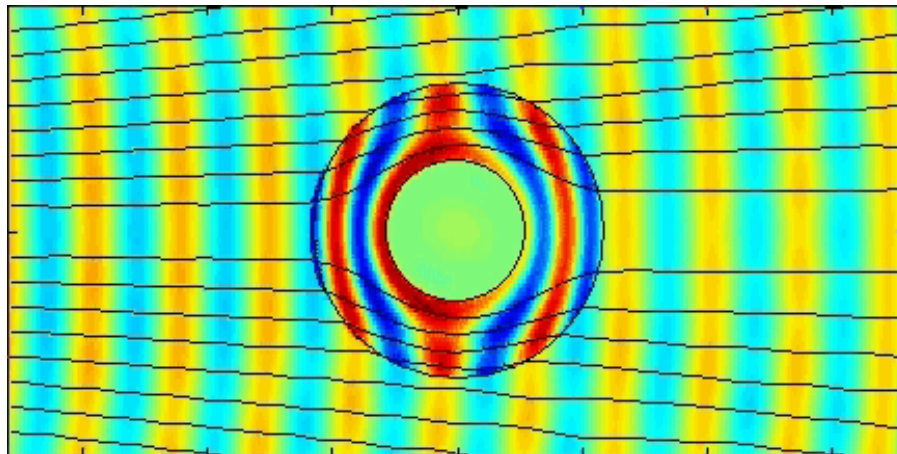
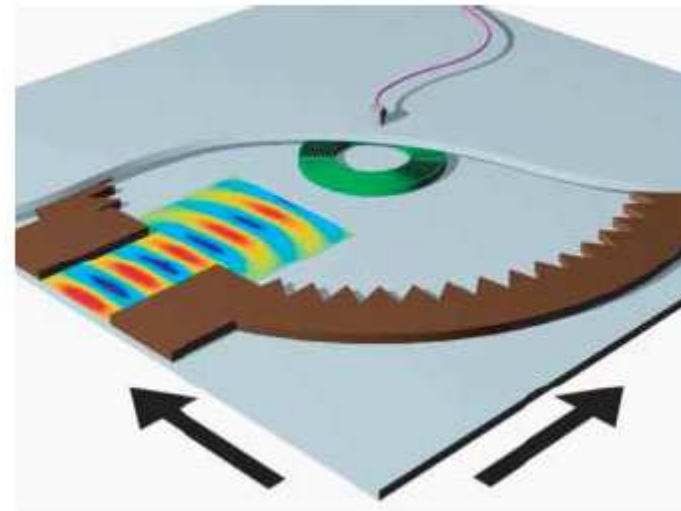
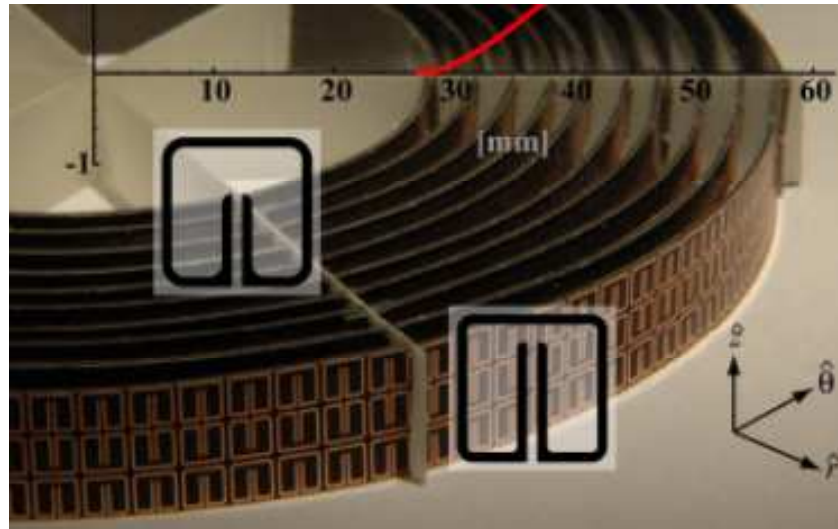


- Experimental verification already reported

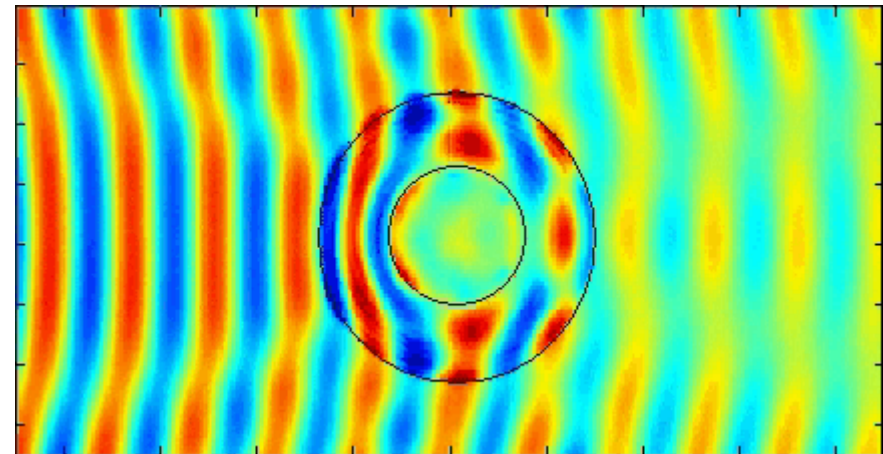
[www.sciencexpress.org](http://www.sciencexpress.org) / 25 May 2006



# First Experimental Results (Schurig 2006)



**Simulations**

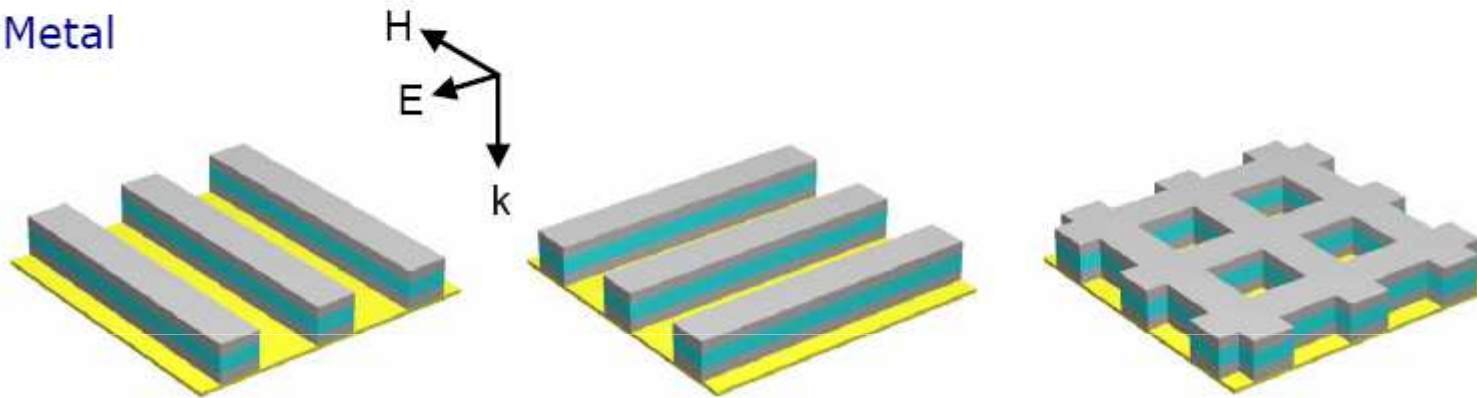


**Measurements**

## Negative permeability and negative permittivity

■ Dielectric

□ Metal



Nanostrip pair (TM)

$\mu < 0$  (resonant)

Nanostrip pair (TE)

$\epsilon < 0$  (non-resonant)

Fishnet

$\epsilon$  and  $\mu < 0$

Figure taken from V. Shalaev (Purdue University) presentation slides *S. Zhang, et al., PRL (2005)*

Problem 1 All these structures are again extremely narrowband!

## Negative Refractive Index in Optics: State of the Art

<i>Year and Research group</i>	<i>1st time posted and publication</i>	<i>Refractive index, <math>n'</math></i>	<i>Wavelength <math>\lambda</math></i>	<i>Figure of Merit <math>F= n /n''</math></i>	<i>Structure used</i>
<b>2005:</b>					
<i>Purdue</i>	April 13 (2005) arXiv:physics/0504091 Opt. Lett. (2005)	-0.3	1.5 $\mu\text{m}$	0.1	Paired nanorods
<i>UNM &amp; Columbia</i>	April 28 (2005) arXiv:physics/0504208 Phys. Rev. Lett. (2005)	-2	2.0 $\mu\text{m}$	0.5	Nano-fishnet with round voids
<b>2006:</b>					
<i>UNM &amp; Columbia</i>	J. of OSA B (2006)	-4	1.8 $\mu\text{m}$	2.0	Nano-fishnet with round voids
<i>Karlsruhe &amp; ISU</i>	OL (2006) OL (2007)	-1 -1	1.4 $\mu\text{m}$ 1.4 $\mu\text{m}$	3.0 2.5	Nano-fishnet 3-layer nanofishnet
<i>Karlsruhe &amp; ISU</i>	OL (2006)	-0.6	780 nm	0.5	Nano-fishnet
<i>Purdue</i>	OL (2007)	-0.9 -1.1	770 nm 810nm	0.7 1.3	Nano-fishnet

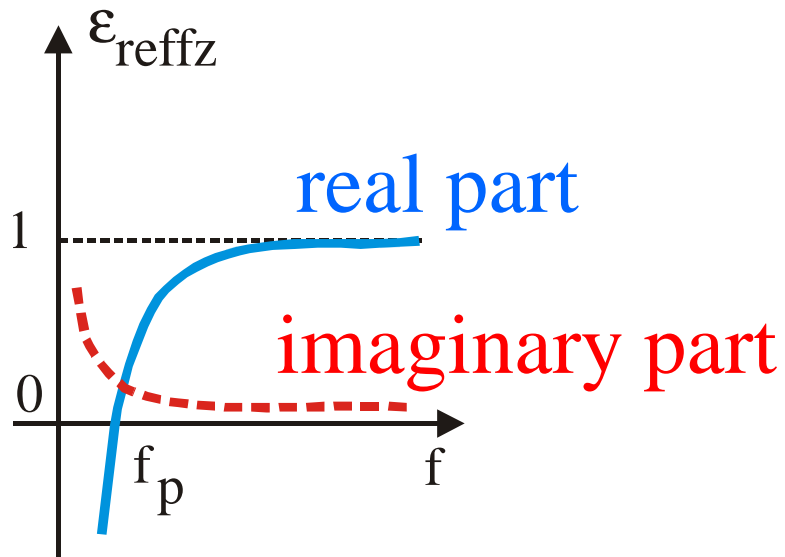
see review: Nature Photonics v. 1, 41 (2007)

Problem 2 All these structures have significant loss !

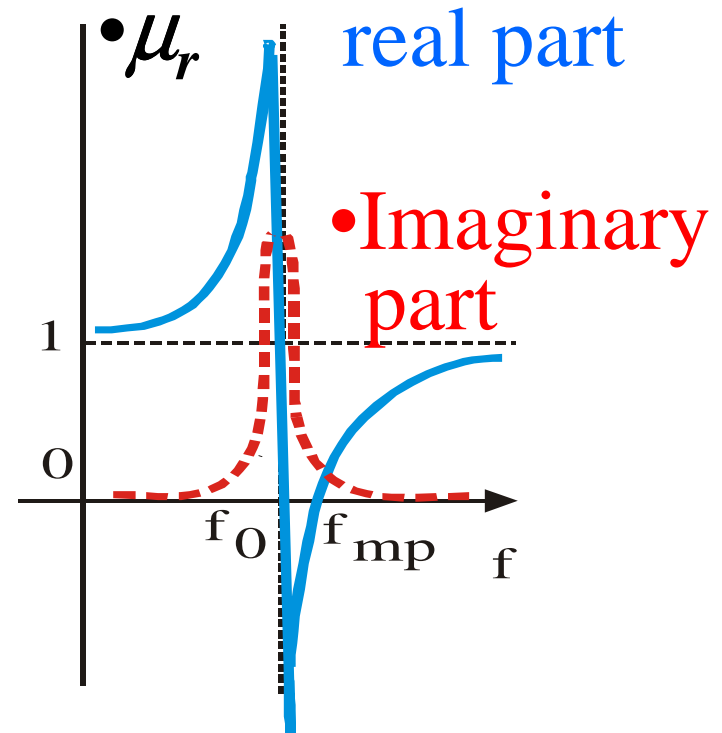
Figure taken from V. Shalaev (Purdue University ) presentation slides

# Dispersion models of passive materials (metamaterials) with $\epsilon_r < 1$ or $\mu_r < 1$

## • Drude model

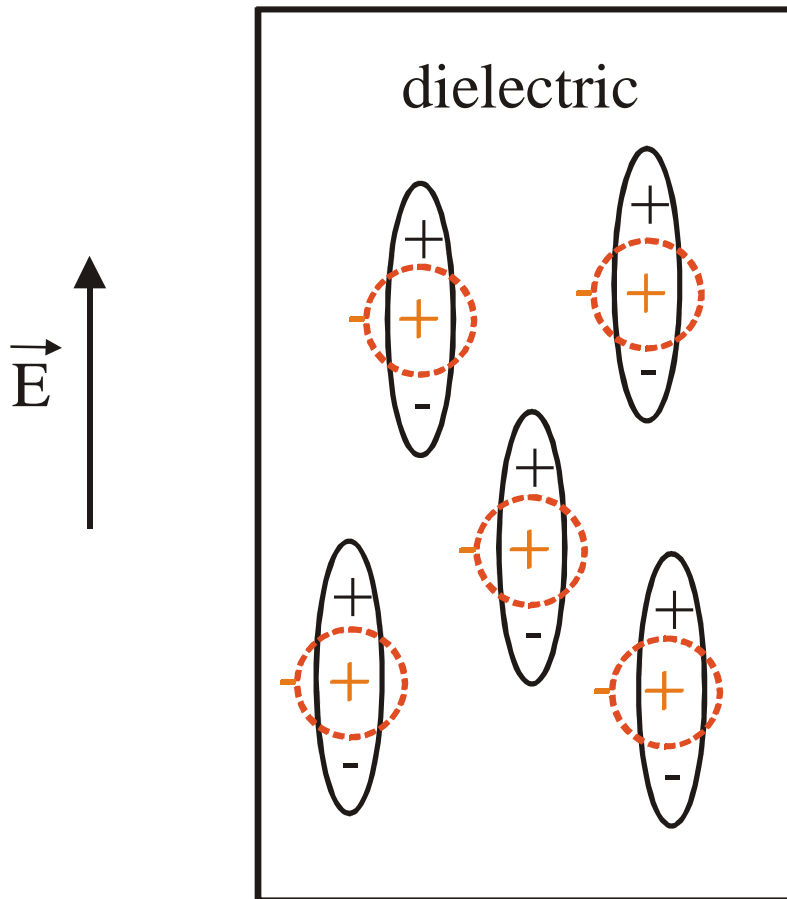


## • Lorentz model

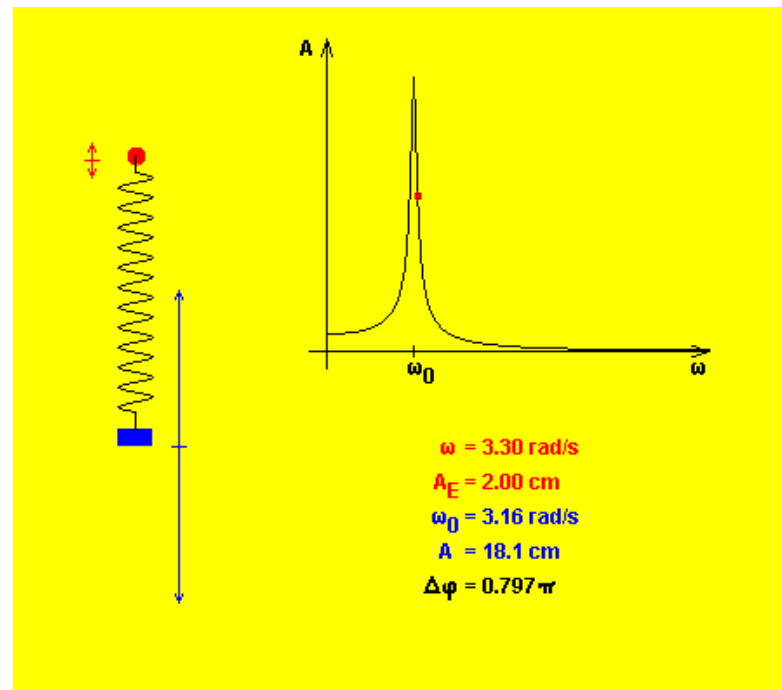
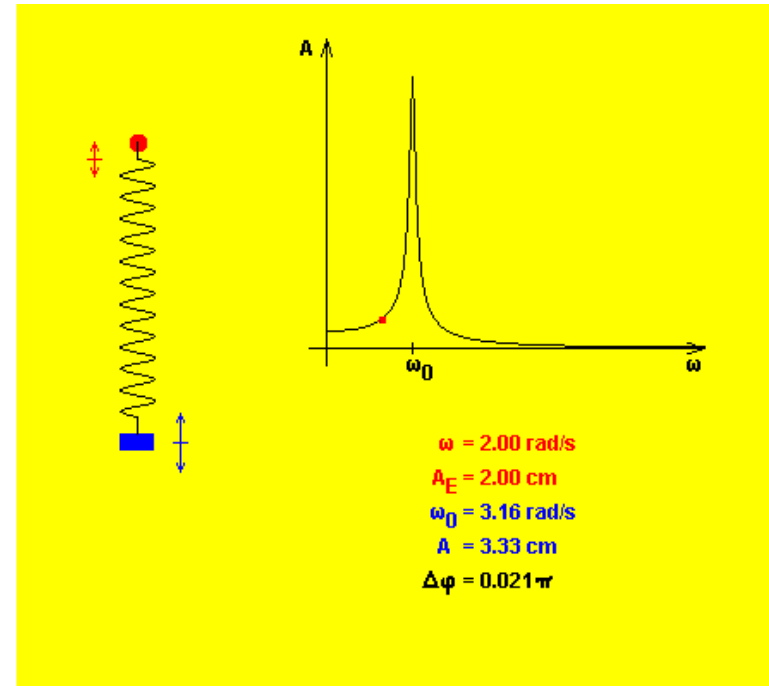


Both of these models are highly dispersive for  $0 < \epsilon_r < 1$  (or  $0 < \mu_r < 1$ )

# Resonance is always present in passive metamaterials – WHY? )



•URL: <http://www.walter-fendt.de/ph14e/resonance.htm>  
 © Walter Fendt, September 9, 1998



# Can one go around the dispersion-energy limitations?

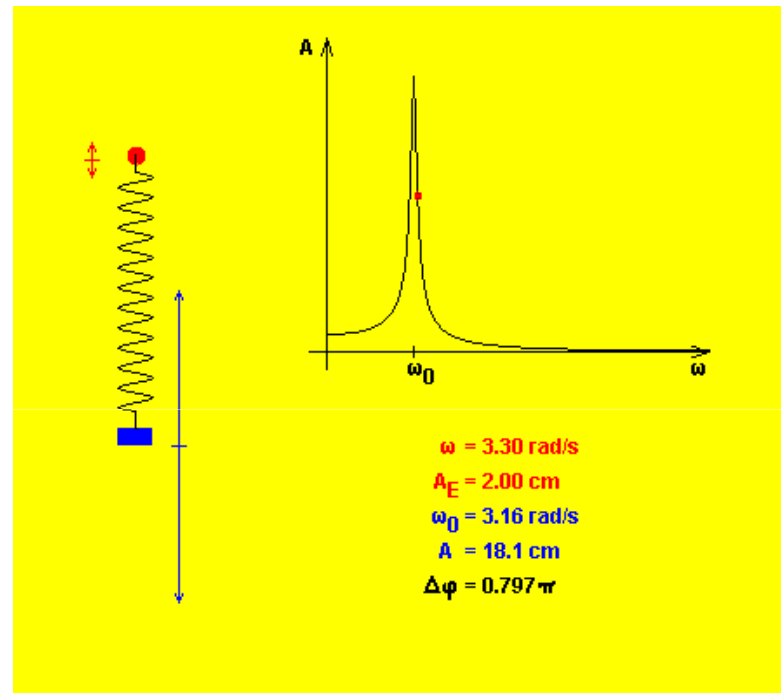
$$W = W_e + W_m = \frac{1}{2} \epsilon_0 \frac{\partial(\omega \epsilon_r)}{\partial \omega} E^2 + \frac{1}{2} \mu_0 \frac{\partial(\omega \mu_r)}{\partial \omega} H^2$$

$$\frac{\partial(\omega \epsilon_r)}{\partial \omega} > 0, \frac{\partial(\omega \mu_r)}{\partial \omega} > 0.$$

$W < 0, ???$

The only way is to use active medium (i.e. to introduce active elements) !

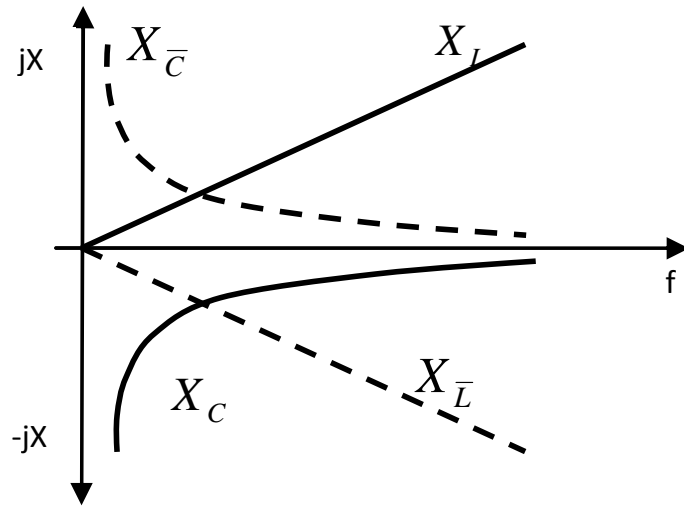
# Active medium should introduce ‘assisting negative restoring force’ !



$F_{as}$

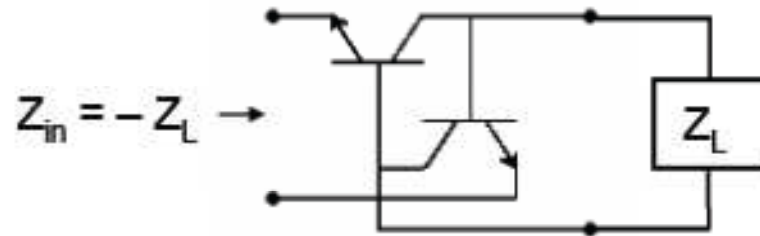
• One would need a ‘negative spring’ or a ‘negative mass’

# •Going Active - Non-Foster reactive elements (negative C and negative L)



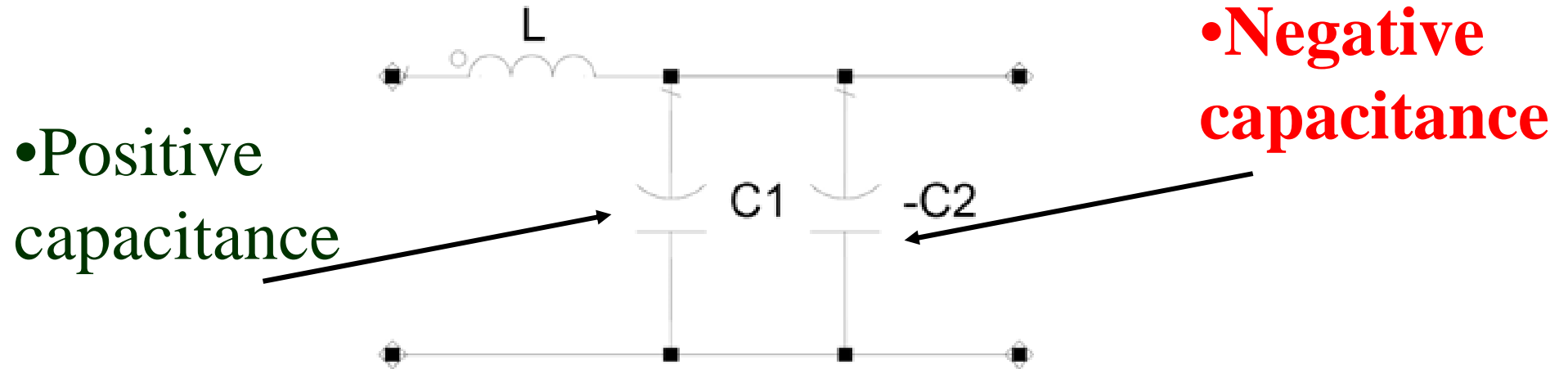
Floating negative impedance  
(Linvill, 1953)

$$Z_{in} = \frac{V_{in}}{I_{in}} = \frac{-V_l}{I_l} = -Z_l$$





# New idea : Introduction of negative capacitance into TL (Hrabar et al , APS 2008)

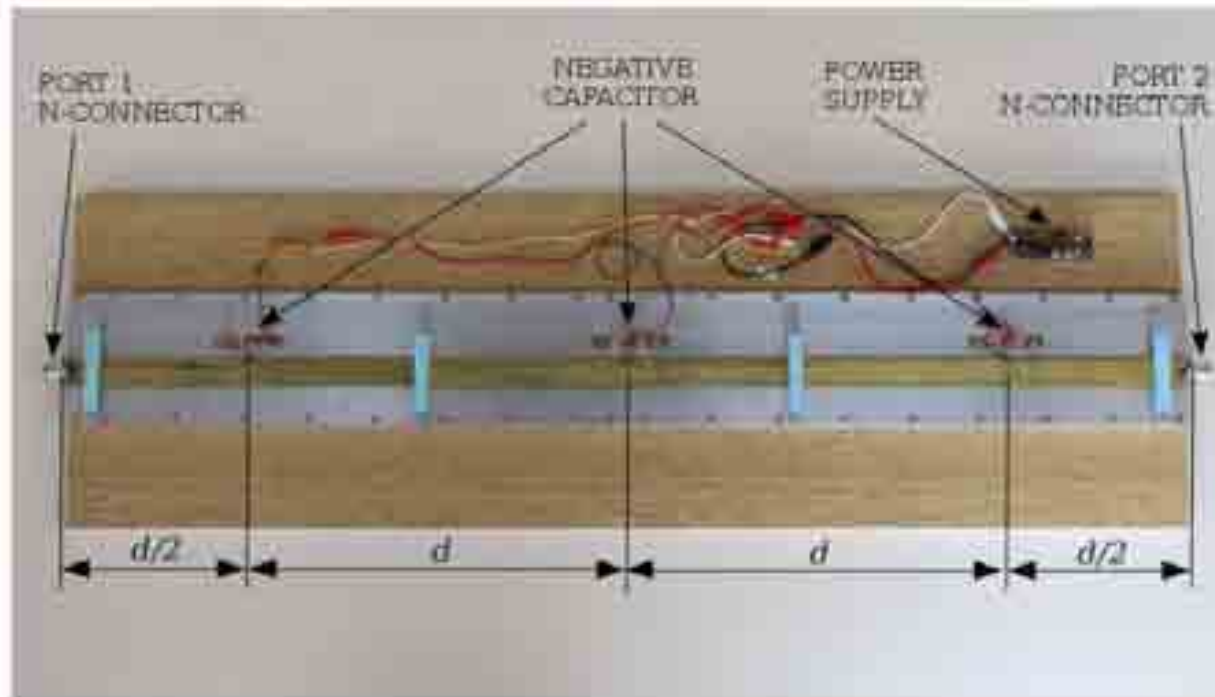
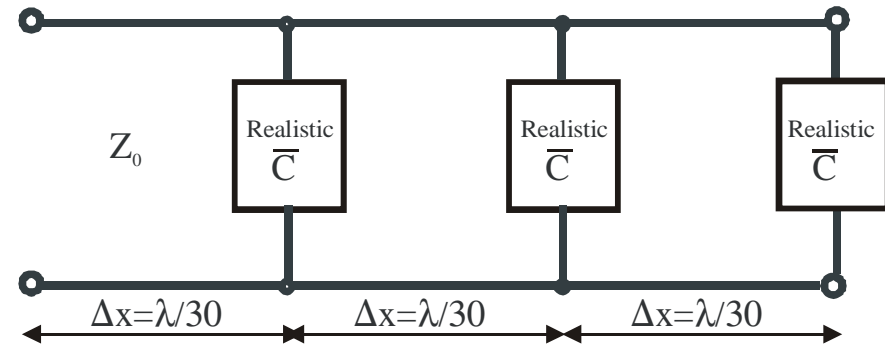
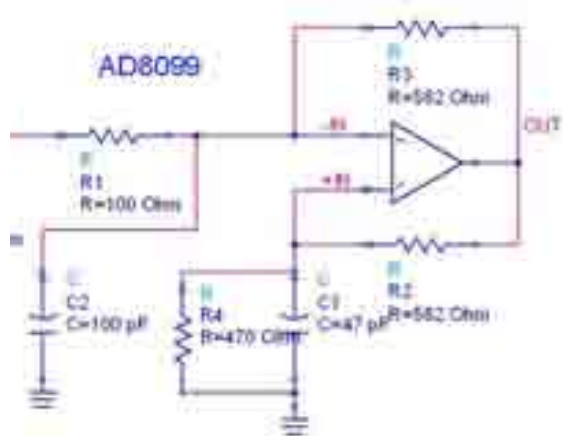


$$C = C_1 + C_2$$

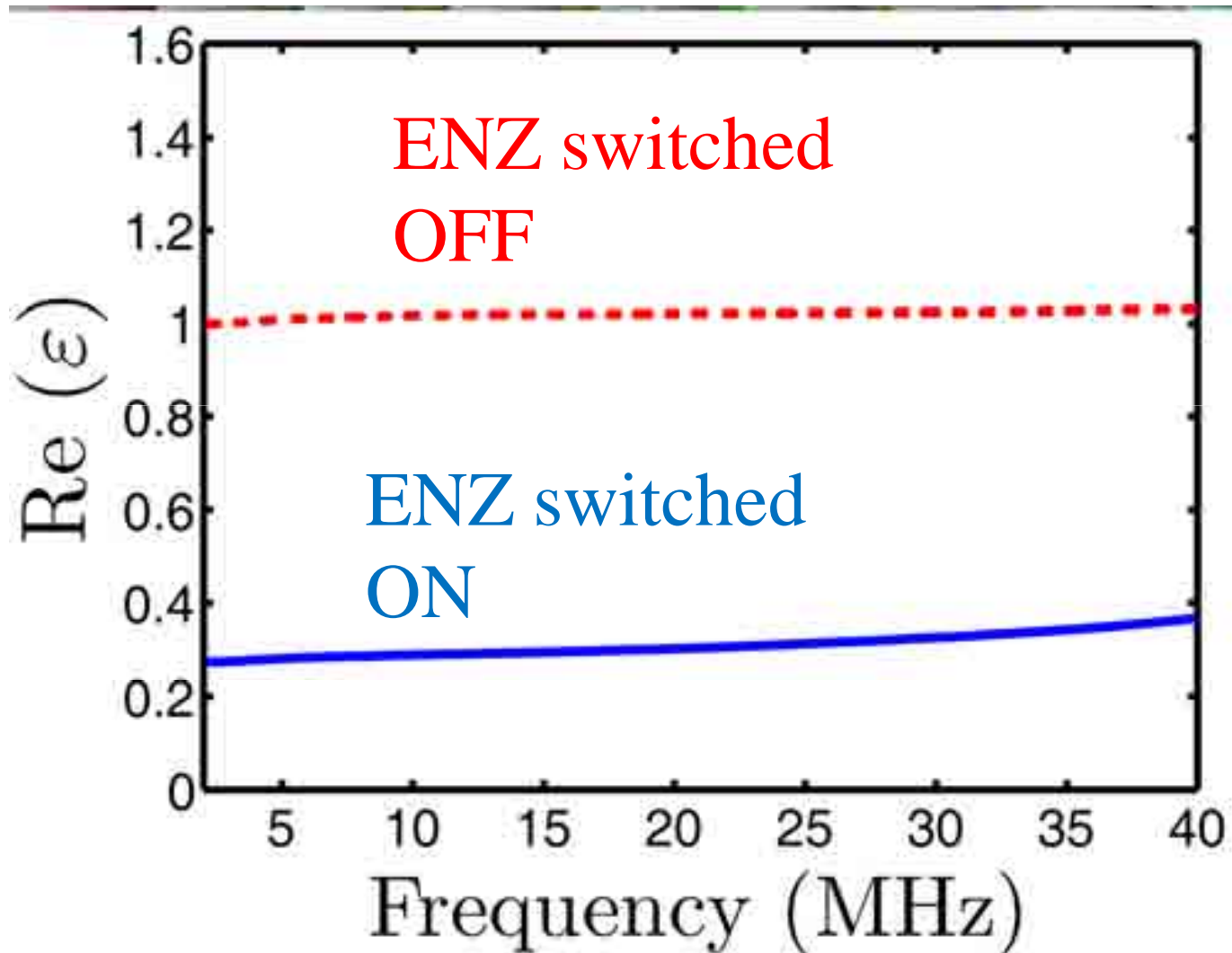
$$C_e = C_+ + C_- = |C_+| - |C_-|$$

$$\epsilon_e = \epsilon_0 + \epsilon_- = \epsilon_0 - |\epsilon_-| \Rightarrow 0 < \epsilon < 1$$

- Experimental RF Active ENZ TL with Three Unit Cells ( $l = 1 \text{ m}$ ), (Hrabar et.al 2011)



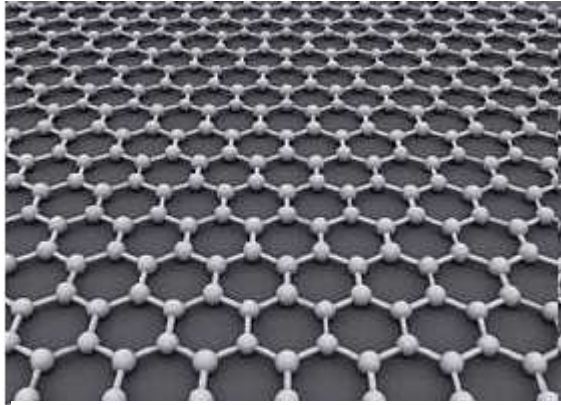
• **Measurement of effective permittivity of ENZ Active TL with Three Unit Cells (Hrabar et.al 2011)**



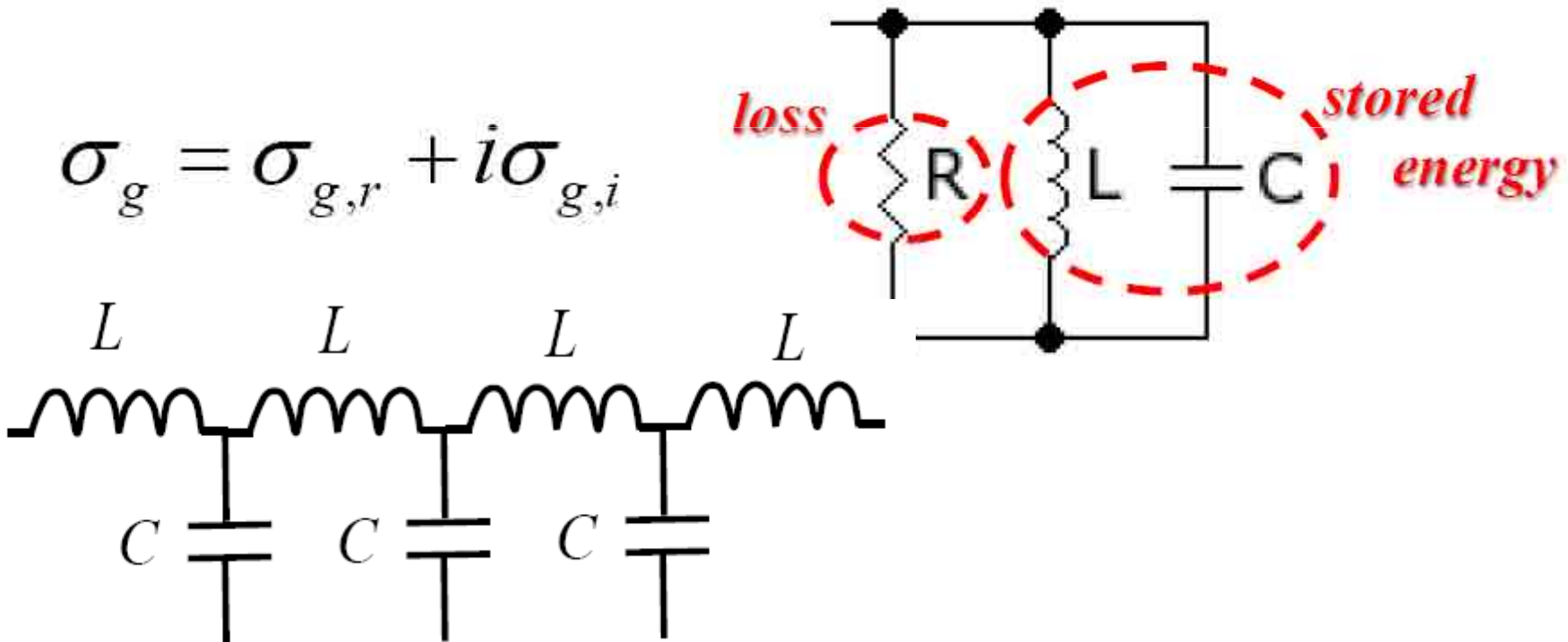
# **Potential Engineering Applications**

## **VI – Going nano?**

# Graphene-based One-atom-thick Metamaterials

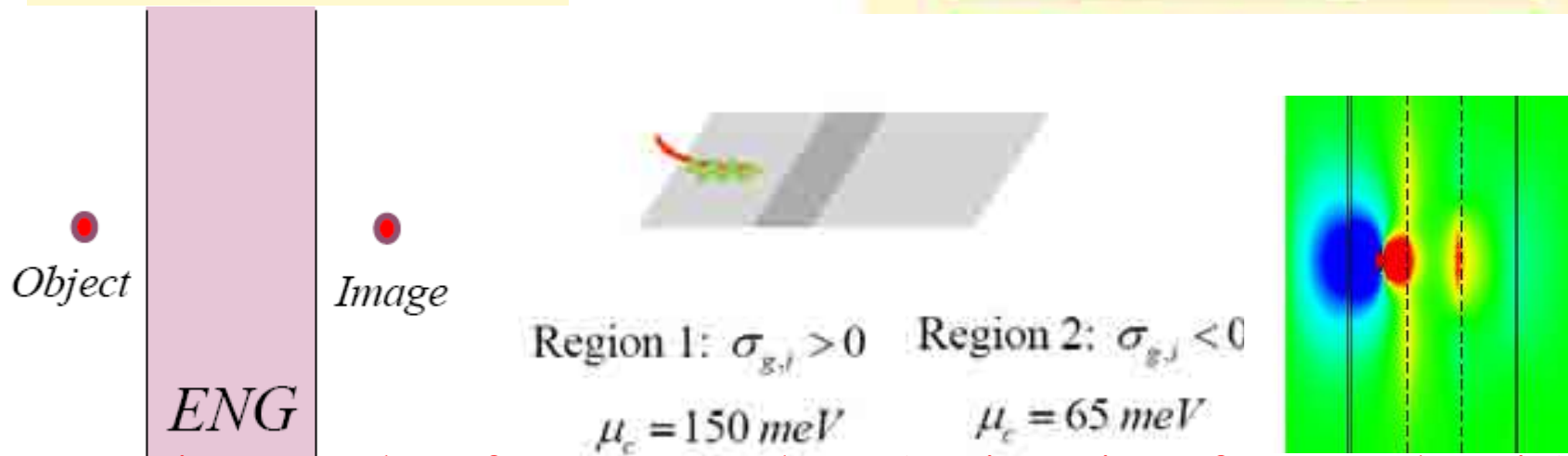
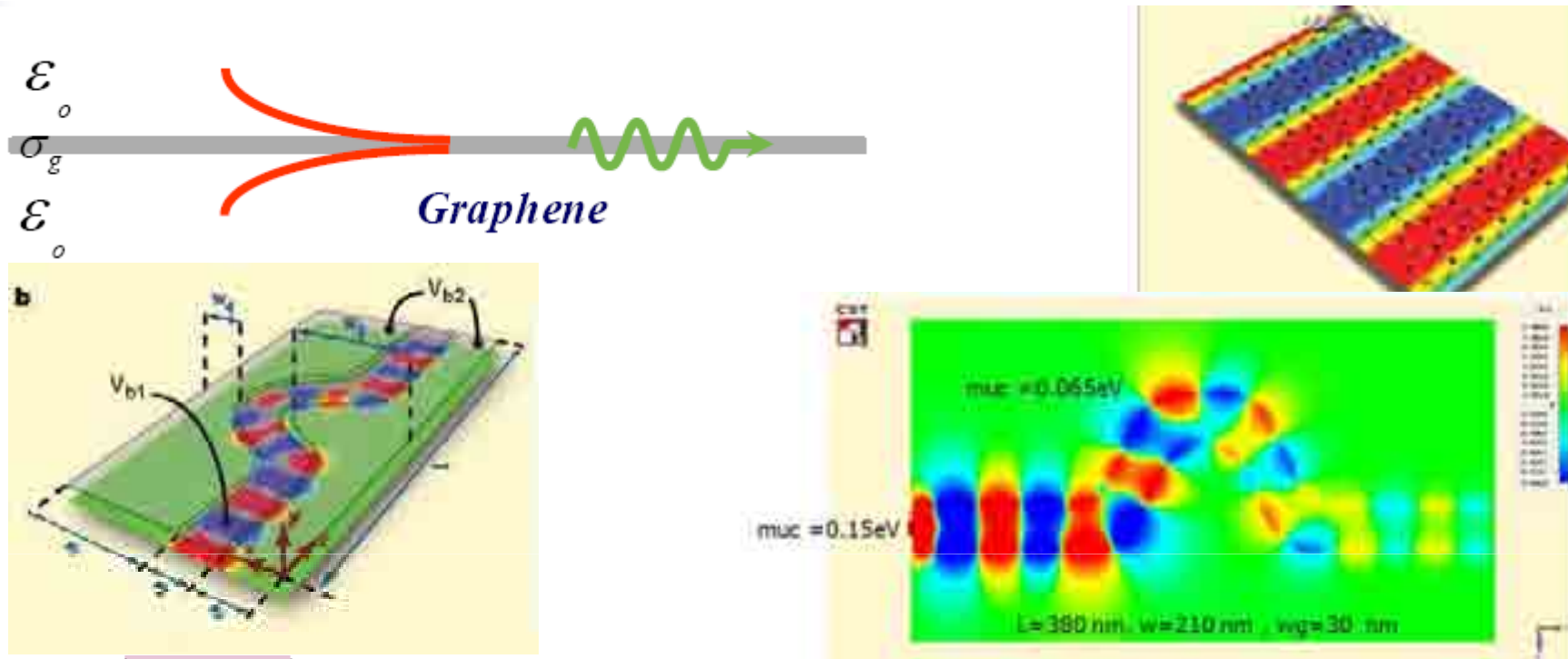


2010 Nobel Prize !



Figures taken from N. Engheta (University of Pennsylvania)  
presentation slides

# Graphene-based Metamaterials



Figures taken from N. Engheta (University of Pennsylvania) presentation slides

# What's next?

- Nanotechnology + metamaterials (plasmonics, photonics, lasing, gain materials, superluminal materials, graphene, nano-spheres and nano-films)
- Gravity, matter, acoustics + metamaterials (black hole metamaterials, seismic metamaterials, acoustic lenses)

# Conclusions

- Metamaterials offer new exciting physical phenomena, which are very often counter-intuitive !
- In author's opinion, this area is not in its infant phase any more and applications, that make use of new intriguing phenomena are expected to come in coming years!
- It is an interdisciplinary field which needs extensive collaboration of different worlds (math, physics, engineering, at least!)